CIRCLE

1. Three circles of radii a, b, c(a < b < c) touch each other externally. If they have x-axis as a common tangent, then :

(1) $\frac{1}{\sqrt{a}}=\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{c}}$

(2) a, b, c are in A. P.

(3) $\sqrt{a},\sqrt{b},\sqrt{c}$ are in A. P.

(4) $\frac{1}{\sqrt{b}}=\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{c}}$

2. If the circles $x^2 + y^2 - 16x-20y + 164 = r^2$ and $(x-4)^2 + (y - 7)^2 = 36$ intersect at two distinct points, then:

(1) $0 < r < 1$ (2) $1 < r < 11$
(3) $r > 11$ (4) $r = 11$

3. If a circle C passing through the point (4,0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, –1), then the radius of C is :

(1) $\sqrt{57}$ (2) 4 (3) $2\sqrt{5}$ (4) 5

4. If the area of an equilateral triangle inscribed in the circle, $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq. units then c is equal to :

(1) 20 (2) 25 (3) 13 (4) –25

5. A square is inscribed in the circle $x^2 + y^2 - 6x + 8y - 103 = 0$ with its sides parallel to the corrdinate axes. Then the distance of the vertex of this square which is nearest to the origin is :-

(1) 13 (2) $\sqrt{137}$ (3) 6 (4) $\sqrt{41}$

6. The straight line x + 2y = 1 meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is :

(1) $\frac{\sqrt{5}}{4}$ (2) $\frac{\sqrt{5}}{2}$ (3) $2\sqrt{5}$ (4) $4\sqrt{5}$

7. Two circles with equal radii are intersecting at the points (0, 1) and (0, –1). The tangent at the point (0, 1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is :

(1) 1 (2) $\sqrt{2}$ (3) $2\sqrt{2}$ (4) 2

8. A circle cuts a chord of length 4a on the x-axis and passes through a point on the y-axis, distant 2b from the origin. Then the locus of the centre of this circle, is :-

(1) A hyperbola (2) A parabola
(3) A straight line (4) An ellipse

9. If a variable line, $3x+4y-\lambda=0$ is such that the two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2+y^2-18x-2y+78 = 0$ are on its opposite sides, then the set of all values of λ is the interval :-

(1) [12, 21] (2) (2, 17)
(3) (23, 31) (4) [13, 23]

10. Let C_1 and C_2 be the centres of the circles $x^2+y^2-2x-2y-2 = 0$ and $x^2+y^2-6x-6y+14 = 0$ respectively. If P and Q are the points of intersection of these circles, then the area (in sq. units) of the quadrilateral PC_1QC_2 is :

(1) 8 (2) 6 (3) 9 (4) 4

11. Let O(0, 0) and A(0, 1) be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is :

(1) $8x^2 - 9y^2 + 9y = 18$
(2) $9x^2 + 8y^2 - 8y = 16$
(3) $8x^2 + 9y^2 - 9y = 18$
(4) $9x^2 - 8y^2 + 8y = 16$

12. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is :

(1) 320 (2) 160
(3) 105 (4) 210

13. The tangent and the normal lines at the point $\left(\sqrt{3},1\right)$ to the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The area of this triangle (in square units) is :

(1) $\frac{1}{3}$ (2) $\frac{4}{\sqrt{3}}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{2}{\sqrt{3}}$

14. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is

(1) $x^2 + y^2 - 2xy = 0$
(2) $x^2 + y^2 - 16x^2y^2 = 0$
(3) $x^2 + y^2 - 4x^2y^2 = 0$
(4) $x^2 + y^2 - 2x^2y^2 = 0$

15. The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point :-

(1) $(-4, 6)$ (2) $(6, -2)$
(3) $(-6, 4)$ (4) $(4, -2)$

16. If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2+y^2)+2Kx+3y-1=0$, $(K \in R)$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q for :

(1) exactly two values of K
(2) exactly one value of K
(3) no value of K.
(4) infinitely many values of K

17. The line x = y touches a circle at the point (1, 1). If the circle also passes through the point (1, –3), then its radius is :

(1) $3\sqrt{2}$ (2) 3 (3) $2\sqrt{2}$ (4) 2

18. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is :

(1) $y = \sqrt{1+4x}$, $x \geq 0$
(2) $x = \sqrt{1+4y}$, $y \geq 0$
(3) $x = \sqrt{1+2y}$, $y \geq 0$
(4) $y = \sqrt{1+2x}$, $x \geq 0$

19. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is :

