COMPLEX NUMBER

1. Let A $=\left\{0\in\left(-\frac{\pi}{2},\pi\right):\frac{3+2i\sin\theta}{1-2i\sin\theta}\text{ is purely imaginary}\right\}$.

Then the sum of the elements in A is :

(1) $\frac{5\pi}{6}$ (2) $\frac{2\pi}{3}$

(3) $\frac{3\pi}{4}$ (4) π

2. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z=3+6iz_0^{81}-3iz_0^{93}$, then arg z is equal to:

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$

(3) 0 (4) $\frac{\pi}{6}$

3. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1| = 4|z_2|$.

If $z=\frac{3z_1}{2z_2}+\frac{2z_2}{3z_1}$ then:

(1) $|z|=\frac{1}{2}\sqrt{\frac{17}{2}}$ (2) Re(z) = 0

(3) $|z|=\sqrt{\frac{5}{2}}$ (4) Im(z) = 0

4. Let $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$. If R(z) and I[z] respectively denote the real and imaginary parts of z, then :

(1) R(z) > 0 and I(z) > 0

(2) R(z) < 0 and I(z) > 0

(3) R(z) = –3

(4) I(z) = 0

5. Let $\left(-2-\frac{1}{3}i\right)^3=\frac{x+iy}{27}\left(i=\sqrt{-1}\right)$, where x and y are real numbers, then y – x equals :

(1) –85 (2) 85

(3) –91 (4) 91

6. Let z be a complex number such that $|z| + z = 3 + i$ (where $i=\sqrt{-1}$). Then |z| is equal to :-

(1) $\frac{5}{4}$ (2) $\frac{\sqrt{41}}{4}$ (3) $\frac{\sqrt{34}}{3}$ (4) $\frac{5}{3}$

7. If $\frac{z-\alpha}{z+\alpha}(\alpha\in R)$ is a purely imaginary number and |z| = 2, then a value of α is :

(1) 1 (2) 2

(3) $\sqrt{2}$ (4) $\frac{1}{2}$

8. Let Z_1 and Z_2 be two complex numbers satisfying $|Z_1| = 9$ and $|Z_2-3-4i|=4$. Then the minimum value of $|Z_1-Z_2|$ is :

(1) 0 (2) 1

(3) $\sqrt{2}$ (4) 2

9. If $z=\frac{\sqrt{3}}{2}+\frac{i}{2}\left(i=\sqrt{-1}\right)$,

then $\left(1+iz+z^5+iz^8\right)^9$ is equal to

(1) –1 (2) 1

(3) 0 (4) $\left(-1+2i\right)^9$

10. All the points in the set

$S=\left\{\frac{\alpha+i}{\alpha-i}:\alpha\in R\right\}$ $\left(i=\sqrt{-1}\right)$ lie on a

(1) circle whose radius is 1.

(2) straight line whose slope is 1.

(3) straight line whose slope is –1

(4) circle whose radius is $\sqrt{2}$.

11. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in R,

$\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to

(1) y^3 (2) $y^3 - 1$

(3) $y(y^2 - 1)$ (4) $y(y^2 - 3)$

12. Let $z\in C$ be such that $|z| < 1$. If $\omega=\frac{5+3z}{5(1-z)}$, then:-

(1) $5Im(\omega) < 1$ (2) $4Im(\omega) > 5$

(3) $5Re(\omega) > 1$ (4) $5Re(\omega) > 4$

13. If $a > 0$ and $z=\frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to :

(1) $-\frac{3}{5}-\frac{1}{5}i$ (2) $-\frac{1}{5}+\frac{3}{5}i$

(3) $-\frac{1}{5}-\frac{3}{5}i$ (4) $\frac{1}{5}-\frac{3}{5}i$

14. If z and w are two complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$, then :

(1) $\bar{z}w=i$ (2) $\bar{z}w=-i$

(3) $z\bar{w}=\frac{1-i}{\sqrt{2}}$ (4) $z\bar{w}=\frac{-1+i}{\sqrt{2}}$

15. The equation $|z-i|=|z-1|$, $i=\sqrt{-1}$, represents:

(1) the line through the origin with slope –1.

(2) a circle of radius 1.

(3) a circle of radius $\frac{1}{2}$.

(4) the line through the origin with slope 1.

