3D

1. The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y-axis also passes through the point :

(1) (–3, 0, –1) (2) (3, 3, –1)
(3) (3, 2, 1) (4) (–3, 1, 1)

2. The equation of the line passing through (–4, 3, 1), parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is:

(1) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

(2) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(3) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

(4) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

3. The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is:

(1) $x + 2y - 2z = 0$ (2) $x - 2y + z = 0$
(3) $5x + 2y - 4z = 0$ (4) $3x + 2y - 3z = 0$

4. If the lines $x = ay+b$, $z = cy + d$ and $x=a'z + b'$, $y = c'z + d'$ are perpendicular, then:

(1) $cc' + a + a' = 0$ (2) $aa' + c + c' = 0$
(3) $ab' + bc' + 1 = 0$ (4) $bb' + cc' + 1 = 0$

5. The plane passing through the point (4, –1, 2) and parallel to the lines $\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2}$ and $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$ also passes through the point :

(1) (–1, –1, –1) (2) (–1, –1, 1)
(3) (1, 1, –1) (4) (1, 1, 1)

6. Let A be a point on the line $\vec{r} = (1-3\mu)\hat{i} + (\mu-1)\hat{j} + (2+5\mu)\hat{k}$ and B(3, 2, 6) be a point in the space. Then the value of μ for which the vector $\overline{AB}$ is parallel to the plane $x - 4y + 3z = 1$ is :

(1) $\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) $\frac{1}{4}$ (4) $\frac{1}{8}$

7. The plane which bisects the line segment joining the points (–3,–3,4) and (3,7,6) at right angles, passes through which one of the following points ?

(1) (4, –1,7) (2) (4,1,–2)
(3) (–2,3,5) (4) (2,1,3)

8. On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane, $x + y + z = 2$?

(1) $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$

(2) $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$

(3) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$

(4) $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$

9. The direction ratios of normal to the plane through the points (0, –1, 0) and (0, 0, 1) and making an anlge $\frac{\pi}{4}$ with the plane $y–z+5=0$ are:

(1) $2\sqrt{3}$, 1, –1 (2) 2, $\sqrt{2}$, $-\sqrt{2}$
(3) 2, –1, 1 (4) $\sqrt{2}$, 1, –1

10. The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also containing its projection on the plane $2x + 3y - z = 5$, contains which one of the following points ?

(1) (2, 0, –2) (2) (–2, 2, 2)
(3) (0, –2, 2) (4) (2, 2, 0)

11. If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to :-

(1) 5 (2) 17 (3) 12 (4) 7

12. Two lines $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates :-

(1) (2, 4, 7) (2) (–2, 4, 7)
(3) (2, –4, –7) (4) (2, –4, 7)

13. The perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is:

(1) $11/\sqrt{6}$ (2) $6\sqrt{11}$ (3) 11 (4) $11\sqrt{6}$

14. A tetrahedron has vertices P(1, 2, 1), Q(2, 1, 3), R(–1,1,2) and O(0, 0, 0). The angle between the faces OPQ and PQR is :

(1) $\cos^{-1}\left(\frac{9}{35}\right)$ (2) $\cos^{-1}\left(\frac{19}{35}\right)$

(3) $\cos^{-1}\left(\frac{17}{31}\right)$ (4) $\cos^{-1}\left(\frac{7}{31}\right)$

15. If an angle between the line, $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z-3}{-2}$ and the plane, $x-2y-kz=3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is:

(1) $-\frac{5}{3}$ (2) $\sqrt{\frac{3}{5}}$ (3) $\sqrt{\frac{5}{3}}$ (4) $-\frac{3}{5}$

16. Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point (–1, –1, 1). Then S is equal to :

(1) $\{\sqrt{3}\}$ (2) $\{\sqrt{3} - \sqrt{3}\}$
(3) {1 , –1} (4) {3, –3}

17. The length of the perpendicular from the point (2, –1, 4) on the straight line, $\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}$ is :

(1) less than 2
(2) greater than 3 but less than 4
(3) greater than 4
(4) greater than 2 but less than 3

18. The equation of a plane containing the line of intersection of the planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and passing through the point (1, 1, 0) is :

