ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

Chapter 03 Contents

JEE (MAIN) TOPICWISE SOLUTION OF TEST PAPERS JANUARY & APRIL 2019

MATHEMATICS

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JEE (MAIN) TOPICWISE SOLUTION OF TEST PAPERS JANUARY & APRIL 2019

MATHEMATICS

JANUARY AND APRIL 2019 ATTEMPT (MATHEMATICS)

COMPOUND ANGLE

1. Ans. (1)

We have,

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$

$= 3(1 - \sin 2\theta)^2 + 6(1 + \sin 2\theta) + 4\sin^6\theta$

$= 3(1 - 2\sin 2\theta + \sin^2 2\theta) + 6 + 6\sin 2\theta + 4\sin^6\theta$

$= 9 + 12\sin^2\theta \cdot \cos^2\theta + 4(1 - \cos^2\theta)^3$

$= 13 - 4\cos^6\theta$

2. Ans. (3)

$$2\sin\frac{\pi}{2^{10}}\cos\frac{\pi}{2^{10}}\ldots\ldots\cos\frac{\pi}{2^2}$$

$$\frac{1}{2^9}\sin\frac{\pi}{2} = \frac{1}{512}$$

Option (3)

3. Ans. (4)

$f_4(x) - f_6(x)$

$$= \frac{1}{4}\left(\sin^4 x + \cos^4 x\right) - \frac{1}{6}\left(\sin^6 x + \cos^6 x\right)$$

$$= \frac{1}{4}\left(1 - \frac{1}{2}\sin^2 2x\right) - \frac{1}{6}\left(1 - \frac{3}{4}\sin^2 2x\right) = \frac{1}{12}$$

4. Ans. (1)

$$y = 3\cos\theta + 5\left(\sin\theta\frac{\sqrt{3}}{2} - \cos\theta\frac{1}{2}\right)$$

$$\frac{5\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta$$

$$y_{max} = \sqrt{\frac{75}{4} + \frac{1}{4}} = \sqrt{19}$$

5. Official Ans. by NTA (4)

Sol. $0 < \alpha + \beta = \frac{\pi}{2}$ and $\frac{-\pi}{4} < \alpha - \beta < \frac{\pi}{4}$

if $\cos(\alpha + \beta) = \frac{3}{5}$ then $\tan(\alpha + \beta) = \frac{4}{3}$

and if $\sin(\alpha - \beta) = \frac{5}{13}$ then $\tan(\alpha - \beta) = \frac{5}{12}$

(since $\alpha - \beta$ here lies in the first quadrant)

Now $\tan(2\alpha) = \tan\{(\alpha + \beta) + (\alpha - \beta)\}$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta).\tan(\alpha - \beta)} = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3}.\frac{5}{12}} = \frac{63}{16}$$

6. Official Ans. by NTA (2)

Sol. $\frac{1}{2}\left(2\cos^2 10° - 2\cos 10°\cos 50° + 2\cos^2 50°\right)$

$$\Rightarrow \frac{1}{2}\left(1 + \cos 20° - \left(\cos 60° + \cos 40°\right) + 1 + \cos 100°\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{3}{2} + \cos 20° + 2\sin 70°\sin\left(-30°\right)\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{3}{2} + \cos 20° - \sin 70°\right)$$

$\Rightarrow \frac{3}{4}$ Ans. (2)

7. Official Ans. by NTA (4)

Sol. (sin 10º sin 30º sin 70º) sin 30º

$$\frac{1}{4}(\sin 30°)^2 = \frac{1}{4}\cdot\frac{1}{4} = \frac{1}{16}$$

QUADRATIC EQUATION

1. Ans. (3)

We have

$(x + 1)^2 + 1 = 0$

$\Rightarrow (x + 1)^2 - (i)^2 = 0$

$\Rightarrow (x + 1 + i)(x + 1 - i) = 0$

$\therefore x = \underbrace{-(1+i)}_{\alpha(\text{let})}\ \underbrace{-(1-i)}_{\beta(\text{let})}$

So, $\alpha^{15} + \beta^{15} = (\alpha^2)^7\alpha + (\beta^2)^7\beta$

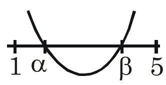
$= -128(-i + 1 + i + 1)$

$= -256$

2. Ans. (1)

$x^2 - mx + 4 = 0$

$\alpha, \beta \in [1,5]$



(1) $D > 0 \Rightarrow m^2 - 16 > 0$

$\Rightarrow m \in (-\infty,-4) \cup (4,\infty)$

(2) $f(1) \geq 0 \Rightarrow 5 - m \geq 0 \Rightarrow m \in (-\infty,5]$

(3) $f(5) \geq 0 \Rightarrow 29 - 5m \geq 0 \Rightarrow m \in \left(-\infty, \frac{29}{5}\right]$

(4) $1 < \frac{-b}{2a} < 5 \Rightarrow 1 < \frac{m}{2} < 5 \Rightarrow m \in (2,10)$

$\Rightarrow m \in (4,5)$

No option correct : Bonus

* If we consider $\alpha, \beta \in (1,5)$ then option (1) is correct.

3. **Ans. (3)**

$6x^2 - 11x + \alpha = 0$

given roots are rational

$\Rightarrow$ D must be perfect square

$\Rightarrow 121 - 24\alpha = \lambda^2$

$\Rightarrow$ maximum value of α is 5

$\alpha = 1 \Rightarrow \lambda \notin I$

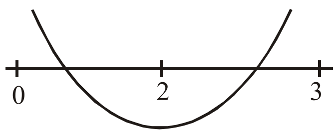
$\alpha = 2 \Rightarrow \lambda \notin I$

$\alpha = 3 \Rightarrow \lambda \in I$ $\Rightarrow$ 3 integral values

$\alpha = 4 \Rightarrow \lambda \in I$

$\alpha = 5 \Rightarrow \lambda \in I$

4. **Ans. (1)**



Let $f(x) = (c - 5)x^2 - 2cx + c - 4$

$\therefore f(0)f(2) < 0$ .....(1)

& $f(2)f(3) < 0$ .....(2)

from (1) & (2)

$(c - 4)(c - 24) < 0$

& $(c - 24)(4c - 49) < 0$

$\Rightarrow \frac{49}{4} < c < 24$

$\therefore$ s = {13, 14, 15, ... 23}

Number of elements in set S = 11

5. **Ans. (1)**

$\alpha + \beta = \lambda - 3$

$\alpha\beta = 2 - \lambda$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (\lambda - 3)^2 - 2(2 - \lambda)$

$= \lambda^2 + 9 - 6\lambda - 4 + 2\lambda$

$= \lambda^2 - 4\lambda + 5$

$= (\lambda - 2)^2 + 1$

$\therefore \lambda = 2$

Option (1)

6. **Ans. (3)**

$81x^2 + kx + 256 = 0$; $x = \alpha, \alpha^3$

$\Rightarrow \alpha^4 = \frac{256}{81} \Rightarrow \alpha = \pm\frac{4}{3}$

Now $-\frac{k}{81} = \alpha + \alpha^3 = \pm\frac{100}{27} \Rightarrow k = \pm 300$

7. **Ans. (1)**

$D = (1+ \sin\theta \cos\theta)^2 - 4\sin\theta\cos\theta$

$= (1 - \sin\theta \cos\theta)^2$

$\Rightarrow$ roots are $\beta = \operatorname{cosec}\theta$ and $\alpha = \cos\theta$

$\Rightarrow \sum_{n=0}^{\infty}\left(\alpha^n + \left(-\frac{1}{\beta}\right)^n\right) = \sum_{n=0}^{\infty}(\cos\theta)^n + \sum_{n=0}^{n}(-\sin\theta)^n$

$= \frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

8. **Ans. (2)**

$3m^2x^2 + m(m - 4)x + 2 = 0$

$\lambda + \frac{1}{\lambda} = 1$, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1$, $\alpha^2 + \beta^2 = \alpha\beta$

$(\alpha + \beta)^2 = 3\alpha\beta$

$\left(-\frac{m(m-4)}{3m^2}\right)^2 = \frac{3(2)}{3m^2}$, $\frac{(m-4)^2}{9m^2} = \frac{6}{3m}$

$(m - 4)^2 = 18$, $m = 4 \pm \sqrt{18},\ 4 \pm 3\sqrt{2}$

9. **Ans. (2)**

Exprsssion is always positve it

$2m + 1 > 0 \Rightarrow m > -\frac{1}{2}$ & $D < 0$

$\Rightarrow m^2 - 6m - 3 < 0$

$3 - \sqrt{12} < m < 3 + \sqrt{12}$ .... (iii)