(1) $\frac{60}{13}$ (2) $\frac{120}{13}$ (3) $\frac{13}{2}$ (4) $\frac{13}{5}$

20. A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point :

(1) (3, 10) (2) (2,3) (3) (1,5) (4) (3,5)

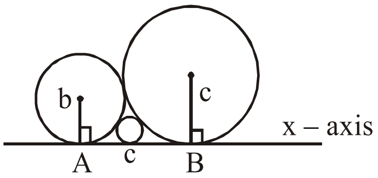
SOLUTION

1. Ans. (1)

AB = AC + CB

$$\sqrt{(b+c)^2-(b-c)^2}$$

$$=\sqrt{(b+a)^2-(b-a)^2}+\sqrt{(a+c)^2-(a-c)^2}$$



$$\sqrt{bc}=\sqrt{ab}+\sqrt{ac}$$

$$\frac{1}{\sqrt{a}}=\frac{1}{\sqrt{c}}+\frac{1}{\sqrt{b}}$$

2. Ans. (2)

$x^2 + y^2 - 16x - 20y + 164 = r^2$

$A(8,10),\ R_1 = r$

$(x-4)^2 + (y-7)^2 = 36$

$B(4,7),\ R_2 = 6$

$|R_1 - R_2| < AB < R_1 + R_2$

$\Rightarrow 1 < r < 11$

3. Ans. (4)

$x^2 + y^2 + 4x - 6y - 12 = 0$

Equation of tangent at (1, –1)

$x - y + 2(x+1) - 3(y-1) - 12 = 0$

$3x - 4y - 7 = 0$

$\therefore$ Equation of circle is

$(x^2 + y^2 + 4x - 6y - 12) + \lambda(3x - 4y - 7) = 0$

It passes through (4, 0) :

$(16 + 16 - 12) + \lambda(12 - 7) = 0$

$\Rightarrow 20 + \lambda(5) = 0$

$\Rightarrow \lambda = -4$

$\therefore (x^2+y^2+4x-6y-12)-4(3x-4y-7)=0$

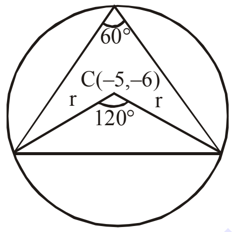
or $x^2 + y^2 - 8x + 10y + 16 = 0$

Radius = $\sqrt{16+25-16}=5$

4. Ans. (2)

$$3\left(\frac{1}{2}r^2.\sin120°\right)=27\sqrt{3}$$

$$\frac{r^2}{2}\frac{\sqrt{3}}{2}=\frac{27\sqrt{3}}{3}$$



$$r^2=\frac{108}{3}=36$$

Radius = $\sqrt{25+36-C}=\sqrt{36}$

$\boxed{C=25}$

$\therefore$ Option (2)

5. Ans. (4)

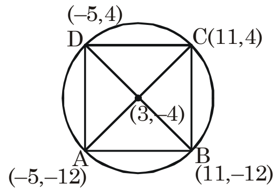
$R=\sqrt{9+16+103}=8\sqrt{2}$

OA = 13

$OB=\sqrt{265}$

$OC=\sqrt{137}$

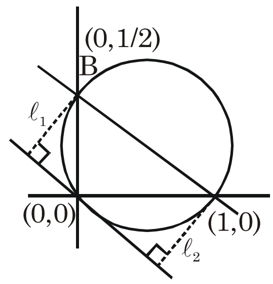
$OD=\sqrt{41}$



6. Ans. (2)

Equation of circle

$$(x-1)(x-0)+(y-0)\left(y-\frac{1}{2}\right)=0$$



$$\Rightarrow x^2+y^2-x-\frac{y}{2}=0$$

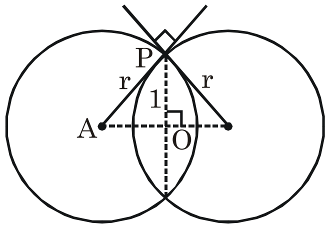
Equation of tangent of origin is 2x + y = 0

$$\ell_1+\ell_2=\frac{2}{\sqrt{5}}+\frac{1}{2\sqrt{5}}$$

$$=\frac{4+1}{2\sqrt{5}}=\frac{\sqrt{5}}{2}$$

7. Ans. (4)

In ΔAPO



$$\left(\frac{\sqrt{2}\,r}{2}\right)^2 + 1^2 = r^2$$

$$\Rightarrow \boxed{r=\sqrt{2}}$$

So distance between centres = $\sqrt{2}\,r$ = 2

8. Ans. (2)

Let equation of circle is

$x^2 + y^2 + 2fx + 2fy + e = 0$, it passes through (0, 2b)