(1) $x + 3y + z = 4$ (2) $x - y - z = 0$
(3) $x - 3y - 2z = -2$ (4) $2x - z = 2$

19. The vector equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$ is :

(1) $\vec{r}\times(\hat{i}+\hat{k})+2=0$

(2) $\vec{r}.(\hat{i}-\hat{k})-2=0$

(3) $\vec{r}.(\hat{i}-\hat{k})+2=0$

(4) $\vec{r}\times(\hat{i}-\hat{k})+2=0$

20. If a point R(4,y,z) lies on the line segment joining the points P(2,–3,4) and Q(8,0,10), then the distance of R from the origin is :

(1) $2\sqrt{14}$ (2) 6
(3) $\sqrt{53}$ (4) $2\sqrt{21}$

21. A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$, also passes through the point

(1) $(-\sqrt{2},1,-4)$ (2) $(\sqrt{2},1,4)$
(3) $(\sqrt{2},-1,4)$ (4) $(-\sqrt{2},-1,-4)$

22. If the line, $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ meets the plane, $x+2y+3z=15$ at a point P, then the distance of P from the origin is

(1) $\frac{9}{2}$ (2) $2\sqrt{5}$ (3) $\frac{\sqrt{5}}{2}$ (4) $\frac{7}{2}$

23. The vertices B and C of a ΔABC lie on the line, $\frac{x+2}{3}=\frac{y-1}{0}=\frac{z}{4}$ such that BC = 5 units. Then the area (in sq. units) of this triangle, given that the point A(1, –1, 2), is :-

(1) $2\sqrt{34}$ (2) $\sqrt{34}$ (3) 6 (4) $5\sqrt{17}$

24. Let P be the plane, which contains the line of intersection of the planes, $x+y+z-6=0$ and $2x+3y+z+5=0$ and it is perpendicular to the xy-plane. Then the distance of the point (0, 0, 256) from P is equal to :-

(1) $63\sqrt{5}$ (2) $205\sqrt{5}$

(3) $17/\sqrt{5}$ (4) $11/\sqrt{5}$

25. If the system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x+3y+\lambda z=\mu,\ (\lambda,\mu\in R)$, has infinitely many solutions, then the value of $\lambda+\mu$ is :

(1) 12 (2) 10 (3) 9 (4) 7

26. Let A(3, 0, –1), B (2, 10, 6) and C(1, 2, 1) be the vertices of a triangle and M be the midpoint of AC. If G divides BM in the ratio, 2 : 1, then $\cos(\angle GOA)$ (O being the origin) is equal to :

(1) $\frac{1}{\sqrt{30}}$ (2) $\frac{1}{6\sqrt{10}}$

(3) $\frac{1}{\sqrt{15}}$ (4) $\frac{1}{2\sqrt{15}}$

27. If the length of the perpendicular from the point $(\beta, 0, \beta)$ $(\beta\neq 0)$ to the line, $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to :

(1) –1 (2) 2 (3) –2 (4) 1

28. If Q(0, –1, –3) is the image of the point P in the plane $3x - y + 4z = 2$ and R is the point (3, –1, –2), then the area (in sq. units) of ΔPQR is :

(1) $\frac{\sqrt{65}}{2}$ (2) $\frac{\sqrt{91}}{4}$ (3) $2\sqrt{13}$ (4) $\frac{\sqrt{91}}{2}$

29. A perpendicular is drawn from a point on the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}$ to the plane $x + y + z = 3$ such that the foot of the perpendicular Q also lies on the plane $x - y + z = 3$. Then the co-ordinates of Q are :

(1) (2, 0, 1) (2) (4, 0, –1)

(3) (–1, 0, 4) (4) (1, 0, 2)

30. If the plane $2x - y + 2z + 3 = 0$ has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes $4x - 2y + 4z + \lambda = 0$ and $2x - y + 2z + \mu = 0$, respectively, then the maximum value of $\lambda + \mu$ is equal to :

(1) 15 (2) 5

(3) 13 (4) 9

31. If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the palne $2x + 3y - z + 13 = 0$ at a point P and the plane $3x + y + 4z = 16$ at a point Q, then PQ is equal to :