$\therefore$ Common interval is

$3 - \sqrt{12} < m < 3 + \sqrt{12}$

$\therefore$ Intgral value of m {0,1,2,3,4,5,6}

10. **Official Ans. by NTA (3)**

Sol. $(x - 1)^2 + 1 = 0 \Rightarrow x = 1 + i, 1 - i$

$\therefore \left(\frac{\alpha}{\beta}\right)^n = 1 \Rightarrow (\pm i)^n = 1$

$\therefore$ n (least natural number) = 4

11. Official Ans. by NTA (3)

Sol. $\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0$

$\left|\sqrt{x}-2\right|+\left(\sqrt{x}\right)^2-4\sqrt{x}+2=0$

$\left|\sqrt{x}-2\right|^2+\left|\sqrt{x}-2\right|-2=0$

$\left|\sqrt{x}-2\right|=-2$ (not possible) or $\left|\sqrt{x}-2\right|=1$

$\sqrt{x}-2=1,-1$

$\sqrt{x}=3,1$

$x = 9, 1$

Sum = 10

12. Official Ans. by NTA (1)

Sol. $D<0$

$4(1+3m)^2-4(1+m^2)(1+8m)<0$

$\Rightarrow m(2m-1)^2>0 \Rightarrow m>0$

13. Official Ans. by NTA (2)

ALLEN Ans. (2) or (Bonus)

Sol. In given question p, q $\in$ R. If we take other root as any real number α, then quadratic equation will be

$x^2-(\alpha+2-\sqrt{3})x+\alpha.\left(2-\sqrt{3}\right)=0$

Now, we can have none or any of the options can be correct depending upon 'α'

Instead of p, q $\in$ R it should be p, q $\in$ Q then other root will be $2+\sqrt{3}$

$\Rightarrow p=-\left(2+\sqrt{3}-2-\sqrt{3}\right)=-4$

and $q=\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)=1$

$\Rightarrow p^2-4q-12=(-4)^2-4-12$

$=16-16=0$

Option (2) is correct

14. Official Ans. by NTA (4)

Sol. $SOR=\dfrac{3}{m^2+1}\Rightarrow (S.O.R)_{max}=3$

when $m = 0$

$x^2-3x+1=0 \rightarrow \alpha, \beta$

$\alpha+\beta=3$

$\alpha\beta=1$

$|\alpha^3-\beta^2| = ||\alpha-\beta|(\alpha^2+\beta^2+\alpha\beta)|$

$=\left|\sqrt{(\alpha-\beta)^2-\alpha\beta}\;\;((\alpha+\beta)^2-\alpha\beta)\right|$

$=\left|\sqrt{9-4}\,(9-1)\right|$

$=\sqrt{5}\times 8$

15. Official Ans. by NTA (4)

Sol. $\dfrac{\alpha^{12}+\beta^{12}}{\left(\dfrac{1}{\alpha^{12}}+\dfrac{1}{\beta^{12}}\right)(\alpha-\beta)^{24}}=\dfrac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$

$=\dfrac{(\alpha\beta)^{12}}{\left[(\alpha+\beta)^2-4\alpha\beta\right]^{12}}=\left[\dfrac{\alpha\beta}{(\alpha+\beta)^2-4\alpha\beta}\right]^{12}$

$=\left(\dfrac{-2\sin\theta}{\sin^2\theta+8\sin\theta}\right)^{12}=\dfrac{2^{12}}{(\sin\theta+8)^{12}}$

SEQUENCE & PROGRESSION

1. Ans. (4)

$\dfrac{b}{r}, b, br \rightarrow$ G.P. $\quad (|r| \neq 1)$

given $a+b+c=xb$

$\Rightarrow b/r+b+br=xb$

$\Rightarrow b=0$ (not possible)

or $1+r+\dfrac{1}{r}=x \Rightarrow x-1=r+\dfrac{1}{r}$

$\Rightarrow x-1>2$ or $x-1<-2$

$\Rightarrow x>3$ or $x<-1$

So x can't be '2'

2. **Ans. (4)**

$S = a_1 + a_2 + ... + a_{30}$

$S = \frac{30}{2}[a_1 + a_{30}]$

$S = 15(a_1 + a_{30}) = 15(a_1 + a_1 + 29d)$

$T = a_1 + a_3 + ... + a_{29}$

$= (a_1) + (a_1 + 2d) ..... + (a_1 + 28d)$

$= 15a_1 + 2d(1 + 2 + ... + 14)$

$T = 15a_1 + 210\,d$

Now use S – 2T = 75

$\Rightarrow 15(2a_1 + 29d) - 2(15a_1 + 210\,d) = 75$

$\Rightarrow d = 5$

Given $a_5 = 27 = a_1 + 4d \Rightarrow a_1 = 7$

Now $a_{10} = a_1 + 9d = 7 + 9 \times 5 = 52$

3. **Ans. (1)**

$$T_n = \frac{(3+(n-1)\times 3)(1^2+2^2+....+n^2)}{(2n+1)}$$

$$T_n = \frac{3.\frac{n(n+1)(2n+1)}{6}}{2n+1} = \frac{n^2(n+1)}{2}$$

$$S_{15} = \frac{1}{2}\sum_{n=1}^{15}(n^3+n^2) = \frac{1}{2}\left[\left(\frac{15(15+1)}{2}\right)^2 + \frac{15\times16\times31}{6}\right]$$

= 7820

4. **Ans. (2)**

a = A + 6d

b = A + 10d

c = A + 12d

a,b,c are in G.P.

$\Rightarrow (A + 10d)^2 = (A + 6d)(a + 12d)$

$$\Rightarrow \frac{A}{d} = -14$$

$$\frac{a}{c} = \frac{A+6d}{A+12d} = \frac{6+\frac{A}{d}}{12+\frac{A}{d}} = \frac{6-14}{12-14} = 4$$

5. **Ans. (3)**

$$\frac{a}{1-r} = 3 \qquad ...(1)$$

$$\frac{a^3}{1-r^3} = \frac{27}{19} \Rightarrow \frac{27(1-r)^3}{1-r^3} = \frac{27}{19}$$

$\Rightarrow 6r^2 - 13r + 6 = 0$

$$\Rightarrow r = \frac{2}{3} \text{ as } |r| < 1$$

6. **Ans. (3)**

$a_1, a_2, ..., a_{10}$ are in G.P.,

Let the common ratio be r

$$\frac{a_3}{a_1} = 25 \Rightarrow \frac{a_1r^2}{a_1} = 25 \Rightarrow r^2 = 25$$

$$\frac{a_9}{a_5} = \frac{a_1r^8}{a_1r^4} = r^4 = 5^4$$

7. **Ans. (1)**

$a + 18d = 0 \qquad ...(1)$

$$\frac{a+48d}{a+28d} = \frac{-18d+48d}{-18d+28d} = \frac{3}{1}$$

8. **Ans. (2)**

$$\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})} = \frac{1}{\left(x^m+\frac{1}{x^m}\right)\left(y^n+\frac{1}{y^n}\right)} \le \frac{1}{4}$$

using AM ≥ GM

9. **Ans. (4)**

Let terms are $\frac{a}{r}, a, ar \to$ G.P

$\therefore a^3 = 512 \Rightarrow a = 8$

$$\frac{8}{r}+4, 12, 8r \to \text{A.P.}$$

$$24 = \frac{8}{r} + 4 + 8r$$

$$r = 2,\ r = \frac{1}{2}$$

r = 2 (4, 8, 16)

$$r = \frac{1}{2}(16, 8, 4)$$

Sum = 28

10. **Ans. (1)**

$$S_K = \frac{K+1}{2}$$

$$\Sigma S_k^2 = \frac{5}{12}A$$

$$\sum_{K=1}^{10}\left(\frac{K+1}{2}\right)^2 = \frac{2^2+3^2+--+11^2}{4} = \frac{5}{12}A$$

$$\frac{11\times12\times23}{6}-1=\frac{5}{3}A$$

$$505=\frac{5}{3}A,\quad A=303$$

11. Ans (2)

A.M. ≥ G.M.

$$\frac{\sin^4\alpha+4\cos^4\beta+1+1}{4}\geq(\sin^4\alpha.4\cos^4\beta.1.1)^{\frac{1}{4}}$$

$\sin^4\alpha + 4\cos^2\beta + 2 \geq 4\sqrt{2}\sin\alpha\cos\beta$

given that $\sin^4\alpha + 4\cos^4\beta + 2$

$= 4\sqrt{2}\ \sin\alpha\cos\beta$

$\Rightarrow$ A.M. = G.M. $\Rightarrow \sin^4\alpha = 1 = 4\cos^4\beta$

$\sin\alpha = 1, \cos\beta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \sin\beta = \frac{1}{\sqrt{2}}$ as $\beta \in [0, \pi]$

$\cos(\alpha+\beta) - \cos(\alpha-\beta)$

$= -2\sin\alpha\sin\beta$

$= -\sqrt{2}$

12. Ans. (2)

$$S=\left(\frac{3}{4}\right)^3+\left(\frac{6}{4}\right)^3+\left(\frac{9}{4}\right)^3+\left(\frac{12}{4}\right)^3+............15\text{ term}$$

$$=\frac{27}{64}\sum_{r=1}^{15}r^3$$

$$=\frac{27}{64}.\left[\frac{15(15+1)}{2}\right]^2$$

= 225 K (Given in question)

K = 27

13. Official Ans. by NTA (2)

Sol. S_A = sum of numbers between 100 & 200 which are divisible by 7.