$\Rightarrow 0 + 4b^2 + 2g \times 0 + 4f + c = 0$

$\Rightarrow 4b^2 + 4f + c = 0$ ...(i)

$2\sqrt{g^2 - c} = 4a$ ...(ii)

$g^2 - c = 4a^2 \Rightarrow c = \left(g^2 - 4a^2\right)$

Putting in equation (1)

$\Rightarrow 4b^2 + 4f + g^2 - 4a^2 = 0$

$\Rightarrow x^2 + 4y + 4(b^2 - a^2) = 0$, it represent a parabola.

9. Ans. (1)

Centre of circles are opposite side of line

$(3 + 4 - \lambda)(27 + 4 - \lambda) < 0$

$(\lambda - 7)(\lambda - 31) < 0$

$\lambda \in (7, 31)$

distance from S_1

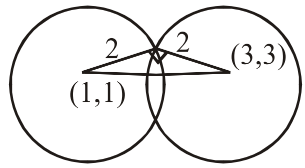
$$\left|\frac{3+4-\lambda}{5}\right| \geq 1 \Rightarrow \lambda \in (-\infty, 2] \cup [(12, \infty]$$

distance from S_2

$$\left|\frac{27+4-\lambda}{5}\right| \geq 2 \Rightarrow \lambda \in (-\infty, 21] \cup [41, \infty)$$

so $\lambda \in [12, 21]$

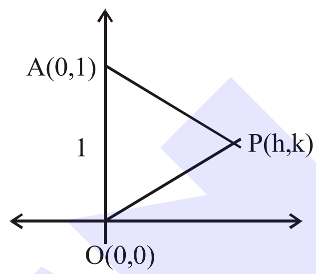
10. Ans. (4)



$$\text{Area} = 2 \times \frac{1}{2}.4 = 2$$

11. Official Ans. by NTA (2)

Sol.



AP + OP + AO = 4

$$\sqrt{h^2 + (k-1)^2} + \sqrt{h^2 + k^2} + 1 = 4$$

$$\sqrt{h^2 + (k-1)^2} + \sqrt{h^2 + k^2} = 3$$

$$h^2 + (k-1)^2 = 9 + h^2 + k^2 - 6\sqrt{h^2 + k^2}$$

$$-2k - 8 = -6\sqrt{h^2 + k^2}$$

$$k + 4 = 3\sqrt{h^2 + k^2}$$

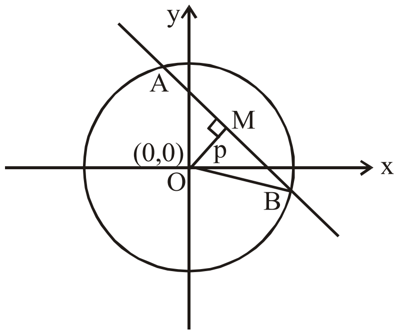
$k^2 + 16 + 8k = 9(h^2 + k^2)$

$9h^2 + 8k^2 - 8k - 16 = 0$

Locus of P is $9x^2 + 8y^2 - 8y - 16 = 0$

12. Official Ans. by NTA (4)

Sol.



$p=\frac{n}{\sqrt{2}}$, but $\frac{n}{\sqrt{2}}<4 \Rightarrow n=1, 2, 3, 4, 5.$

Length of chord AB $=2\sqrt{16-\frac{n^2}{2}}$

$=\sqrt{64-2n^2}=\ell(\text{say})$

For $n=1$, $\ell^2=62$

$n=2$, $\ell^2=56$

$n=3$, $\ell^2=46$

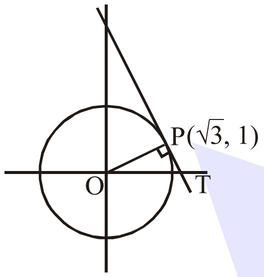
$n=4$, $\ell^2=32$

$n=5$, $\ell^2=14$

$\therefore$ Required sum $=62+56+46+32+14=210$

13. Official Ans. by NTA (4)

Sol.



Given $x^2+y^2=4$

equation of tangent

$\Rightarrow \sqrt{3}x+y=4$ ...(1)

Equation of normal

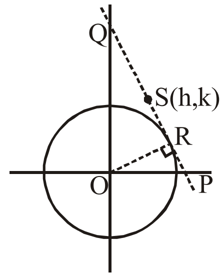
$x-\sqrt{3}y=0$ ...(2)

Coordinate of $T\left(\frac{4}{\sqrt{3}},0\right)$

$\therefore$ Area of triangle $=\frac{2}{\sqrt{3}}$

14. Official Ans. by NTA (3)

Sol.