16. Let $z \in C$ with $Im(z) = 10$ and it satisfies $\frac{2z-n}{2z+n} = 2i-1$ for some natural number n. Then :

(1) n = 20 and Re(z) = – 10

(2) n = 20 and Re(z) = 10

(3) n = 40 and Re(z) = – 10

(4) n = 40 and Re(z) = 10

SOLUTION

1. Ans. (2)

Given $z = \frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely img

so real part becomes zero.

$$z = \left(\frac{3+2i\sin\theta}{1-2i\sin\theta}\right)\times\left(\frac{1+2i\sin\theta}{1+2i\sin\theta}\right)$$

$$z = \frac{(3-4\sin^2\theta)+i(8\sin\theta)}{i+4\sin^2\theta}$$

Now Re(z) = 0

$$\frac{3-4\sin^2\theta}{1+4\sin^2\theta} = 0$$

$$\sin^2\theta = \frac{3}{4}$$

$$\sin\theta = \pm\frac{\sqrt{3}}{2} \Rightarrow \theta = -\ -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$$

$$\because \theta \in \left(-\frac{\pi}{2}, \pi\right)$$

then sum of the elements in A is

$$-\frac{\pi}{3}+\frac{\pi}{3}+\frac{2\pi}{3} = \frac{2\pi}{3}$$

2. Ans. (1)

$z_0 = \omega$ or ω^2 (where ω is a non-real cube root of unity)

$z = 3 + 6i(\omega)^{81} - 3i(\omega)^{93}$

$z = 3 + 3i$

$\Rightarrow \arg z = \frac{\pi}{4}$

3. Ans. (Bonus)

$3|z_1| = 4|z_2|$

$$\Rightarrow \frac{|z_1|}{|z_2|} = \frac{4}{3}$$

$$\Rightarrow \frac{|3z_1|}{|2z_2|} = 2$$

Let $\frac{3z_1}{2z_2} = a = 2\cos\theta + 2i\sin\theta$

$$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1} = a + \frac{1}{a}$$

$$= \frac{5}{2}\cos\theta + \frac{3}{2}i\sin\theta$$

Now all options are incorrect

Remark :

There is a misprint in the problem actual problem should be :

"Let z_1 and z_2 be any non-zero complex number such that $3|z_1| = \mathbf{2}|z_2|$.

If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$, then"

Given

$3|z_1| = 2|z_2|$

Now $\left|\frac{3z_1}{2z_2}\right| = 1$

Let $\frac{3z_1}{2z_2} = a = \cos\theta + i\sin\theta$

$$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$$

$$= a + \frac{1}{a} = 2\cos\theta$$

$\therefore$ Im(z) = 0

Now option (4) is correct.

4. Ans. (4)

$$z=\left(\frac{\sqrt{3}+i}{2}\right)^5+\left(\frac{\sqrt{3}-i}{2}\right)^5$$

$$z=\left(e^{i\pi/6}\right)^5+\left(e^{-i\pi/6}\right)^5$$

$$=e^{i5\pi/6}+e^{-i5\pi/6}$$

$$=\cos\frac{5\pi}{6}+i\frac{\sin 5\pi}{6}+\cos\left(\frac{-5\pi}{6}\right)+i\sin\left(\frac{-5\pi}{6}\right)$$

$$=2\cos\frac{5\pi}{6}<0$$

$I(z)=0$ and $Re(z)<0$

Option (4)

5. Ans. (4)

$$\left(-2-\frac{i}{3}\right)^3=-\frac{(6+i)^3}{27}$$

$$=\frac{-198-107i}{27}=\frac{x+iy}{27}$$

Hence, $y-x=198-107=91$

6. Ans. (4)

$|z|+z=3+i$

$z=3-|z|+i$

Let $3-|z|=a\Rightarrow |z|=(3-a)$

$$\Rightarrow z=a+i\Rightarrow |z|=\sqrt{a^2+1}$$

$$\Rightarrow 9+a^2-6a=a^2+1\Rightarrow a=\frac{8}{6}=\frac{4}{3}$$

$$\Rightarrow |z|=3-\frac{4}{3}=\frac{5}{3}$$

7. Ans. (2)

$$\frac{z-\alpha}{z+\alpha}+\frac{\bar{z}-\alpha}{\bar{z}+\alpha}=0$$

$$z\bar{z}+z\alpha-\alpha\bar{z}-\alpha^2+z\bar{z}-z\alpha+\bar{z}\alpha-\alpha^2=0$$