$\Rightarrow S_A = 105 + 112 + ... + 196$

$$S_A=\frac{14}{2}[105+196]=2107$$

S_B = Sum of numbers between 100 & 200 which are divisible by 13.

S_B = 104 + 117 + …. + 195 =

$$\frac{8}{2}[104+195]=1196$$

S_C = Sum of numbers between 100 & 200 which are divisible by both 7 & 13.

$S_C = 182$

$\Rightarrow$ H.C.F. (91, n) > 1 $= S_A + S_B - S_C =$ 3121

14. Official Ans. by NTA (2)

Sol. $S=\sum_{k=1}^{20}\frac{1}{2^k}$

$$S=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{3^2}+...+\frac{20}{2^{20}}$$

$$S\times\frac{1}{2}=\frac{1}{2^2}+\frac{2}{2^3}+...+\frac{19}{2^{20}}+\frac{20}{2^{21}}$$

$$\Rightarrow\left(1-\frac{1}{2}\right)S=\frac{1}{2}+\frac{1}{2^2}+...+\frac{1}{2^{20}}-\frac{20}{2^{21}}$$

$$\Rightarrow S=2-\frac{11}{2^{19}}$$

15. Official Ans. by NTA (3)

Sol. a, b, c in G.P.

say a, ar, ar^2

satisfies $ax^2 + 2bx + c = 0 \Rightarrow x = -r$

x = –r is the common root, satisfies second equation $d(-r)^2 + 2e(-r) + f = 0$

$$\Rightarrow d.\frac{c}{a}-\frac{2ce}{b}+f=0$$

$$\Rightarrow\frac{d}{a}+\frac{f}{c}=\frac{2e}{b}$$

16. Official Ans. by NTA (1)

Sol. $S_n=50n+\frac{n(n-7)}{2}A$

$T_n = S_n - S_{n-1}$

$$=50n+\frac{n(n-7)}{2}A-50(n-1)-\frac{(n-1)(n-8)}{2}A$$

$= 50 + \frac{A}{2}[n^2 - 7n - n^2 + 9n - 8]$

$= 50 + A(n - 4)$

$d = T_n - T_{n-1}$

$= 50 + A(n - 4) - 50 - A(n - 5)$

$= A$

$T_{50} = 50 + 46A$

$(d, A_{50}) = (A, 50+46A)$

17. Official Ans. by NTA (1)

Sol. $a - d + a + a + d = 33 \Rightarrow a = 11$

$a(a^2 - d^2) = 1155$

$121 - d^2 = 105$

$d^2 = 16 \Rightarrow d = \pm 4$

If d = 4 then I[st] term = 7

If d = –4 then I[st] term = 15

$T_{11} = 7 + 40 = 47$

OR $T_{11} = 15 - 40 = -25$

18. Official Ans. by NTA (2)

Sol. $T_r = r(2r - 1)$

$S = \Sigma 2r^2 - \Sigma r$

$$S = \frac{2.n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

$$S_{11} = \frac{2}{6}.(11)(12)(23) - \frac{11(12)}{2} = (44)(23) - 66 = 946$$

19. Official Ans. by NTA (1)

Sol. $$T_n = \frac{(3+(n-1)\times 2)(1^3+2^3+\ldots+n^3)}{(1^2+2^2+\ldots+n^2)}$$

$$= \frac{3}{2}n(n+1) = \frac{n(n+1)(n+2)-(n-1)n(n+1)}{2}$$

$$\Rightarrow S_n = \frac{n(n+1)(n+2)}{2}$$

$\Rightarrow S_{10} = 660$

20. Official Ans. by NTA (3)

Sol. $a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 114$

$$\Rightarrow \frac{6}{2}(a_1 + a_{16}) = 114$$

$\Rightarrow a_1 + a_{16} = 38$

$$\text{So, } a_1 + a_6 + a_{11} + a_{16} = \frac{4}{2}(a_1 + a_{16})$$

$= 2 \times 38 = 76$

21. Official Ans. by NTA (4)

Sol. $$\text{Sum} = \sum_{n=1}^{15} \frac{1^3+2^3+\ldots.n^3}{1+2+\ldots.+n} - \frac{1}{2}.\frac{15.16}{2}$$

$$= \sum_{n=1}^{15} \frac{n(n+1)}{2} - 60$$

$$= \sum_{n=1}^{15} \frac{n(n+1)(n+2-(n-1))}{6} - 60$$

$$= \frac{15.16.17}{6} - 60 = 620$$

22. Official Ans. by NTA (2)

Sol. $b = ar$

$c = ar^2$

3a, 7b and 15 c are in A.P.

$\Rightarrow 14b = 3a + 15c$

$\Rightarrow 14(ar) = 3a + 15\,ar^2$

$\Rightarrow 14r = 3 + 15r^2$

$\Rightarrow 15r^2 - 14r + 3 = 0 \Rightarrow (3r-1)(5r-3) = 0$

$r = \frac{1}{3}, \frac{3}{5}$.

Only acceptable value is $r = \frac{1}{3}$, because

$r \in \left(0, \frac{1}{2}\right]$

$\therefore$ c. d $= 7b - 3a = 7ar - 3a = \frac{7}{3}a - 3a = -\frac{2}{3}a$

$\therefore$ 4[th] term $= 15\,c - \frac{2}{3}a = \frac{15}{9}a - \frac{2}{3}a = a$

23. Official Ans. by NTA (3)

Sol. $375x^2 - 25x - 2 = 0$

$$\alpha+\beta = \frac{25}{375},\quad \alpha\beta = \frac{-2}{375}$$

$\Rightarrow$ ($\alpha + \alpha^2$ + ... upto infinite terms) + ($\beta + \beta^2$ + ... upto infinite terms)

$$= \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta} = \frac{1}{12}$$

24. Official Ans. by NTA (1)

Sol. $2\{2a+3d\} = 16$

$3(2a + 5d) = -48$

$2a + 3d = 8$

$2a + 5d = -16$

$\boxed{d = -12}$

$S_{10} = 5\,\{44 - 9 \times 12\}$

$= -320$

25. Official Ans. by NTA (1)

Sol. $a_1 + a_7 + a_{16} = 40$

$a + a + 6d + a + 15d = 40$

$\Rightarrow 3a + 21d = 40 \quad \Rightarrow \boxed{a + 7d = \frac{40}{3}}$

$$S_{15}=\frac{15}{2}(2a+14d)=15(a+7d)$$

$$S_{15}=15\times\frac{40}{3}\Rightarrow 200 \quad \boxed{S_{15}=200}$$

26. Official Ans. by NTA (1)

Sol. $\alpha x^2 + 2\beta x + \gamma = 0$

Let $\beta = \alpha t$, $\gamma = \alpha t^2$

$\therefore\ \alpha x^2 + 2\alpha tx + \alpha t^2 = 0$

$\Rightarrow x^2 + 2tx + t^2 = 0$

$\Rightarrow (x + t)^2 = 0$

$\Rightarrow x = -t$

it must be root of equation $x^2 + x - 1 = 0$

$\therefore\ t^2 - t - 1 = 0$ (1)

Now

$\alpha(\beta + \gamma) = \alpha^2(t + t^2)$

Option 1 $\beta\gamma = \alpha t\ .\ \alpha t^2 = \alpha^2 t^3 = a^2 (t^2 + t)$

(from equation 1)

