AREA UNDER THE CURVE

1. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is :

(1) $\frac{14}{3}$ (2) $\frac{56}{3}$ (3) $\frac{8}{3}$ (4) $\frac{32}{3}$

2. The area of the region $A=\left[(x,y):0\le y\le x|x|+1 \text{ and } -1\le x\le 1\right]$ in sq. units, is :

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) 2 (4) $\frac{4}{3}$

3. If the area enclosed between the curves $y=kx^2$ and $x=ky^2$, $(k>0)$, is 1 square unit. Then k is:

(1) $\frac{1}{\sqrt{3}}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\sqrt{3}$

4. The tangent to the curve, $y = xe^{x^2}$ passing through the point (1,e) also passes through the point :

(1) $\left(\frac{4}{3},2e\right)$ (2) (2,3e) (3) $\left(\frac{5}{3},2e\right)$ (4) (3,6e)

5. The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$:-

(1) $\frac{5}{4}$ (2) $\frac{9}{8}$ (3) $\frac{3}{4}$ (4) $\frac{7}{8}$

6. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is :

(1) $\frac{15}{4}$ (2) $\frac{15}{2}$ (3) $\frac{21}{2}$ (4) $\frac{17}{4}$

7. The area (in sq. units) of the region $A = \{(x, y) \in R \times R | 0 \le x \le 3, 0 \le y \le 4, y \le x^2 + 3x\}$ is :

(1) $\frac{53}{6}$ (2) $\frac{59}{6}$

(3) 8 (4) $\frac{26}{3}$

8. Let $S(\alpha) = \{(x,y) : y^2 \le x, 0 \le x \le \alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a λ, $0 < \lambda < 4$, $A(\lambda) : A(4) = 2 : 5$, then λ equals

(1) $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$ (2) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

(3) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (4) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$

9. The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is

(1) $\frac{10}{3}$ (2) $\frac{9}{2}$

(3) $\frac{31}{6}$ (4) $\frac{13}{6}$

10. The area (in sq. units) of the region

$A=\{(x,y):\frac{y^2}{2}\le x\le y+4\}$ is :-

(1) $\frac{53}{3}$ (2) 18 (3) 30 (4) 16

11. The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is :

(1) $\frac{3}{2}-\frac{1}{\log_e 2}$ (2) $\frac{1}{2}$

(3) $\log_e 2+\frac{3}{2}$ (4) $\frac{3}{2}$

12. If the area (in sq. units) of the region $\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$ is $a\sqrt{2}+b$, then a – b is equal to :

(1) $\frac{8}{3}$ (2) $\frac{10}{3}$ (3) 6 (4) $-\frac{2}{3}$

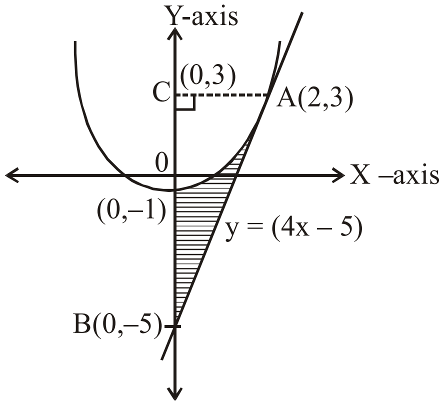
13. If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$, then λ is equal to :

(1) 24 (2) 48

(3) $4\sqrt{3}$ (4) $2\sqrt{6}$

SOLUTION

1. **Ans. (3)**



Equation of tangent at (2,3) on

$y = x^2 - 1$, is $y = (4x - 5)$ ....(i)

∴ Required shaded area

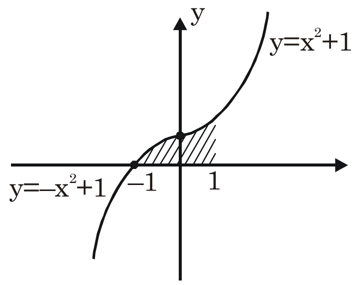
$= \text{ar}(\Delta ABC) - \int_{-1}^{3} \sqrt{y+1}\, dy$