(1) $2\sqrt{14}$ (2) $\sqrt{14}$ (3) $2\sqrt{7}$ (4) 14

32. The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines

$\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+2\hat{j}-\hat{k})$ and

$\vec{r}=(\hat{i}+\hat{j})+\mu(-\hat{i}+\hat{j}-2\hat{k})$ is :

(1) $\sqrt{3}$ (2) $\frac{1}{\sqrt{3}}$

(3) $\frac{1}{3}$ (4) 3

33. A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point:

(1) (2,4,1)

(2) (2, – 4, 1)

(3) (1, 4, – 1)

(4) (1, – 4, 1)

SOLUTION

1. Ans. (3)

Equation of plane

$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$

$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z - 1 + 4\lambda = 0$

dr's of normal of the plane are

$1 + 2\lambda,\ 1 + 3\lambda,\ 1 - \lambda$

Since plane is parallel to y - axis, $1 + 3\lambda = 0$

$\Rightarrow \lambda = -1/3$

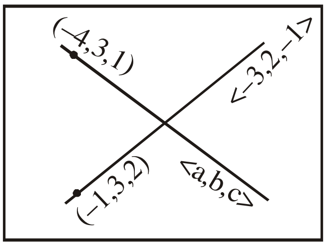
So the equation of plane is

$x + 4z - 7 = 0$

Point (3, 2, 1) satisfies this equation

Hence Answer is (3)

2. Ans. (2)



Normal vector of plane containing two intersecting lines is parallel to vector.

$$(\vec{V}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 1 \\ -3 & 2 & -1 \end{vmatrix}$$

$= -2\hat{i} + 6\hat{k}$

$\therefore$ Required line is parallel to vector

$$(\vec{V}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 0 & 6 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$$

$\Rightarrow$ Required equation of line is

$$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$

3. Ans. (2)

Vector along the normal to the plane containing the lines

$\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$

is $\left(8\hat{i} - \hat{j} - 10\hat{k}\right)$

vector perpendicular to the vectors $2\hat{i} + 3\hat{j} + 4\hat{k}$ and $8\hat{i} - \hat{j} - 10\hat{k}$ is $26\hat{i} - 52\hat{j} + 26\hat{k}$

so, required plane is

$26x - 52y + 26z = 0$

$x - 2y + z = 0$

4. Ans. (2)

Line $x = ay + b,\ z = cy + d \Rightarrow \frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$

Line $x = a'z + b',\ y = c'z + d'$

$\Rightarrow \frac{x-b'}{a'} = \frac{y-d'}{c'} = \frac{z}{1}$

Given both the lines are perpendicular

$\Rightarrow aa' + c' + c = 0$

5. Ans. (4)

Let $\vec{n}$ be the normal vector to the plane passing through (4,–1, 2) and parallel to the lines L_1 & L_2

then $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$

$\therefore\ \vec{n} = -7\hat{i} - 7\hat{j} + 7\hat{k}$

$\therefore$ Equation of plane is

$-1(x - 4) - 1(y + 1) + 1(z - 2) = 0$

$\therefore\ x + y - z - 1 = 0$

Now check options

6. Ans. (3)

Let point A is

$(1 - 3\mu)\hat{i} + (\mu - 1)\hat{j} + (2 + 5\mu)\hat{k}$

and point B is (3, 2, 6)

then $\overrightarrow{AB} = (2 + 3\mu)\hat{i} + (3 - \mu)\hat{j} + (4 - 5\mu)\hat{k}$

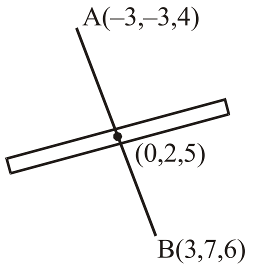
which is parallel to the plane $x - 4y + 3z = 1$

$\therefore\ 2 + 3\mu - 12 + 4\mu + 12 - 15\mu = 0$

$8\mu = 2$

$\mu = \frac{1}{4}$

7. Ans. (2)



$\vec{n} = 3\hat{i} + 5\hat{j} + \hat{k}$

p : 3(x – 0) + 5 (y – 2) + 1 (z – 5) = 0

3x + 5y + z = 15

$\therefore$ Option (2)