TRIGONOMETRIC EQUATION

1. Ans. (1)

$\sin x - \sin 2x + \sin 3x = 0$

$\Rightarrow (\sin x + \sin 3x) - \sin 2x = 0$

$\Rightarrow 2\sin x.\cos x - \sin 2x = 0$

$\Rightarrow \sin 2x(2\cos x - 1) = 0$

$\Rightarrow \sin 2x = 0$ or $\cos x=\frac{1}{2}$

$\Rightarrow x=0,\frac{\pi}{3}$

2. Ans. (1)

$\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4},\ \theta\in\left(0,\frac{\pi}{2}\right)$

$\Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow 4\cos^4 2\theta - 4\cos^2 2\theta + 1 = 0$

$\Rightarrow (2\cos^2 2\theta - 1)^2 = 0$

$\Rightarrow \cos^2 2\theta=\frac{1}{2}=\cos^2\frac{\pi}{4}$

$\Rightarrow 2\theta=n\pi\pm\frac{\pi}{4}$, $n\in I$

$\Rightarrow \theta=\frac{n\pi}{2}\pm\frac{\pi}{8}$

$\Rightarrow \theta=\frac{\pi}{8},\frac{\pi}{2}-\frac{\pi}{8}$

Sum of solutions $\frac{\pi}{2}$

3. Official Ans. by NTA (3)

Sol. $2(1-\sin^2\theta)+3\sin\theta=0$

$\Rightarrow 2\sin^2\theta-3\sin\theta-2=0$

$\Rightarrow (2\sin\theta+1)(\sin\theta-2)=0$

$\Rightarrow \sin\theta=-\frac{1}{2};\sin\theta=2(\text{reject})$

roots : $\pi+\frac{\pi}{6},2\pi-\frac{\pi}{6},-\frac{\pi}{6},-\pi+\frac{\pi}{6}$

$\Rightarrow$ sum of values $=2\pi$

4. Official Ans. by NTA (1)

Sol. $2^{\sqrt{\sin^2 x-2\sin x+5}}\ .\ 4^{-\sin^2 y}\le 1$

$\Rightarrow 2^{\sqrt{(\sin x-1)^2+4}}\le 2^{2\sin^2 y}$

$\Rightarrow \sqrt{(\sin x-1)^2+4}\le 2\sin^2 y$

$\Rightarrow \sin x=1$ and $|\sin y|=1$

5. Official Ans. by NTA (1)

Sol. $1 + \sin^4 x = \cos^2 3x$

$\sin x = 0\ \&\ \cos 3x = 1$

$0, 2\pi, -2\pi, -\pi, \pi$

6. Official Ans. by NTA (1)

Sol. $\cos 2x + \alpha\sin x = 2\alpha - 7$

$\Rightarrow 2\sin^2 x - \alpha\sin x + 2\alpha - 8 = 0$

$\sin^2 x-\frac{\alpha}{2}\sin x+\alpha-4=0$

$\Rightarrow \sin x=2$ (rejected) or $\sin x=\frac{\alpha-4}{2}$

$\Rightarrow \left|\frac{\alpha-4}{2}\right|\le 1$

$\Rightarrow \alpha\in[2,6]$

SOLUTION OF TRIANGLE

1. **Ans. (4)**

r = 1 is obviously true.

Let 0 < r < 1

$\Rightarrow \quad r + r^2 > 1$

$\Rightarrow r^2 + r - 1 > 0$

$$\left(r-\frac{-1-\sqrt{5}}{2}\right)\left(r-\left(\frac{-1+\sqrt{5}}{2}\right)\right)$$

$$\Rightarrow \; r-\frac{-1-\sqrt{5}}{2} \text{ or } r>\frac{-1+\sqrt{5}}{2}$$

$$r\in\left(\frac{\sqrt{5}-1}{2},1\right)$$

$$\frac{\sqrt{5}-1}{2}<r<1$$

When r > 1

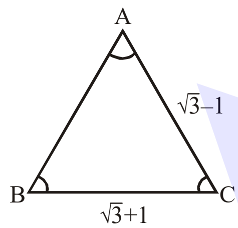
$$\Rightarrow \frac{\sqrt{5}+1}{2}>\frac{1}{r}>1$$

$$\Rightarrow \; r\in\left(\frac{\sqrt{5}-1}{2},\frac{\sqrt{5}+1}{2}\right)$$

Now check options

2. **Ans. (1)**

A + B = 120°



$$\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

$$=\frac{\sqrt{3}+1-\sqrt{3}+1}{2(\sqrt{3})}\cot(30^\circ) = \frac{1}{\sqrt{3}}.\sqrt{3}=1$$

$$\frac{A-B}{2}=45^\circ \quad \Rightarrow \quad A-B=90^\circ$$

$$A+B=120^\circ$$

$$2A = 210^\circ$$

$$A = 105^\circ$$

$$B = 15^\circ$$

$\therefore$ Option (1)

3. **Ans. (2)**

Given a + b = x and ab = y

If $x^2 - c^2 = y \Rightarrow (a + b)^2 - c^2 = ab$

$\Rightarrow a^2 + b^2 - c^2 = -ab$

$$\Rightarrow \frac{a^2+b^2-c^2}{2ab}=-\frac{1}{2}$$

$$\Rightarrow \cos C=-\frac{1}{2}$$

$$\Rightarrow \angle C=\frac{2\pi}{3}$$

$$R=\frac{c}{2\sin C} = \frac{c}{\sqrt{3}}$$

4. **Ans. (3)**

$b + c = 11\lambda,\ c + a = 12\lambda,\ a + b = 13\lambda$

$\Rightarrow a = 7\lambda,\ b = 6\lambda,\ c = 5\lambda$

(using cosine formula)

$$\cos A=\frac{1}{5},\cos B=\frac{19}{35},\cos C=\frac{5}{7}$$

$\alpha : \beta : \gamma \Rightarrow 7 : 19 : 25$

5. **Official Ans. by NTA (3)**

Sol. a < b < c are in A.P.

$\angle C = 2\angle A$ (Given)

$\Rightarrow \sin C = \sin 2A$

$\Rightarrow \sin C = 2\sin A.\cos A$

$$\Rightarrow \frac{\sin C}{\sin A}=2\cos A$$

$$\Rightarrow \frac{c}{a}=2\frac{b^2+c^2-a^2}{2bc}$$

put $a = b - \lambda,\ c = b + \lambda,\ \lambda > 0$

$$\Rightarrow \lambda=\frac{b}{5}$$

$$\Rightarrow a=b-\frac{b}{5}=\frac{4}{5}b, c=b+\frac{b}{5}=\frac{6b}{5}$$

$\Rightarrow$ required ratio = 4 : 5 : 6

6. **Official Ans. by NTA (3)**

Sol. $\angle B=\frac{\pi}{3}$, by sine Rule

$$\sin A = \frac{1}{2}$$

$\Rightarrow A = 30^\circ, \quad a = 2,\ b = 2\sqrt{3},\ c = 4$

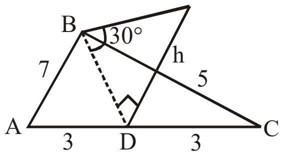
$$\Delta = \frac{1}{2}\times 2\sqrt{3}\times 2 = 2\sqrt{3} \text{ sq. cm}$$

HEIGHT & DISTANCE

1. **Ans. (2)**

$BD = h\cot 30° = h\sqrt{3}$

So, $7^2 + 5^2 = 2((h\sqrt{3})^2 + 3^2)$

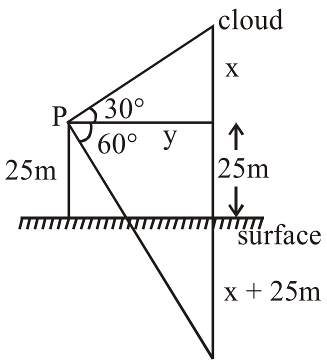


$\Rightarrow 37 = 3h^2 + 9.$

$\Rightarrow 3h^2 = 28$

$\Rightarrow h = \sqrt{\frac{28}{3}} = \frac{2}{3}\sqrt{21}$

2. **Ans (2)**



$\tan 30° = \frac{x}{y} \Rightarrow y = \sqrt{3}\,x$ .....(i)

$\tan 60° = \frac{25 + x + 25}{y}$

$\Rightarrow \sqrt{3}\,y = 50 + x$

$\Rightarrow 3x = 50 + x$

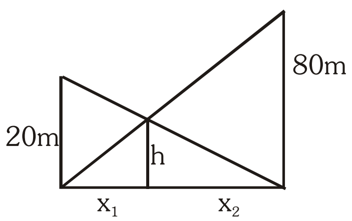
$\Rightarrow x = 25$ m

$\therefore$ Height of cloud from surface

$= 25 + 25 = 50$m

3. **Official Ans. by NTA (3)**

Sol.