$= \frac{1}{2}.(8).(2) - \frac{2}{3}\left((y+1)^{3/2}\right)_{-1}^{3}$

$= 8 - \frac{16}{3} = \frac{8}{3}$ (square units)

2. **Ans. (3)**

The graph is a follows



$\int_{-1}^{0}\left(-x^2+1\right)dx + \int_{0}^{1}\left(x^2+1\right)dx = 2$

3. **Ans. (1)**

Area bounded by $y^2 = 4ax$ & $x^2 = 4by$,

a, b ≠ 0 is $\left|\frac{16ab}{3}\right|$

by using formula : $4a = \frac{1}{k} = 4b, k > 0$

Area $= \left|\frac{16.\frac{1}{4k}.\frac{1}{4k}}{3}\right| = 1$

$\Rightarrow k^2 = \frac{1}{3}$

$\Rightarrow k = \frac{1}{\sqrt{3}}$

4. Ans. (1)

$y = xe^{x^2}$

$\left.\dfrac{dy}{dx}\right|_{(1,e)} = \left(e \cdot e^{x^2} \cdot 2x + e^{x^2}\right)\Big|_{(1,e)} = 2 \cdot e + e = 3e$

T : $y - e = 3e(x - 1)$

$y = 3ex - 3e + e$

$y = (3e)x - 2e$

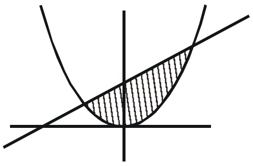
$\left(\dfrac{4}{3}, 2e\right)$ lies on it

Option (1)

5. Ans. (2)

$x = 4y - 2$ & $x^2 = 4y$

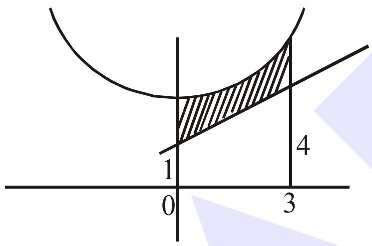
$\Rightarrow x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$



$x = 2, -1$

So, $\displaystyle\int_{-1}^{2}\left(\frac{x+2}{4} - \frac{x^2}{4}\right)dx = \frac{9}{8}$

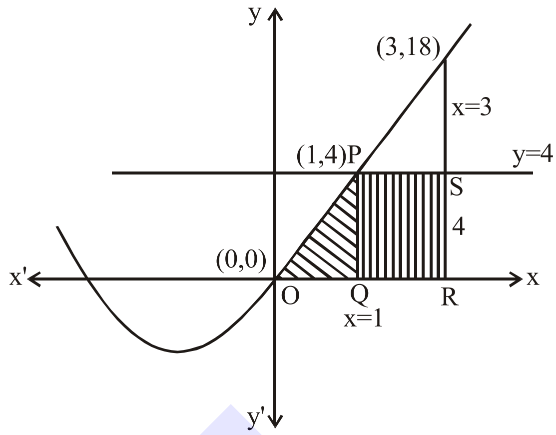
6. Ans. (2)



Req. area = $\displaystyle\int_0^3 (x^2+2)dx - \frac{1}{2}.5.3 = 9 + 6 - \frac{15}{2} = \frac{15}{2}$

7. Official Ans. by NTA (2)

Sol.



Required Area

$= \displaystyle\int_0^1 (x^2 + 3x)dx + \text{Area of rectangle PQRS}$

$= \dfrac{11}{6} + 8 = \dfrac{59}{6}$

8. Official Ans. by NTA (2)

Sol. $S(\alpha) = \{(x,y) : y^2 \le x, 0 \le x \le \alpha\}$

$A(\alpha) = 2\displaystyle\int_0^\alpha \sqrt{x}dx = 2\alpha^{\frac{3}{2}}$

$A(4) = 2 \times 4^{3/2} = 16$

$A(\lambda) = 2 \times \lambda^{3/2}$

$\dfrac{A(\lambda)}{A(4)} = \dfrac{2}{5} \Rightarrow \lambda = 4.\left(\dfrac{4}{25}\right)^{1/3}$