8. Ans. (3)

General point on the given line is

$x = 2\lambda + 4$

$y = 2\lambda + 5$

$z = \lambda + 3$

Solving with plane,

$2\lambda + 4 + 2\lambda + 5 + \lambda + 3 = 2$

$5\lambda + 12 = 2$

$5\lambda = -10$

$\boxed{\lambda = -2}$

$\therefore$ Option (3)

9. Ans. (2, 4)

Let the equation of plane be

a(x – 0) + b(y + 1) + c(z – 0) = 0

It passes through (0,0,1) then

b + c = 0 ...(1)

Now $\cos\frac{\pi}{4} = \frac{a(0)+b(1)+c(-1)}{\sqrt{2}\sqrt{a^2+b^2+c^2}}$

$\Rightarrow a^2 = -2bc$ and $b = -c$

we get $a^2 = 2c^2$

$\Rightarrow a = \pm\sqrt{2}\,c$

$\Rightarrow$ direction ratio (a, b, c) = $\left(\sqrt{2}, -1, 1\right)$ or $\left(\sqrt{2}, 1, -1\right)$

10. Ans. (1)

The normal vector of required plane

$= \left(2\hat{i} - \hat{j} + 3\hat{k}\right)\times\left(2\hat{i} + 3\hat{j} - \hat{k}\right)$

$= -8\hat{i} + 8\hat{j} + 8\hat{k}$

So, direction ratio of normal is (–1, 1, 1)

So required plane is

–(x – 3) + (y + 2) + (z – 1) = 0

$\Rightarrow$ –x + y + z + 4 = 0

Which is satisfied by (2, 0, –2)

11. Ans. (4)

Normal vector of plane

$$= \begin{vmatrix} i & j & k \\ 2 & -5 & 0 \\ 4 & -4 & 4 \end{vmatrix} = -4\left(5\hat{i} + 2\hat{j} - 3\hat{k}\right)$$

equation of plane is 5(x–7)+ 2y–3(z– 6) = 0

5x + 2y – 3z = 17

12. Ans. (3)

Point on L_1 $(\lambda + 3, 3\lambda - 1, -\lambda + 6)$

Point on L_2 $(7\mu - 5, -6\mu + 2, 4\mu + 3)$

$\Rightarrow \lambda + 3 = 7\mu - 5$ ...(i)

$3\lambda - 1 = -6\mu + 2$ ...(ii)

$\Rightarrow \lambda = -1, \mu = 1$

point R(2,–4,7)

Reflection is (2,–4,–7)

13. Ans. (1)

$$\begin{vmatrix} i & j & k \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$$

$\hat{i}(35-28) - \hat{j}(21.7) + \hat{k}(12-5)$

$7\hat{i} - 14\hat{j} + 7\hat{k}$

$\hat{i} - 2\hat{j} + \hat{k}$

1(x + 2) – 2(y – 2) + 1 (z+15) = 0

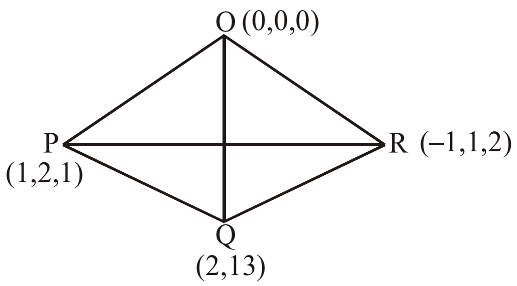
x – 2y + z + 11 = 0

$$\frac{11}{\sqrt{4+1+1}} = \frac{11}{\sqrt{6}}$$

14. Ans. (2)

$\overrightarrow{OP}\times\overrightarrow{OQ}=(\hat{i}+2\hat{j}+\hat{k})\times(2\hat{i}+\hat{j}+3\hat{k})$

$5\hat{i}-\hat{j}-3\hat{k}$



$\overrightarrow{PQ}\times\overrightarrow{PR}=(\hat{i}-\hat{j}+2\hat{k})\times(-2\hat{i}-\hat{j}+\hat{k})$