by similar triangle

$\frac{h}{x_1} = \frac{80}{x_1 + x_2}$ ...(1)

by $\frac{h}{x_2} = \frac{20}{x_1 + x_2}$ ..(2)

by (1) and (2)

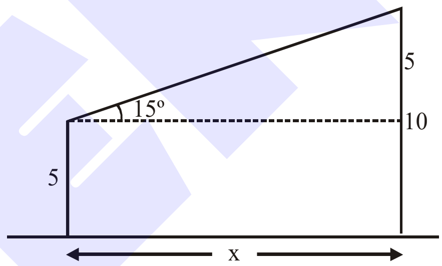
$\frac{x_2}{x_1} = 4$ or $x_2 = 4x_1$

$\Rightarrow \frac{h}{x_1} = \frac{80}{5x_1}$

or h = 16m

4. **Official Ans. by NTA (3)**

Sol. $\tan 15° = \frac{5}{x}$



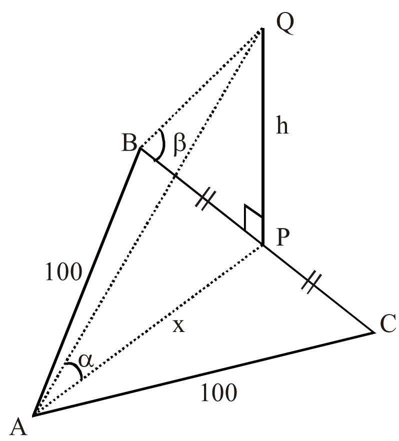
$2 - \sqrt{13} = \frac{5}{x}$

$x = 5(2 + \sqrt{3})$

5. **Official Ans. by NTA (3)**

Sol. $\cot \alpha = 3\sqrt{2}$

& $\text{cosec}\, \beta = 2\sqrt{2}$



So, $\frac{x}{h} = 3\sqrt{2}$ ...(i)

And $\dfrac{h}{\sqrt{10^4 - x^2}} = \dfrac{1}{\sqrt{7}}$ ...(ii)

So, from (i) & (ii)

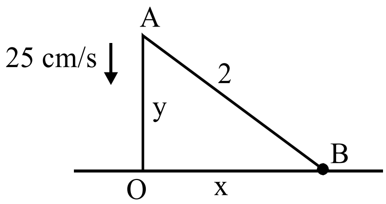
$\Rightarrow \dfrac{h}{\sqrt{10^4 - 18h^2}} = \dfrac{1}{\sqrt{7}}$

$\Rightarrow 25h^2 = 100 \times 100$

$\Rightarrow h = 20.$

6. Official Ans. by NTA (3)

Sol.



$x^2 + y^2 = 4 \left(\dfrac{dy}{dt} = -25\right)$

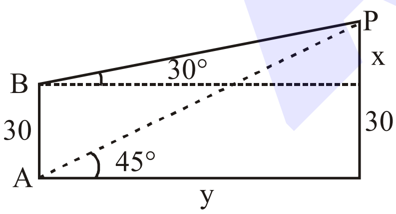
$x\dfrac{dx}{dt} + y\dfrac{dy}{dt} = 0$

$\sqrt{3}\dfrac{dx}{dt} - 1(25) = 0$

$\dfrac{dx}{dt} = \dfrac{25}{\sqrt{3}}\ \text{cm/sec}$

7. Official Ans. by NTA (2)

Sol.



$\tan 45° = 1 = \dfrac{x+30}{y} \Rightarrow x + 30 = y$ (i)

$\tan 30° = \dfrac{1}{\sqrt{3}} = \dfrac{x}{y} \Rightarrow x = \dfrac{y}{\sqrt{3}}$ (ii)

from (i) and (ii) $y = 15\left(3+\sqrt{3}\right)$

DETERMINANT

1. Ans. (2)

$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2-1 \end{vmatrix} = a^2 - 3$

$D_1 = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 3 & 2 \\ a+1 & 3 & a^2-1 \end{vmatrix} = a^2 - a + 1$

$D_2 = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 5 & 2 \\ 2 & a+1 & a^2-1 \end{vmatrix} = a^2 - 3$

$D_3 = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 5 \\ 2 & 3 & a+1 \end{vmatrix} = a - 4$

D = 0 at $|a| = \sqrt{3}$ but $D_3 = \pm\sqrt{3} - 4 \neq 0$

So the system is Inconsistant for $|a| = \sqrt{3}$

2. Ans. (2)

$P_1 \equiv x - 4y + 7z - g = 0$

$P_2 \equiv 3x - 5y - h = 0$

$P_3 \equiv -2x + 5y - 9z - k = 0$

Here $\Delta = 0$

$2P_1 + P_2 + P_3 = 0$ when $2g + h + k = 0$

3. Ans. (4)

$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 2 & \alpha-1 \end{vmatrix} = (\alpha-1) - 4 = (\alpha - 5)$

for infinite solutions D = 0 $\Rightarrow \alpha = 5$

$D_x = 0 \Rightarrow \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ -1 & -1 & 3 \\ \beta-15 & -2 & 5 \end{vmatrix} = 0$

$\Rightarrow 2 + \beta - 15 = 0 \Rightarrow \beta - 13 = 0$

on $\beta = 13$ we get $D_y = D_z = 0$

$\alpha = 5, \beta = 13$

4. Ans. (3)

$$\det A = \begin{vmatrix} -2 & 4+d & \sin\theta-2 \\ 1 & \sin\theta+2 & d \\ 5 & 2\sin\theta-d & -\sin\theta+2+2d \end{vmatrix}$$

$(R_1 \to R_1 + R_3 - 2R_2)$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & \sin\theta+2 & d \\ 5 & 2\sin\theta-d & 2+2d-\sin\theta \end{vmatrix}$$

$= (2+\sin\theta)(2+2d-\sin\theta) - d(2\sin\theta - d)$

$= 4 + 4d - 2\sin\theta + 2\sin\theta + 2d\sin\theta - \sin^2\theta - 2d\sin\theta + d^2$

$= d^2 + 4d + 4 - \sin^2\theta$

$= (d+2)^2 - \sin^2\theta$

For a given d, minimum value of

$\det(A) = (d+2)^2 - 1 = 8$

$\Rightarrow d = 1$ or -5

5. Ans. (1)

Apply

$C_3 \to C_3 - C_2$

$C_2 \to C_2 - C_1$

We get $D = 0$

Option (1)

6. Ans. (4)

$$\begin{vmatrix} 1 & 3 & 7 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix} = 0$$

$(8 - 7\cos 2\theta) - 3(-2 - 7\sin 3\theta) + 7(-\cos 2\theta - 4\sin 3\theta) = 0$

$14 - 7\cos 2\theta + 21\sin 3\theta - 7\cos 2\theta - 28\sin 3\theta = 0$

$14 - 7\sin 3\theta - 14\cos 2\theta = 0$

$14 - 7(3\sin\theta - 4\sin^3\theta) - 14(1 - 2\sin^2\theta) = 0$

$-21\sin\theta + 28\sin^3\theta + 28\sin^2\theta = 0$

$7\sin\theta\,[-3 + 4\sin^2\theta + 4\sin\theta] = 0$

$\sin\theta,\ 4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3 = 0$

$2\sin\theta(2\sin\theta + 3) - 1(2\sin\theta + 3) = 0$

$\sin\theta = \frac{-3}{2}$; $\sin\theta = \frac{1}{2}$

Hence, 2 solutions in $(0, \pi)$

Option (4)

7. Ans. (1)

$P_1 : 2x + 2y + 3z = a$

$P_2 : 3x - y + 5z = b$

$P_3 : x - 3y + 2z = c$

We find

$P_1 + P_3 = P_2 \Rightarrow a + c = b$

8. Ans. (4)

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 2c & c-a-b \end{vmatrix}$$

$= (a + b + c)(a + b + c)^2$

$\Rightarrow x = -2(a + b + c)$

9. Ans. (3)

For unique solution

$$\Delta \neq 0 \Rightarrow \begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0 \Rightarrow \alpha + \beta \neq -2$$

10. Ans. (2)

$$\begin{vmatrix} \lambda-1 & 2 & 2 \\ 1 & 2-\lambda & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0 \Rightarrow (\lambda-1)^3 = 0 \Rightarrow \lambda = 1$$