Let the mid point be S(h,k)

$\therefore$ P(2h,0) and Q(0,2k)

equation of PQ : $\frac{x}{2h}+\frac{y}{2k}=1$

$\because$ PQ is tangent to circle at R(say)

$\therefore$ OR = 1 $\Rightarrow \left|\frac{-1}{\sqrt{\left(\frac{1}{2h}\right)^2+\left(\frac{1}{2k}\right)^2}}\right|=1$

$\Rightarrow \frac{1}{4h^2}+\frac{1}{4k^2}=1$

$\Rightarrow x^2+y^2-4x^2y^2=0$

Aliter :

tangent to circle

$x\cos\theta+y\sin\theta=1$

P : $(\sec\theta, 0)$

Q : $(0, \operatorname{cosec}\theta)$

$2h=\sec\theta \Rightarrow \cos\theta=\frac{1}{2h}$ & $\sin\theta=\frac{1}{2k}$

$\frac{1}{(2x)^2}+\frac{1}{(2y)^2}=1$

15. Official Ans. by NTA (2)

Sol. Circle touches internally

$C_1(0, 0)$; $r_1=2$

C_2 : $(-3, -4)$; $r_2=7$

$C_1C_2=|r_1-r_2|$

$S_1-S_2=0 \Rightarrow$ eqn. of common tangent

$6x+8y-20=0$

$3x+4y=10$

$(6, -2)$ satisfy it

16. Official Ans. by NTA (3)

Sol. Equation of common chord

$$4kx + \frac{1}{2}y + k + \frac{1}{2} = 0 \quad ....(1)$$

and given line is $4x + 5y - k = 0$ .....(2)

On comparing (1) & (2), we get

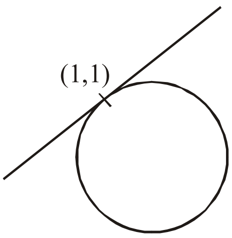
$$k = \frac{1}{10} = \frac{k + \frac{1}{2}}{-k}$$

$\Rightarrow$ No real value of k exist

17. Official Ans. by NTA (1)

ALLEN Ans. (3)

Sol.



Equation of circle can be written as $(x-1)^2 + (y-1)^2 + \lambda(x-y) = 0$

It passes through $(1, -3)$

$16 + \lambda(4) = 0 \Rightarrow \lambda = -4$

So $(x-1)^2 + (y-1)^2 - 4(x-y) = 0$

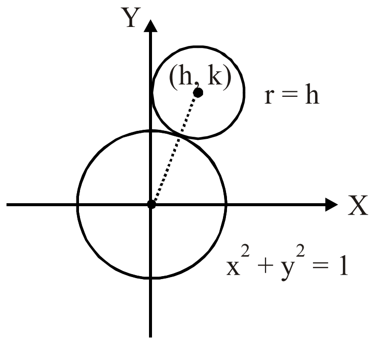
$\Rightarrow x^2 + y^2 - 6x + 2y + 2 = 0$

$\Rightarrow r = 2\sqrt{2}$

(correct key is 3)

18. Official Ans. by NTA (4)

Sol.



$$\sqrt{h^2 + k^2} = |h| + 1$$

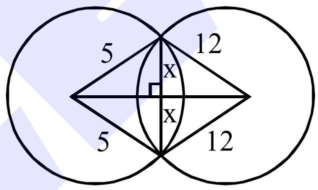
$\Rightarrow x^2 + y^2 = x^2 + 1 + 2x$

$\Rightarrow y^2 = 1 + 2x$

$\Rightarrow y = \sqrt{1+2x}$; $x \geq 0$.

19. Official Ans. by NTA (2)

Sol.



Let length of common chord = 2x

$$\sqrt{25 - x^2} + \sqrt{144 - x^2} = 13$$

after solving

$$x = \frac{12 \times 5}{13}$$

$$2x = \frac{120}{13}$$

20. Official Ans. by NTA (1)

Sol. Equaiton of circles are

$$\begin{cases} (x-3)^2 + (y-5)^2 = 25 \\ (x-3)^2 + (y+5)^2 = 25 \end{cases}$$

$$\Rightarrow \begin{cases} x^2 + y^2 - 6x - 10y + 9 = 0 \\ x^2 + y^2 - 6x + 10y + 9 = 0 \end{cases}$$