$\hat{i}-5\hat{j}-3\hat{k}$

$$\cos\theta=\frac{5+5+9}{\left(\sqrt{25+9+1}\right)^2}=\frac{19}{35}$$

15. Ans (3)

DR's of line are 2, 1, –2

normal vector of plane is $\hat{i}-2\hat{j}-k\hat{k}$

$$\sin\alpha=\frac{(2\hat{i}+\hat{j}-2\hat{k}).(\hat{i}-2\hat{j}-k\hat{k})}{3\sqrt{1+4+k^2}}$$

$$\sin\alpha=\frac{2k}{3\sqrt{k^2+5}} \quad ....(1)$$

$$\cos\alpha=\frac{2\sqrt{2}}{3} \quad ....(2)$$

$(1)^2+(2)^2=1\Rightarrow k^2=\frac{5}{3}$

16. Ans (2)

All four points are coplaner so

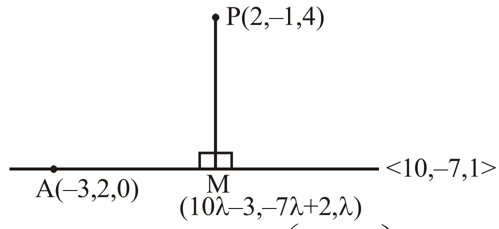
$$\begin{vmatrix}1-\lambda^2 & 2 & 0\\ 2 & -\lambda^2+1 & 0\\ 2 & 2 & -\lambda^2-1\end{vmatrix}=0$$

$(\lambda^2+1)^2(3-\lambda^2)=0$

$\lambda=\pm\sqrt{3}$

17. Official Ans. by NTA (2)

Sol.



$$\equiv\left(2,\frac{-3}{2},\frac{1}{2}\right)$$

Now, $\overrightarrow{MP}.\left(10\hat{i}-7\hat{j}+\hat{k}\right)=0$

$\Rightarrow\quad\lambda=\frac{1}{2}$

$\therefore$ Length of perpendicular

$$\left(=PM\right)=\sqrt{0+\frac{1}{4}+\frac{49}{4}}$$

$$=\sqrt{\frac{50}{4}}=\sqrt{\frac{25}{2}}=\frac{5}{\sqrt{2}},$$

which is greater than 3 but less than 4.

18. Official Ans. by NTA (2)

Sol. The required plane is

$(2x-y-4)+\lambda(y+2z-4)=0$

it passes through (1, 1, 0)

$\Rightarrow\quad(2-1-4)+\lambda(1-4)=0$

$\Rightarrow\quad-3-3\lambda=0\quad\Rightarrow\quad\lambda=-1$

$\Rightarrow\quad x-y-z=0$

19. Official Ans. by NTA (3)

Sol. Let the plane be

$(x+y+z-1)+\lambda(2x+3y+4z-5)=0$

$\Rightarrow(2\lambda+1)x+(3\lambda+1)y+(4\lambda+1)z-(5\lambda+1)=0$

$\perp$ to the plane $x-y+z=0$

$\Rightarrow\lambda=-\frac{1}{3}$

$\Rightarrow$ the required plane is $x-z+2=0$

20. Official Ans. by NTA (1)

Sol. $\frac{4}{2}=\frac{-y}{y+3}=\frac{10-z}{z-4}$

$\Rightarrow z=6\ \&\ y=-2$

$\Rightarrow R(4,-2,6)$

dist. from origin $=\sqrt{16+4+36}=2\sqrt{14}$

21. Official Ans. by NTA (2)

Sol. Let $ax + by + cz = 1$ be the equation of the plane

$\Rightarrow 0 - b + 0 = 1$

$\Rightarrow b = -1$

$0 + 0 + c = 1$

$\Rightarrow c = 1$

$$\cos\theta = \left|\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right|$$

$$\frac{1}{\sqrt{2}} = \frac{|0-1-1|}{\sqrt{(a^2+1+1)}\sqrt{0+1+1}}$$

$\Rightarrow a^2 + 2 = 4$

$\Rightarrow a = \pm\sqrt{2}$

$\Rightarrow \pm\sqrt{2}x - y + z = 1$

Now for –sign

$-\sqrt{2}.\sqrt{2} - 1 + 4 = 1$

option (2)