11. Official Ans. by NTA (1)

Sol. For non-trivial solution

$D = 0$

$$\begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0 \Rightarrow 2c^3 - 3c^2 - 1 = 0$$

$\Rightarrow\ (c + 1)^2 (2c - 1) = 0$

$\therefore$ Greatest value of c is $\frac{1}{2}$

12. Official Ans. by NTA (3)

Sol. $x - 2y + kz = 1$ ...(1)

$2x + y + z = 2$ ...(2)

$3x - y - kz = 3$ ...(3)

(1) +(3)

$\Rightarrow 4x - 3y = 4$

13. Official Ans. by NTA (3)

Sol. $\begin{vmatrix} 2 & 3 & -1 \\ 1 & K & -2 \\ 2 & -1 & 1 \end{vmatrix} = 0$

By solving $K = \frac{9}{2}$

$2x + 3y - z = 0$ ...(1)

$x + \frac{9}{2}y - 2z = 0$ ...(2)

$2x - y + z = 0$ ...(3)

(1)-(3) $\Rightarrow 4y - 2z = 0$

$2y = z$ ...(4)

$\boxed{\frac{y}{z} = \frac{1}{2}}$ ...(5)

put z from eqn. (4) into (1)

$2x + 3y - 2y = 0$

$2x + y = 0$

$\boxed{\frac{x}{y} = -\frac{1}{2}}$ ...(6)

$\frac{(6)}{(5)}$ $\boxed{\frac{z}{x} = -4}$

$\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + K = \frac{1}{2}$

14. Official Ans. by NTA (2)

Sol. $\Delta_1 = f(\theta) = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = -x^3$

and $\Delta_2 = f(2\theta) = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix} = -x^3$

So $\Delta_1 + \Delta_2 = -2x^3$

15. Official Ans. by NTA (2)

Sol. $D = 0$

$\begin{vmatrix} 1 & 1 & 1 \\ 4 & \lambda & -\lambda \\ 3 & 2 & -4 \end{vmatrix} = 0 \Rightarrow \lambda = 3$

16. Official Ans. by NTA (3)

Sol. By expansion, we get

$-5x^3 + 30x - 30 + 5x = 0$

$\Rightarrow -5x^3 + 35x - 30 = 0$

$\Rightarrow x^3 - 7x + 6 = 0$, All roots are real

So, sum of roots = 0

17. Official Ans. by NTA (3)

Sol. $R_1 \to R_1 - R_2$

$\begin{vmatrix} 1 & -1 & 0 \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$

$R_2 \to R_2 - R_3$

$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$

$\Rightarrow (1 + 4\cos 6\theta) + \sin^2\theta + 1(\cos^2\theta) = 0$

$1 + 2\cos 6\theta = 0 \Rightarrow \cos 6\theta = -1/2$

$6\theta = \frac{2\pi}{3} \Rightarrow \boxed{\theta = \frac{\pi}{9}}$

18. Official Ans. by NTA (2)

Sol. $[\sin\theta]x + [-\cos\theta]y = 0$ and $[\cos\theta]x + y = 0$

for infinite many solution

$\begin{vmatrix} [\sin\theta] & [-\cos\theta] \\ [\cos\theta] & 1 \end{vmatrix} = 0$

ie $[\sin\theta] = -[\cos\theta][\cot\theta]$ (1)

when $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \Rightarrow \sin\theta \in \left(0, \frac{1}{2}\right)$

$-\cos\theta \in \left(0, \frac{1}{2}\right)$

$\cot\theta \in \left(-\frac{1}{\sqrt{3}}, 0\right)$

when $\theta \in \left(\pi, \frac{7\pi}{6}\right) \Rightarrow \sin\theta \in \left(-\frac{1}{2}, 0\right)$

$-\cos\theta \in \left(\frac{\sqrt{3}}{2}, 1\right)$

$\cot\theta \in \left(\sqrt{3}, \infty\right)$

when $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ then equation (i) satisfied there fore infinite many solution.

when $\theta \in \left(\pi, \frac{7\pi}{6}\right)$ then equation (i) not satisfied there fore infinite unique solution.

STRAIGHT LINE

1. Ans. (4)

Given set of lines px + qy + r = 0

given condition 3p + 2q + 4r = 0

$\Rightarrow \frac{3}{4}p + \frac{1}{2}q + r = 0$

$\Rightarrow$ All lines pass through a fixed point $\left(\frac{3}{4}, \frac{1}{2}\right)$.

2. Ans. (4)

Equation of AB is

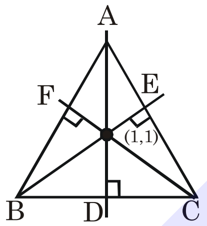
3x – 2y + 6 = 0

equation of AC is

4x + 5y – 20 = 0

Equation of BE is

2x + 3y – 5 = 0



Equation of CF is 5x – 4y – 1 = 0

$\Rightarrow$ Equation of BC is 26x – 122y = 1675

3. Ans. (4)

Let A(α,0) and B(0,β)

be the vertices of the given triangle AOB

$\Rightarrow |\alpha\beta| = 100$

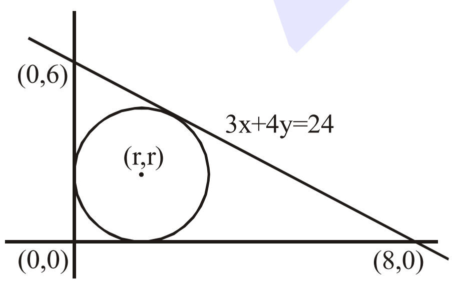
$\Rightarrow$ Number of triangles

= 4 × (number of divisors of 100)

= 4 × 9 = 36

4. Ans. (2)

$\left|\frac{3r + 4r - 24}{5}\right| = r$



7r – 24 = ±5r

2r = 24 or 12r + 24

r = 14, r = 2

then incentre is (2, 2)

5. Ans. (2)

Let the centroid of ΔPQR is (h, k) & P is (α, β), then

$\frac{\alpha + 1 + 3}{3} = h$ and $\frac{\beta + 4 - 2}{3} = k$

α = (3h – 4) β = (3k – 4)

Point P(α, β) lies on line 2x – 3y + 4 = 0

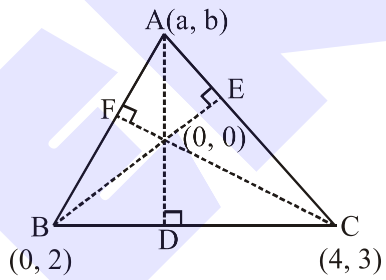
∴ 2(3h – 4) – 3(3k – 2) + 4 = 0

$\Rightarrow$ locus is 6x – 9y + 2 = 0

6. Ans. (2)

$m_{BD} \times m_{AD} = -1 \Rightarrow \left(\frac{3-2}{4-0}\right) \times \left(\frac{b-0}{a-0}\right) = -1$

$\Rightarrow$ b + 4a = 0 ……(i)



$m_{AB} \times m_{CF} = -1 \Rightarrow \left(\frac{(b-2)}{a-0}\right) \times \left(\frac{3}{4}\right) = -1$

$\Rightarrow$ 3b – 6 = –4a $\Rightarrow$ 4a + 3b = 6 ……(ii)

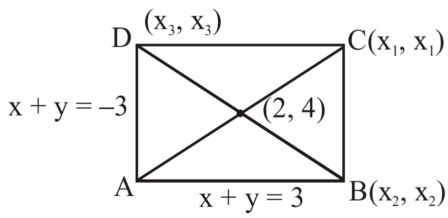
From (i) and (ii)

$a = \frac{-3}{4}$, b = 3

∴ II[nd] quadrant.

