BINOMIAL THEOREM

1. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to :
(1) 14 (2) 6 (3) 4 (4) 8

2. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is
(1) 12 (2) 15 (3) 10 (4) 14

3. $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$, then k equals :
(1) 200 (2) 50 (3) 100 (4) 400

4. If the third term in the binomial expansion of $\left(1+x^{\log_2 x}\right)^5$ equals 2560, then a possible value of x is:
(1) $2\sqrt{2}$ (2) $\frac{1}{8}$ (3) $4\sqrt{2}$ (4) $\frac{1}{4}$

5. The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is :
(1) $\sqrt{5}$ (2) 4 (3) $2\sqrt{2}$ (4) 3

6. If $\sum_{r=0}^{25}\left\{{}^{50}C_r \cdot {}^{50-r}C_{25-r}\right\} = K\left({}^{50}C_{25}\right)$, then K is equal to :
(1) $2^{25}-1$ (2) $(25)^2$ (3) 2^{25} (4) 2^{24}

7. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equals 5670 is :
(1) 6 (2) 8 (3) 0 (4) 4

8. The value of r for which
${}^{20}C_r\,{}^{20}C_0 + {}^{20}C_{r-1}\,{}^{20}C_1 + {}^{20}C_{r-2}\,{}^{20}C_2 + \ldots\, {}^{20}C_0\,{}^{20}C_r$
is maximum, is
(1) 20 (2) 15 (3) 11 (4) 10

9. Let $(x+10)^{50}+(x-10)^{50} = a_0 + a_1x + a_2x^2 + \ldots + a_{50}x^{50}$, for all $x \in R$, then $\frac{a_2}{a_0}$ is equal to:-
(1) 12.50 (2) 12.00 (3) 12.75 (4) 12.25

10. Let $S_n = 1 + q + q^2 + \ldots\ldots + q^n$ and $T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + \ldots\ldots + \left(\frac{q+1}{2}\right)^n$, where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2.S_1 + \ldots\ldots + {}^{101}C_{101}.S_{100} = \alpha T_{100}$, then α is equal to :-
(1) 2^{100} (2) 200 (3) 2^{99} (4) 202

11. A ratio of the 5^{th} term from the beginning to the 5^{th} term from the end in the binomial expansion of $\left(2^{1/3}+\frac{1}{2(3)^{1/3}}\right)^{10}$ is :
(1) $1 : 4(16)^{\frac{1}{3}}$ (2) $1 : 2(6)^{\frac{1}{3}}$
(3) $2(36)^{\frac{1}{3}} : 1$ (4) $4(36)^{\frac{1}{3}} : 1$

12. The total number of irrational terms in the binomial expansion of $\left(7^{1/5}-3^{1/10}\right)^{60}$ is :
(1) 55 (2) 49 (3) 48 (4) 54

13. The sum of the series
$2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + \ldots + 62.{}^{20}C_{20}$ is equal to :
(1) 2^{24} (2) 2^{25}
(3) 2^{26} (4) 2^{23}

14. The sum of the co-efficients of all even degree terms in x in the expansion of $\left(x+\sqrt{x^3-1}\right)^6+\left(x-\sqrt{x^3-1}\right)^6, (x>1)$ is equal to :
(1) 32 (2) 26 (3) 29 (4) 24

15. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10} x}}}+x^{\frac{1}{12}}\right)^6$ is equal to 200, and x > 1, then the value of x is :

(1) 10^3 (2) 100 (3) 10^4 (4) 10

16. If the fourth term in the binomial expansion of $\left(\frac{2}{x}+x^{\log_8 x}\right)^6$ $(x > 0)$ is 20×8^7, then a value of x is :

(1) 8 (2) 8^2 (3) 8^{-2} (4) 8^3

17. If some three consecutive in the binomial expansion of $(x + 1)^n$ is powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficient is :-

(1) 964 (2) 625 (3) 227 (4) 232

18. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 - 3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to :

(1) (28, 315) (2) (–54, 315)
(3) (–21, 714) (4) (24, 861)

19. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2+\frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is :

(1) 35 (2) 38 (3) 23 (4) 58

20. The coefficient of x^{18} in the product $(1+x)(1-x)^{10}(1 + x + x^2)^9$ is :

(1) –84 (2) 84 (3) 126 (4) –126

21. If ${}^{20}C_1 + (2^2)\,{}^{20}C_2 + (3^2)\,{}^{20}C_3 + \ldots\ldots + (20^2)\,{}^{20}C_{20} = A(2^\beta)$, then the ordered pair (A, β) is equal to:

(1) (420, 18) (2) (380, 19)
(3) (380, 18) (4) (420, 19)

22. The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to :

(1) 36 (2) – 108 (3) – 72 (4) – 36

SOLUTION

1. Ans. (4)

$$\frac{2^{403}}{15}=\frac{2^3.(2^4)^{100}}{15}=\frac{8}{15}(15+1)^{100}$$

$$=\frac{8}{15}(15\lambda+1)=8\lambda+\frac{8}{15}$$

$\because$ 8λ is integer

$\Rightarrow$ fractional part of $\frac{2^{403}}{15}$ is $\frac{8}{15}\Rightarrow k=8$

2. Ans. (2)

$(1-t^6)^3\,(1-t)^{-3}$

$(1-t^{18}-3t^6+3t^{12})\,(1-t)^{-3}$

$\Rightarrow$ cofficient of t^4 in $(1-t)^{-3}$ is

${}^{3+4-1}C_4={}^6C_2=15$

3. Ans. (3)

$$\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i+{}^{20}C_{i-1}}\right)^3=\frac{k}{21}$$

$$\Rightarrow \sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{21}C_i}\right)^3=\frac{k}{21}$$

$$\Rightarrow \sum_{i=1}^{20}\left(\frac{i}{21}\right)^3=\frac{k}{21}$$

$$\Rightarrow \frac{1}{(21)^3}\left[\frac{20(21)}{2}\right]^2=\frac{k}{21}$$

$\Rightarrow 100=k$

4. Ans. (4)

$(1+x^{\log_2 x})^5$

$T_3={}^5C_2.(x^{\log_2 x})^2=2560$

$\Rightarrow 10.x^{2\log_2 x}=2560$

$\Rightarrow x^{2\log_2 x}=256$

$\Rightarrow 2(\log_2 x)^2=\log_2 256$

$\Rightarrow 2(\log_2 x)^2=8$

$\Rightarrow (\log_2 x)^2=4 \quad \Rightarrow \log_2 x=2$ or -2

$x=4$ or $\frac{1}{4}$

5. Ans. (2)

$$x^2\left({}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{\lambda}{x^2}\right)^r\right)$$

$$x^2\left[{}^{10}C_r(x)^{\frac{10-r}{2}}(\lambda)^r\,(x)^{-2r}\right]$$

$$x^2\left[{}^{10}C_r\,\lambda^r\,x^{\frac{10-5r}{2}}\right]$$

$\therefore$ $r=2$

Hence, ${}^{10}C_2\,\lambda^2=720$

$\lambda^2=16$

$\lambda=\pm 4$

Option (2)

6. Ans. (3)

$$\sum_{r=0}^{25}{}^{50}C_r\,.\,{}^{50-r}C_{25-r}$$

$$=\sum_{r=0}^{25}\frac{50!}{r!\,(50-r)!}\times\frac{(50-r)!}{(25)!\,(25-r)!}$$

$$=\sum_{r=0}^{25}\frac{50!}{25!\,25!}\times\frac{25!}{(25-r)!\,(r!)}$$

$$={}^{50}C_{25}\sum_{r=0}^{25}{}^{25}C_r=\left(2^{25}\right){}^{50}C_{25}$$

$\therefore$ $K=2^{25}$

Option (3)

7. Ans. (3)

$$T_5={}^8C_4\frac{x^{12}}{81}\times\frac{81}{x^4}=5670$$

$\Rightarrow 70x^8=5670$

$\Rightarrow x=\pm\sqrt{3}$

8. Ans. (1)

Given sum = coefficient of x^r in the expansion of $(1+x)^{20}(1+x)^{20}$,

which is equal to ${}^{40}C_r$

It is maximum when $r=20$

9. Ans. (4)

$(10+x)^{50}+(10-x)^{50}$

$\Rightarrow a_2=2.{}^{50}C_2 10^{48},\ a_0=2.10^{50}$

$$\frac{a_2}{a_0}=\frac{{}^{50}C_2}{10^2}=12.25$$

10. Ans. (1)

$^{101}C_1 + {}^{101}C_2S_1 + ... + {}^{101}C_{101}S_{100} = \alpha T_{100}$

$^{101}C_1 + {}^{101}C_2(1+q) + {}^{101}C_3(1+q+q^2) + ...$

$+ {}^{101}C_{101}(1 + q + ... + q^{100})$

$$= 2\alpha \frac{\left(1-\left(\frac{1+q}{2}\right)^{101}\right)}{(1-q)}$$

$\Rightarrow {}^{101}C_1(1-q) + {}^{101}C_2(1-q^2) + ... + {}^{101}C_{101}(1-q^{101})$

