COMPOUND ANGLE

1. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals :

(1) $13 - 4\cos^6\theta$

(2) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

(3) $13 - 4\cos^2\theta + 6\cos^4\theta$

(4) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

2. The value of $\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\cdot .....\cdot\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$ is :

(1) $\frac{1}{256}$ (2) $\frac{1}{2}$

(3) $\frac{1}{512}$ (4) $\frac{1}{1024}$

3. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for k = 1, 2, 3, ... Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to :-

(1) $\frac{5}{12}$ (2) $\frac{-1}{12}$ (3) $\frac{1}{4}$ (4) $\frac{1}{12}$

4. The maximum value of $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is :

(1) $\sqrt{19}$ (2) $\frac{\sqrt{79}}{2}$ (3) $\sqrt{31}$ (4) $\sqrt{34}$

5. If $\cos(\alpha+\beta) = \frac{3}{5}, \sin(\alpha-\beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to :

(1) $\frac{21}{16}$ (2) $\frac{63}{52}$

(3) $\frac{33}{52}$ (4) $\frac{63}{16}$

6. The value of $\cos^2 10° - \cos 10°\cos 50° + \cos^2 50°$ is

(1) $\frac{3}{2}(1+\cos 20°)$

(2) $\frac{3}{4}$

(3) $\frac{3}{4} + \cos 20°$

(4) $\frac{3}{2}$

7. The value of sin 10º sin30º sin50º sin70º is :-

(1) $\frac{1}{36}$ (2) $\frac{1}{32}$

(3) $\frac{1}{18}$ (4) $\frac{1}{16}$

SOLUTION

1. Ans. (1)

We have,

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$

$= 3(1 - \sin 2\theta)^2 + 6(1 + \sin 2\theta) + 4\sin^6\theta$

$= 3(1 - 2\sin 2\theta + \sin^2 2\theta) + 6 + 6\sin 2\theta + 4\sin^6\theta$

$= 9 + 12\sin^2\theta \cdot \cos^2\theta + 4(1 - \cos^2\theta)^3$

$= 13 - 4\cos^6\theta$

2. Ans. (3)

$$2\sin\frac{\pi}{2^{10}}\cos\frac{\pi}{2^{10}}\ldots\ldots\cos\frac{\pi}{2^2}$$

$$\frac{1}{2^9}\sin\frac{\pi}{2} = \frac{1}{512}$$

Option (3)

3. Ans. (4)

$f_4(x) - f_6(x)$

$$= \frac{1}{4}\left(\sin^4 x + \cos^4 x\right) - \frac{1}{6}\left(\sin^6 x + \cos^6 x\right)$$

$$= \frac{1}{4}\left(1 - \frac{1}{2}\sin^2 2x\right) - \frac{1}{6}\left(1 - \frac{3}{4}\sin^2 2x\right) = \frac{1}{12}$$

4. Ans. (1)

$$y = 3\cos\theta + 5\left(\sin\theta\frac{\sqrt{3}}{2} - \cos\theta\frac{1}{2}\right)$$

$$\frac{5\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta$$

$$y_{max} = \sqrt{\frac{75}{4} + \frac{1}{4}} = \sqrt{19}$$

5. Official Ans. by NTA (4)

Sol. $0 < \alpha + \beta = \frac{\pi}{2}$ and $\frac{-\pi}{4} < \alpha - \beta < \frac{\pi}{4}$

if $\cos(\alpha + \beta) = \frac{3}{5}$ then $\tan(\alpha + \beta) = \frac{4}{3}$

and if $\sin(\alpha - \beta) = \frac{5}{13}$ then $\tan(\alpha - \beta) = \frac{5}{12}$

(since $\alpha - \beta$ here lies in the first quadrant)

Now $\tan(2\alpha) = \tan\{(\alpha + \beta) + (\alpha - \beta)\}$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta).\tan(\alpha - \beta)} = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3}.\frac{5}{12}} = \frac{63}{16}$$

6. Official Ans. by NTA (2)

Sol. $\frac{1}{2}\left(2\cos^2 10° - 2\cos 10°\cos 50° + 2\cos^2 50°\right)$

$$\Rightarrow \frac{1}{2}\left(1 + \cos 20° - (\cos 60° + \cos 40°) + 1 + \cos 100°\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{3}{2} + \cos 20° + 2\sin 70°\sin(-30°)\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{3}{2} + \cos 20° - \sin 70°\right)$$

$\Rightarrow \frac{3}{4}$ Ans. (2)

7. Official Ans. by NTA (4)

Sol. $(\sin 10° \sin 30° \sin 70°) \sin 30°$

$$\frac{1}{4}(\sin 30°)^2 = \frac{1}{4}\cdot\frac{1}{4} = \frac{1}{16}$$