Option (2)

7. Ans. (4)



Solving x + y = 3
and x – y = –3 $\Big\}$ A(0, 3)

$\frac{x_1 + 0}{2} = 2$; $x_i = 4$ similarly $y_1 = 5$

C $\Rightarrow$ (4, 5)

Now equation of BC is $x - y = -1$

and equation of CD is $x + y = 9$

Solving $x + y = 9$ and $x - y = -3$

Point D is (3, 6)

Option (4)

8. **Ans. (4)**

co-ordinates of point D are (4,7)

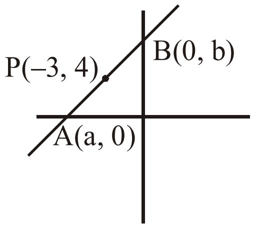
$\Rightarrow$ line AD is $5x - 3y + 1 = 0$

9. **Ans. (4)**

$$\frac{17-\beta}{-8} \times \frac{2}{3} = -1$$

$\beta = 5$

10. **Ans (4)**



Let the line be $\frac{x}{a} + \frac{y}{b} = 1$

$(-3, 4) = \left(\frac{a}{2}, \frac{b}{2}\right)$

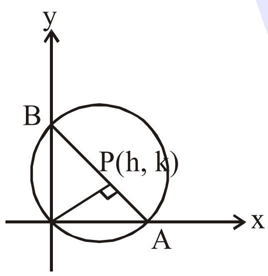
$a = -6,\ b = 8$

equation of line is $4x - 3y + 24 = 0$

11. **Ans. (3)**

Slope of AB $= \frac{-h}{k}$

Equation of AB is $hx + ky = h^2 + k^2$



$A\left(\frac{h^2+k^2}{h}, 0\right),\ B\left(0, \frac{h^2+k^2}{k}\right)$

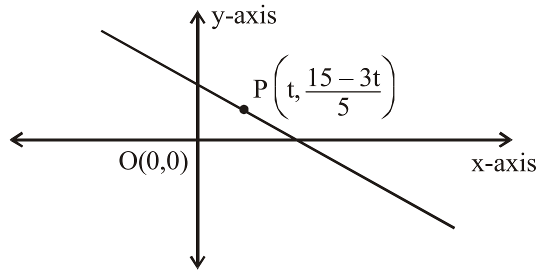
$AB = 2R$

$\Rightarrow (h^2 + k^2)^3 = 4R^2h^2k^2$

$\Rightarrow (x^2 + y^2)^3 = 4R^2x^2y^2$

12. **Official Ans. by NTA (1)**

Sol.



Now, $\left|\frac{15-3t}{5}\right| = |t|$

$\Rightarrow \frac{15-3t}{5} = t$ or $\frac{15-3t}{5} = -t$

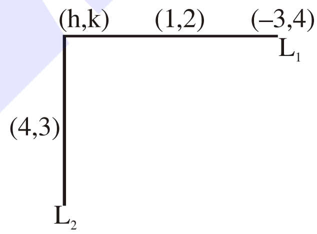
$\therefore\ t = \frac{15}{8}$ or $t = \frac{-15}{2}$

So, $P\left(\frac{15}{8}, \frac{15}{8}\right) \in I^{st}$ quadrant

or $P\left(\frac{-15}{2}, \frac{15}{2}\right) \in II^{nd}$ quadrant

13. **Official Ans. by NTA (3)**

Sol.



equation of L_1 is

$y = -\frac{1}{2}x + \frac{5}{2}$ ...(1)

equation of L_2 is

$y = 2x - 5$ ...(2)

by (1) and (2)

$x = 3$

$y = 1 \Rightarrow h = 3,\ k = 1$

$\frac{k}{h} = \frac{1}{3}$

14. **Official Ans. by NTA (3)**

Sol. $x = 2 + r\cos\theta$

$y = 3 + r\sin\theta$

$\Rightarrow 2 + r\cos\theta + 3 + r\sin\theta = 7$

$\Rightarrow r(\cos\theta + \sin\theta) = 2$

$\Rightarrow \sin\theta + \cos\theta = \frac{2}{r} = \frac{2}{\pm 4} = \pm\frac{1}{2}$

$\Rightarrow 1 + \sin 2\theta = \frac{1}{4}$

$\Rightarrow \sin 2\theta = -\frac{3}{4}$

$\Rightarrow \frac{2m}{1+m^2} = -\frac{3}{4}$

$\Rightarrow 3m^2 + 8m + 3 = 0$

$\Rightarrow m = \frac{-4 \pm \sqrt{7}}{1-7}$

$$\frac{1-\sqrt{7}}{1+\sqrt{7}} = \frac{(1-\sqrt{7})^2}{1-7} = \frac{8-2\sqrt{7}}{-6} = \frac{-4+\sqrt{7}}{3}$$

15. Official Ans. by NTA (4)

Sol. $\left(\frac{-1}{a-1}\right)\left(\frac{-2}{a^2}\right) = -1$

$2 = -(a^2)(a-1)$

$a^3 - a^2 + 2 = 0$

$(a+1)(a^2 - 2a + 2) = 0$

$\therefore\ a = -1$

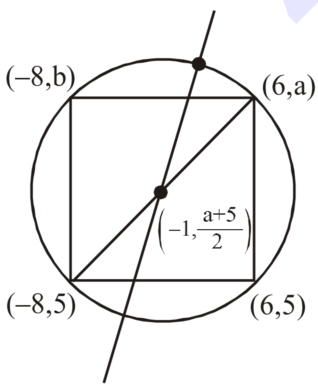
$$\left.\begin{array}{l} L_1 : x - 2y + 1 = 0 \\ L_2 : 2x + y - 1 = 0 \end{array}\right\}$$

$0(0, 0)\quad P\left(\frac{1}{5}, \frac{3}{5}\right)$

$$OP = \sqrt{\frac{1}{25} + \frac{9}{25}} = \sqrt{\frac{10}{25}} = \sqrt{\frac{2}{5}}$$

16. Official Ans. by NTA (2)

Sol.



$$\frac{3(a+5)}{2} = -1 + 7$$

$$a + 5 = \frac{2(6)}{3}$$

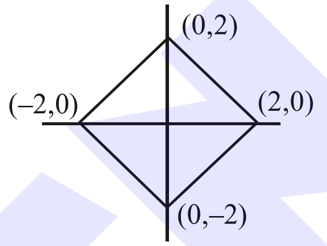
$a = -1$

sides = 6 and 14

$\Rightarrow A = 84$

17. Official Ans. by NTA (1)

Sol. $|x-y| \le 2$ and $|x+y| \le 2$

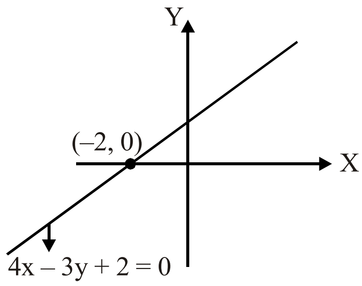


Square whose side is $2\sqrt{2}$

18. Official Ans. by NTA (1)

Sol. Required line is $4x - 3y + \lambda = 0$

$$\left|\frac{\lambda}{5}\right| = \frac{3}{5}$$



$\Rightarrow \lambda = \pm 3.$

So, required equation of line is

$4x - 3y + 3 = 0$ and $4x - 3y - 3 = 0$

(1) $4\left(-\frac{1}{4}\right) - 3\left(\frac{2}{3}\right) + 3 = 0$

19. Official Ans. by NTA (2)

Sol. $2y = 2\sin x \sin(x+2) - 2\sin^2(x+1)$

$2y = \cos 2 - \cos(2x+2) - (1 - \cos(2x+2))$

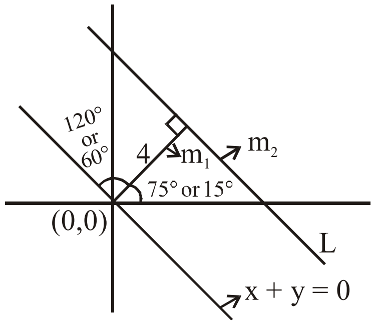
$= \cos 2 - 1$

$2y = -2\sin^2\frac{1}{2}$

$y = -\sin^2\frac{1}{2} \le 0$

20. Official Ans. by NTA (1) or (2)

Sol.