$$= 2\alpha\left(1-\left(\frac{1+q}{2}\right)^{101}\right)$$

$\Rightarrow (2^{101} - 1) - ((1+q)^{101} - 1)$

$$= 2\alpha\left(1-\left(\frac{1+q}{2}\right)^{101}\right)$$

$$\Rightarrow 2^{101}\left(1-\left(\frac{1+q}{2}\right)^{101}\right) = 2\alpha\left(1-\left(\frac{1+q}{2}\right)^{101}\right)$$

$\Rightarrow \alpha = 2^{100}$

11. Ans. (4)

$$\frac{T_5}{T_5^1} = \frac{{}^{10}C_4(2^{1/3})^{10-4}\left(\frac{1}{2(3)^{1/3}}\right)^4}{{}^{10}C_4\left(\frac{1}{2(3^{1/3})}\right)^{10-4}(2^{1/3})^4} = 4.(36)^{1/3}$$

12. Ans. (4)

General term $T_{r+1} = {}^{60}C_r\ 7^{\frac{60-r}{5}}\ 3^{\frac{r}{10}}$

$\therefore$ for rational term, r = 0, 10, 20, 30, 40, 50, 60

$\Rightarrow$ no of rational terms = 7

$\therefore$ number of irrational terms = 54

Also, $-27\times{}^{15}C_3 + 9a\times{}^{15}C_2 - 3b\times{}^{15}C_1 = 0$

$\Rightarrow 9\times{}^{15}C_2\ a - 45\ b - 27\times{}^{15}C_3 = 0$

$\Rightarrow 21a - b - 273 = 0$ ...(ii)

(i) + (ii)

$-24\ a + 672 = 0$

$\Rightarrow a = 28$

So, b = 315

13. Official Ans. by NTA (2)

Sol. $2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + ... + 62.{}^{20}C_{20}$

$$= \sum_{r=0}^{20}(3r+2)\,{}^{20}C_r$$

$$= 3\sum_{r=0}^{20} r.\,{}^{20}C_r + 2\sum_{r=0}^{20}{}^{20}C_r$$

$$= 3\sum_{r=0}^{20} r\left(\frac{20}{r}\right){}^{19}C_{r-1} + 2.2^{20}$$

$= 60.2^{19} + 2.2^{20} = 2^{25}$

14. Official Ans. by NTA (4)

Sol. $\left(x+\sqrt{x^3-1}\right)^6 + \left(x-\sqrt{x^3-1}\right)^6$

$= 2[{}^6C_0x^6 + {}^6C_2x^4(x^3-1) + {}^6C_4x^2(x^3-1)^2 + {}^6C_6(x^3-1)^3]$

$= 2[{}^6C_0x^6 + {}^6C_2x^7 - {}^6C_2x^4 + {}^6C_4x^8 + {}^6C_4x^2 - 2{}^6C_4x^5 + (x^9 - 1 - 3x^6 + 3x^3)]$

$\Rightarrow$ Sum of coefficient of even powers of x

= 2[1 – 15 + 15 + 15 – 1 – 3] = 24

15. Official Ans. by NTA (4)

Sol. $$200 = {}^6C_3\left(x^{\frac{1}{x+\log_{10}x}}\right)^{\frac{3}{2}} \times x^{\frac{1}{4}}$$

$$\Rightarrow 10 = x^{\frac{3}{2(1+\log_{10}x)}+\frac{1}{4}}$$

$$\Rightarrow 1 = \left(\frac{3}{2(1+t)}+\frac{1}{4}\right)t$$

where $t = \log_{10}x$

$\Rightarrow t^2 + 3t - 4 = 0$

$\Rightarrow t = 1, -4$

$\Rightarrow x = 10, 10^{-4}$

$\Rightarrow x = 10$ (As x > 1)