$|z|^2=\alpha^2$, $a=\pm 2$

8. Ans. (1)

$$|z_1|=9\ ,\ |z_2-(3+4i)|=4$$

C_1 (0, 0) radius $r_1=9$

C_2(3, 4), radius $r_2=4$

$C_1C_2=|r_1-r_2|=5$

$\therefore$ Circle touches internally

$$\therefore |z_1-z_2|_{min}=0$$

9. Official Ans. by NTA (1)

Sol. $z=\frac{\sqrt{3}}{2}+\frac{i}{2}=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$

$$\Rightarrow z^5=\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}=\frac{-\sqrt{3}+i}{2}$$

and $z^8=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}=-\left(\frac{1+i\sqrt{3}}{2}\right)$

$$\Rightarrow \left(1+iz+z^5+iz^8\right)^9=\left(1+\frac{i\sqrt{3}}{2}-\frac{1}{2}-\frac{\sqrt{3}}{2}+\frac{i}{2}-\frac{i}{2}+\frac{\sqrt{3}}{2}\right)^9$$

$$=\left(\frac{1+i\sqrt{3}}{2}\right)^9=\cos 3\pi+i\sin 3\pi=-1$$

10. Official Ans. by NTA (1)

Sol. Let $\frac{\alpha+i}{\alpha-i}=z$

$$\Rightarrow \frac{|\alpha+i|}{|\alpha-i|}=|z|$$

$\Rightarrow 1=|z|$

$\Rightarrow$ circle of radius 1

11. Official Ans. by NTA (1)

Sol. Roots of the equation $x^2+x+1=0$ are

$\alpha=\omega$ and $\beta=\omega^2$

where ω, ω^2 are complex cube roots of unity

$$\therefore\ \Delta=\begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$R_1\to R_1+R_2+R_3$

$$\Rightarrow \Delta=y\begin{vmatrix} 1 & 1 & 1 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

Expanding along R_1, we get

$\Delta=y.y^2\Rightarrow D=y^3$

12. Official Ans. by NTA (3)

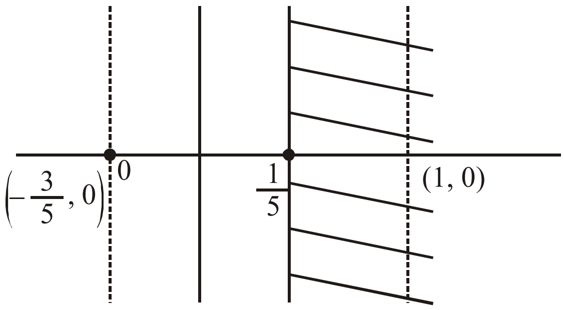
Sol. $|z| < 1$

$5\omega(1 - z) = 5 + 3z$

$5\omega - 5\omega z = 5 + 3z$

$$z = \frac{5\omega - 5}{3 + 5\omega}$$

$$|z| = 5\left|\frac{\omega - 1}{3 + 5\omega}\right| < 1$$



$5|\omega - 1| < |3 + 5\omega|$

$$5|\omega - 1| < 5\left|\omega + \frac{3}{5}\right|$$

$$|\omega - 1| < 5\left|\omega - \left(-\frac{3}{5}\right)\right|$$

13. Official Ans. by NTA (3)

Sol. Given $a > 0$

$$z = \frac{(1+i)^2}{a - i} = \frac{2i(a+i)}{a^2 + 1}$$

Also $|z| = \sqrt{\frac{2}{5}} \Rightarrow \frac{2}{\sqrt{a^2+1}} = \sqrt{\frac{2}{5}} \Rightarrow a = 3$

So $\bar{z} = \frac{-2i(3-i)}{10} = \frac{-1-3i}{5}$

14. Official Ans. by NTA (2)

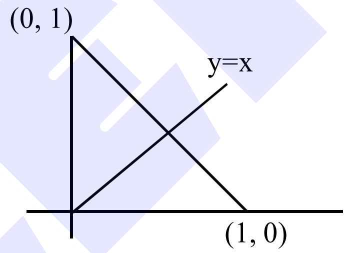
Sol. $|z|.\ |w| = 1 \quad z = re^{i(\theta + \pi/2)}$ and $w = \frac{1}{r}e^{i\theta}$

$$\bar{z}.w = e^{-i(\theta + \pi/2)}.e^{i\theta} = e^{-i(\pi/2)} = -i$$

$$z.\bar{w} = e^{i(\theta+\pi/2)}.e^{-i\theta} = e^{i(\pi/2)} = i$$

15. Official Ans. by NTA (4)

Sol.



$|z - i| = |z - 1|$

$y = x$

16. Official Ans. by NTA (3)

Sol. Put $z = x + 10i$

$\therefore\ 2(x + 10i) - n = (2i - 1)\ .\ [2(x+10i) + n]$

compare real and imginary coefficients

$x = -10,\ n = 40$