9. Official Ans. by NTA (2)

Sol. $x^2 \le y \le x + 2$

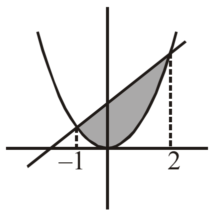
$x^2 = y$; $y = x + 2$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x - 2)(x - 1) = 0$

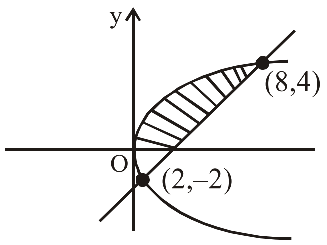
$x = 2, -1$



Area = $\displaystyle\int_{-1}^{2} (x+2) - x^2 \, dx = \frac{9}{2}$

10. Official Ans. by NTA (2)

Sol.



$y^2 = 2x$

$x - y - 4 = 0$

$(x - 4)^2 = 2x$

$x^2 + 16 - 8x - 2x = 0$

$x^2 - 10x + 16 = 0$

$x = 8, 2$

$y = 4, -2$

$$A = \int_{-2}^{4}\left(y + 4 - \frac{y^2}{2}\right)dy$$

$$= \frac{y^2}{2}\Big|_{-2}^{4} + 4y\Big|_{-2}^{4} - \frac{y^3}{6}\Big|_{-2}^{4}$$

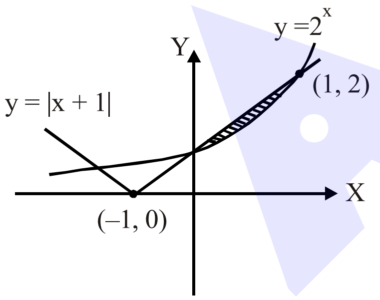
$$= (8-2) + 4(6) - \frac{1}{6}(64+8)$$

$= 6 + 24 - 12 = 18$

11. Official Ans. by NTA (1)

Sol. Required Area

$$\int_0^1\left((x+1) - 2^x\right)dx$$



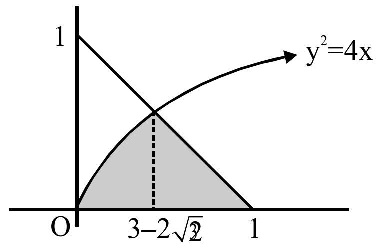
$$= \left(\frac{x^2}{2} + x - \frac{2^x}{\ln 2}\right)_0^1$$

$$= \left(\frac{1}{2} + 1 - \frac{2}{\ln 2}\right) - \left(0 + 0 - \frac{1}{\ln 2}\right)$$

$$= \frac{3}{2} - \frac{1}{\ln 2}$$

12. Official Ans. by NTA (3)

Sol. $\{(x, y) : y^2 \le 4x,\ x + y \le 1,\ x \ge 0,\ y \ge 0\}$



$$A \int_0^{3-2\sqrt{2}} 2\sqrt{x}dx + \frac{1}{2}\left(1 - \left(3 - 2\sqrt{2}\right)\right)\left(1 - (3 - 2\sqrt{2})\right)$$

$$= \frac{2[x^{3/2}]_0^{3-2\sqrt{2}}}{3/2} + \frac{1}{2}\left(2\sqrt{2} - 2\right)\left(2\sqrt{2} - 2\right)$$

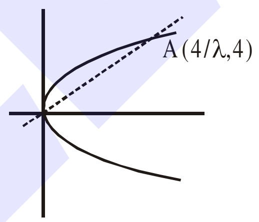
$$= \frac{8\sqrt{2}}{3} + \left(-\frac{10}{3}\right)$$

$a = \frac{8}{3}$, $b = -\frac{10}{3}$

$a - b = 6$

13. Official Ans. by NTA (1)

Sol.



$$\text{Area } = \frac{1}{9} = \int_0^{\frac{4}{\lambda}}\left(\sqrt{4\lambda x} - \lambda x\right)dx$$

$\Rightarrow \lambda = 24$