22. Official Ans. by NTA (1)

Sol. Any point on the given line can be

$(1 + 2\lambda, -1 + 3\lambda, 2 + 4\lambda)$; $\lambda \in R$

Put in plane

$1 + 2\lambda + (-2 + 6\lambda) + (6 + 12\lambda) = 15$

$20\lambda + 5 = 15$

$20\lambda = 10$

$$\lambda = \frac{1}{2}$$

$\therefore$ Point $\left(2, \frac{1}{2}, 4\right)$

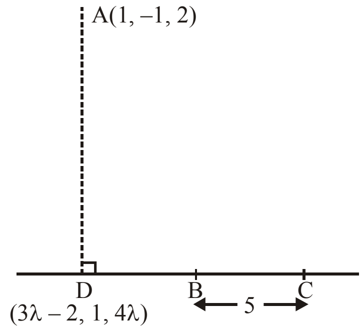
Distance from origin

$$= \sqrt{4 + \frac{1}{4} + 16} = \frac{\sqrt{16+1+64}}{2} = \frac{\sqrt{81}}{2}$$

$$= \frac{9}{2}$$

23. Official Ans. by NTA (2)

Sol.



$\overrightarrow{AD} \cdot (3\hat{i} + 4\hat{k}) = 0$

$3(3\lambda - 3) + 0 + 4(4\lambda - 2) = 0$

$(9\lambda - 9) + (16\lambda - 8) = 0$

$$25\lambda = 17 \Rightarrow \lambda = \frac{17}{25}$$

$$\therefore \overrightarrow{AD} = \left(\frac{51}{25} - 3\right)\hat{i} + 2\hat{j} + \left(\frac{68}{25} - 2\right)\hat{k}$$

$$= \frac{24}{25}\hat{i} + 2\hat{j} + \frac{18}{25}\hat{k}$$

$$|\overrightarrow{AD}| = \sqrt{\frac{576}{625} + 4 + \frac{324}{625}}$$

$$= \sqrt{\frac{900}{625} + 4} = \sqrt{\frac{3400}{625}}$$

$$= \sqrt{34} \cdot \frac{10}{25} = \frac{2}{5}\sqrt{34}$$

Area of $\Delta = \frac{1}{2} \times 5 \times \frac{2\sqrt{34}}{5} = \sqrt{34}$

24. Official Ans. by NTA (4)

Sol. $\lambda(x + y + z - 6) + 2x + 3y + z + 5 = 0$

$(\lambda + 2)x + (\lambda + 3)y + (\lambda + 1)z + 5 - 6\lambda = 0$

$\lambda + 1 = 0 \Rightarrow \lambda = -1$

$P : x + 2y + 11 = 0$

perpendicular distance $= \frac{11}{\sqrt{5}}$

25. Official Ans. by NTA (2)

Sol. $x + 3y + \lambda z - \mu = p(x + y + z - 5) + q(x + 2y + 2z - 6)$

on comparing the coefficient;

$p + q = 1$ and $p + 2q = 3$

$\Rightarrow (p,q) = (-1,2)$

Hence $x + 3y + \lambda z - \mu = x + 3y + 3z - 7$

$\Rightarrow \lambda = 3, \mu = 7$

26. Official Ans. by NTA (3)

Sol. G is the centroid of Δ ABC

G $\equiv$ (2,4,2)

$\overrightarrow{OG} = 2\hat{i} + 4\hat{j} + 2\hat{k}$

$\overrightarrow{OA} = 3\hat{i} - \hat{k}$

$\cos(\angle GOA) = \frac{\overrightarrow{OG}\cdot\overrightarrow{OA}}{|\overrightarrow{OG}||\overrightarrow{OA}|} = \frac{1}{\sqrt{15}}$

27. Official Ans. by NTA (1)

Sol. One of the point on line is P(0, 1, –1) and given point is Q(β, 0, β).