16. Official Ans. by NTA (2)

Sol. $T_4 = T_{3+1} = \binom{6}{3}\left(\frac{2}{x}\right)^3 .\left(x^{\log_8 x}\right)^3$

$$20 \times 8^7 = \frac{160}{x^3}.x^{3\log_8 x}$$

$$8^6 = x^{\log_2 x} - 3$$

$$2^{18} = x^{\log_2 x - 3}$$

$$\Rightarrow 18 = \left(\log_2 x - 3\right)\left(\log_2 x\right)$$

Let $\log_2 x = t$

$$\Rightarrow t^2 - 3t - 18 = 0$$

$$\Rightarrow \left(t-6\right)\left(t+3\right) = 0$$

$$\Rightarrow t = 6, -3$$

$$\log_2 x = 6 \Rightarrow x = 2^6 = 8^2$$

$$\log_2 x = -3 \Rightarrow x = 2^{-3} = 8^{-1}$$

17. Official Ans. by NTA (4)

Sol. $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{15}$

$$\frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{2}{15}$$

$$\frac{r}{n-r+1} = \frac{2}{15}$$

$15r = 2n - 2r + 2$

$$\boxed{17r = 2n + 2}$$

$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15}{70}$$

$$\frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{3}{14}$$

$$\frac{r+1}{n-r} = \frac{3}{14}$$

$14r + 14 = 3n - 3r$

$$3n - 17r = 14$$
$$2n - 17r = -2$$
$$n = 16$$

$17r = 34,\ r = 2$

${}^{16}C_1,\ {}^{16}C_2,\ {}^{16}C_3$

$$\frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16+120+560}{3}$$

$$\frac{680+16}{3} = \frac{696}{3} = 232$$

18. Official Ans. by NTA (1)

Sol. Coefiicient of $x^2 = {}^{15}C_2 \times 9 - 3a\left({}^{15}C_1\right) + b = 0$

$\Rightarrow -45a + b + {}^{15}C_2 \times 9 = 0$ ....(i)

Also, $-27\times{}^{15}C_3 + 9a \times {}^{15}C_2 - 3b\times{}^{15}C_1 = 0$

$\Rightarrow 9 \times {}^{15}C_2\, a - 45\, b - 27 \times {}^{15}C_3 = 0$

$\Rightarrow 21a - b - 273 = 0$ ...(ii)

(i) + (ii)

$-24\, a + 672 = 0$

$\Rightarrow a = 28$

So, $b = 315$

19. Official Ans. by NTA (2)

Sol. $T_r = \sum_{r=0}^{n} {}^{n}C_r x^{2n-2r}.x^{-3r}$

$2n - 5r = 1 \Rightarrow 2n = 5r + 1$

for $r = 15$. $n = 38$

smallest value of n is 38.

20. Official Ans. by NTA (2)

Sol. $(1 + x)(1 - x)^{10}(1 + x + x^2)^9$

$(1 - x^2)(1 - x^3)^9$

${}^9C_6 = 84$

21. Official Ans. by NTA (1)

Sol. $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ... + {}^nC_nx^n$

Diff. w.r.t. x

$\Rightarrow n(1+x)^{n-1} = {}^nC_1 + {}^nC_2(2x) + ...+ {}^nC_n\, n(x)^{n-1}$

Multiply by x both side

$\Rightarrow nx(1+x)^{n-1} = {}^nC_1\, x + {}^nC_2(2x^2) + ...+ {}^nC_n(n\, x^n)$

Diff w.r.t. x

$\Rightarrow n\,[(1+x)^{n-1} + (n-1)x\,(1+x)^{n-2}]$

$= {}^nC_1 + {}^nC_2\, 2^2x + ...\, {}^nC_n\,(n^2)x^{n-1}$

Put $x = 1$ and $n = 20$

$\Rightarrow {}^{20}C_1 + 2^2\, {}^{20}C_2 + 3^2\, {}^{20}C_3 + ... + (20)^2\, {}^{20}C_{20}$

$= 20 \times 2^{18}[2 + 19] = 420(2^{18}) = A(2^{\beta})$

22. Official Ans. by NTA (4)

Sol. $\frac{1}{60}\left(2x^2 - \frac{3}{x^2}\right)^6 - \frac{1}{81}\cdot x^8\left(2x^2 - \frac{3}{x^2}\right)^6$

its general term

$\frac{1}{60}\,{}^6C_r 2^{6-r}(-3)^r x^{12-r} - \frac{1}{81}\,{}^6C_r 2^{6-r}(-3)^r 12^{20-4r}$

for term independent of x, r for I[st] expression is 3 and r for second expression is 5

$\therefore$ term independent of $x = -36$