So, $\overrightarrow{PQ} = \beta\hat{i} - \hat{j} + (\beta+1)\hat{k}$

Hence, $\beta^2 + 1 + (\beta+1)^2 - \frac{(\beta-\beta-1)^2}{2} = \frac{3}{2}$

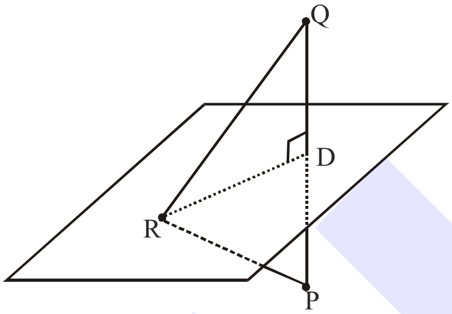
$\Rightarrow 2\beta^2 + 2\beta = 0$

$\Rightarrow \beta = 0, -1$

$\Rightarrow \beta = -1$ (as $\beta \neq 0$)

28. Official Ans. by NTA (4)

Sol. R lies on the plane.



$DQ = \frac{|1-12-2|}{\sqrt{9+1+16}} = \frac{13}{\sqrt{26}} = \sqrt{\frac{13}{2}}$

$\Rightarrow PQ = \sqrt{26}$

Now, $RQ = \sqrt{9+1} = \sqrt{10}$

$\Rightarrow RD = \sqrt{10 - \frac{13}{2}} = \sqrt{\frac{7}{2}}$

Hence, ar(ΔPQR) $= \frac{1}{2}\times\sqrt{26}\times\sqrt{\frac{7}{2}} = \frac{\sqrt{91}}{2}$.

29. Official Ans. by NTA (1)

Sol. Let point P on the line is $(2\lambda+1, -\lambda-1, \lambda)$ foot of perpendicular Q is given by

$\frac{x-2\lambda-1}{1} = \frac{y+\lambda+1}{1} = \frac{z-\lambda}{1} = \frac{-(2\lambda-3)}{3}$

$\because$ Q lies on x + y + z = 3 & x – y + z = 3

$\Rightarrow$ x + z = 3 & y = 0

$y = 0 \Rightarrow \lambda + 1 = \frac{-2\lambda+3}{3} \Rightarrow \lambda = 0$

$\Rightarrow$ Q is (2, 0, 1)

30. Official Ans. by NTA (3)

Sol. 4x – 2y + 4z + 6 = 0

$\frac{|\lambda-6|}{\sqrt{16+4+16}} = \left|\frac{\lambda-6}{6}\right| = \frac{1}{3}$

$|\lambda - 6| = 2$

$\lambda = 8, 4$

$\frac{|\mu-3|}{\sqrt{4+4+1}} = \frac{2}{3}$

$|\mu - 3| = 2$

$\mu = 5, 1$

$\therefore$ Maximum value of $(\mu + \lambda) = 13$.

31. Official Ans. by NTA (1)

Sol. $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1} = \lambda$

$x = 3\lambda + 2,\ y = 2\lambda - 1,\ z = -\lambda + 1$

Intersection with plane 2x + 3y – z + 13 = 0

$2(3\lambda + 2) + 3(2\lambda - 1) - (-\lambda + 1) + 13 = 0$

$13\lambda + 13 = 0$ $\boxed{\lambda = -1}$

$\therefore$ P(–1, –3, 2)

Intersection with plane

3x + y + 4z = 16

$3(3\lambda+2) + (2\lambda-1) + 4(-\lambda+1) = 16$

$\lambda = 1$

Q(5, 1, 0)

$PQ = \sqrt{6^2+4^2+2^2} = \sqrt{56} = 2\sqrt{14}$

32. Official Ans. by NTA (1)

Sol. perpendicular vector to the plane

$$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = -3\hat{i} + 3\hat{j} + 3\hat{k}$$

Eq. of plane

$-3(x-1) + 3(y-1) + 3z = 0$

$\Rightarrow x - y - z = 0$

$$d_{(2,1,4)} = \frac{|2-1-4|}{\sqrt{1^2+1^2+1^2}} = \sqrt{3}$$

33. Official Ans. by NTA (2)

Sol. equation of bisector of angle

$$\frac{2x-y+2z-4}{3} = \pm\frac{x+2y+2z-2}{3}$$

(+) gives $x - 3y = 2$

(–) gives $3x + y + 4z = 6$

therefore option (ii) satisfy