$m_1 = \tan 75° = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

or $m = \tan 15° = \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$m_2 = \dfrac{-1}{m_1} = \dfrac{-\left(\sqrt{3}-1\right)}{\sqrt{3}+1}$

or $m_2 = \dfrac{-1}{m_1} = \dfrac{-\left(\sqrt{3}+1\right)}{\sqrt{3}-1}$

$\Rightarrow y = m_2x + C$

$\Rightarrow y = \dfrac{-\left(\sqrt{3}-1\right)x}{\sqrt{3}+1} + C \Rightarrow L$

or $y = \dfrac{-\left(\sqrt{3}+1\right)x}{\sqrt{3}-1} + C \Rightarrow L$

Distance from origin = 4

$\therefore \left|\dfrac{C}{\sqrt{1+\dfrac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)^2}}}\right| = 4$ or $\left|\dfrac{C}{\sqrt{1+\dfrac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}-1\right)^2}}}\right| = 4$

$\Rightarrow C = \dfrac{8\sqrt{2}}{\left(\sqrt{3}+1\right)}$ or $C = \dfrac{8\sqrt{2}}{\left(\sqrt{3}-1\right)}$

$\Rightarrow \left(\sqrt{3}-1\right)y + \left(\sqrt{3}+1\right)x = 8\sqrt{2}$

or $\left(\sqrt{3}-1\right)x + \left(\sqrt{3}+1\right)y = 8\sqrt{2}$

21. Official Ans. by NTA (2)

Sol. Let $B(\alpha,\beta)$ and $C(\gamma,\delta)$

$\dfrac{\alpha+1}{2} = -1 \Rightarrow \alpha = -3$

$\dfrac{\beta+2}{2} = 1 \Rightarrow \beta = 0$

$\Rightarrow B(-3,0)$

Now $\dfrac{\gamma+1}{2} = 2 \Rightarrow \gamma = 3$

$\dfrac{\delta+2}{2} = 3 \Rightarrow \delta = 4$

$\Rightarrow C(3,4)$

$\Rightarrow$ centroid of triangle is $G\left(\dfrac{1}{3}, 2\right)$

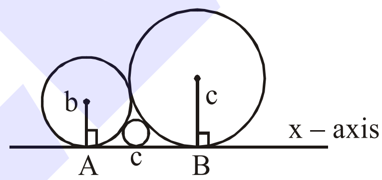
CIRCLE

1. Ans. (1)

AB = AC + CB

$\sqrt{(b+c)^2 - (b-c)^2}$

$= \sqrt{(b+a)^2 - (b-a)^2} + \sqrt{(a+c)^2 - (a-c)^2}$



$\sqrt{bc} = \sqrt{ab} + \sqrt{ac}$

$\dfrac{1}{\sqrt{a}} = \dfrac{1}{\sqrt{c}} + \dfrac{1}{\sqrt{b}}$

2. Ans. (2)

$x^2 + y^2 - 16x - 20y + 164 = r^2$

$A(8,10),\ R_1 = r$

$(x-4)^2 + (y-7)^2 = 36$

$B(4,7),\ R_2 = 6$

$|R_1 - R_2| < AB < R_1 + R_2$

$\Rightarrow 1 < r < 11$

3. Ans. (4)

$x^2 + y^2 + 4x - 6y - 12 = 0$

Equation of tangent at (1, –1)

$x - y + 2(x+1) - 3(y-1) - 12 = 0$

$3x - 4y - 7 = 0$

$\therefore$ Equation of circle is

$(x^2 + y^2 + 4x - 6y - 12) + \lambda(3x - 4y - 7) = 0$

It passes through (4, 0) :

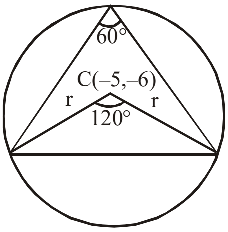
$(16 + 16 - 12) + \lambda(12 - 7) = 0$

$\Rightarrow 20 + \lambda(5) = 0$
$\Rightarrow \lambda = -4$
$\therefore (x^2 + y^2 + 4x - 6y - 12) - 4(3x - 4y - 7) = 0$
or $x^2 + y^2 - 8x + 10y + 16 = 0$
Radius $= \sqrt{16 + 25 - 16} = 5$

4. Ans. (2)

$$3\left(\frac{1}{2}r^2 . \sin 120°\right) = 27\sqrt{3}$$

$$\frac{r^2}{2}\frac{\sqrt{3}}{2} = \frac{27\sqrt{3}}{3}$$



$$r^2 = \frac{108}{3} = 36$$

Radius $= \sqrt{25 + 36 - C} = \sqrt{36}$

$\boxed{C = 25}$

$\therefore$ Option (2)

5. Ans. (4)

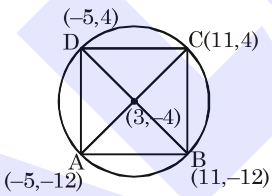
$R = \sqrt{9 + 16 + 103} = 8\sqrt{2}$

OA = 13

$OB = \sqrt{265}$

$OC = \sqrt{137}$

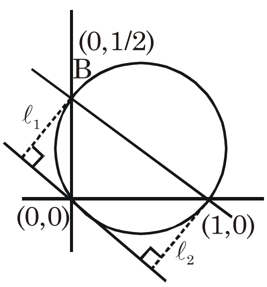
$OD = \sqrt{41}$



6. Ans. (2)

Equation of circle

$$(x - 1)(x - 0) + (y - 0)\left(y - \frac{1}{2}\right) = 0$$



$\Rightarrow x^2 + y^2 - x - \frac{y}{2} = 0$

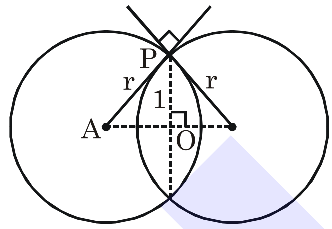
Equation of tangent of origin is $2x + y = 0$

$$\ell_1 + \ell_2 = \frac{2}{\sqrt{5}} + \frac{1}{2\sqrt{5}}$$

$$= \frac{4+1}{2\sqrt{5}} = \frac{\sqrt{5}}{2}$$

7. Ans. (4)

In ΔAPO



$$\left(\frac{\sqrt{2}\,r}{2}\right)^2 + 1^2 = r^2$$

$\Rightarrow \boxed{r = \sqrt{2}}$

So distance between centres $= \sqrt{2}\,r = 2$

8. Ans. (2)

Let equation of circle is

$x^2 + y^2 + 2fx + 2fy + e = 0$, it passes through (0, 2b)

$\Rightarrow 0 + 4b^2 + 2g \times 0 + 4f + c = 0$

$\Rightarrow 4b^2 + 4f + c = 0$ ...(i)

$2\sqrt{g^2 - c} = 4a$ ...(ii)

$g^2 - c = 4a^2 \Rightarrow c = (g^2 - 4a^2)$

Putting in equation (1)

$\Rightarrow 4b^2 + 4f + g^2 - 4a^2 = 0$

$\Rightarrow x^2 + 4y + 4(b^2 - a^2) = 0$, it represent a parabola.

9. Ans. (1)

Centre of circles are opposite side of line

$(3 + 4 - \lambda)(27 + 4 - \lambda) < 0$

$(\lambda - 7)(\lambda - 31) < 0$

$\lambda \in (7, 31)$

distance from S_1

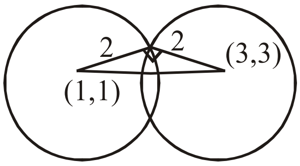
$$\left|\frac{3 + 4 - \lambda}{5}\right| \geq 1 \Rightarrow \lambda \in (-\infty, 2] \cup [(12, \infty]$$

distance from S_2

$$\left|\frac{27 + 4 - \lambda}{5}\right| \geq 2 \Rightarrow \lambda \in (-\infty, 21] \cup [41, \infty)$$

so $\lambda \in [12, 21]$

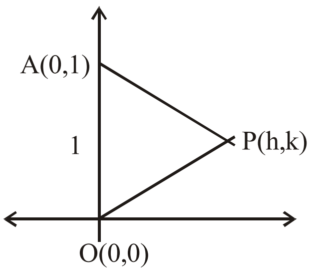
10. Ans. (4)



Area = $2 \times \frac{1}{2}.4 = 2$

11. Official Ans. by NTA (2)

Sol.



AP + OP + AO = 4

$\sqrt{h^2+(k-1)^2}+\sqrt{h^2+k^2}+1=4$

$\sqrt{h^2+(k-1)^2}+\sqrt{h^2+k^2}=3$

$h^2+(k-1)^2=9+h^2+k^2-6\sqrt{h^2+k^2}$

$-2k-8=-6\sqrt{h^2+k^2}$

$k+4=3\sqrt{h^2+k^2}$

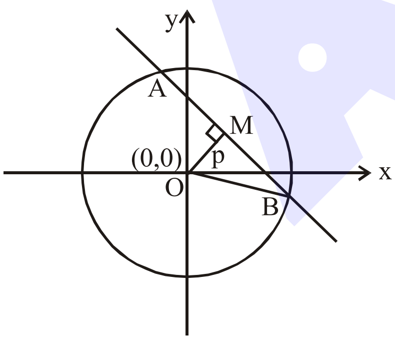
$k^2 + 16 + 8k = 9(h^2 + k^2)$

$9h^2 + 8k^2 - 8k - 16 = 0$

Locus of P is $9x^2 + 8y^2 - 8y - 16 = 0$

12. Official Ans. by NTA (4)

Sol.



$p=\frac{n}{\sqrt{2}}$, but $\frac{n}{\sqrt{2}}<4 \Rightarrow n = 1, 2, 3, 4, 5.$

Length of chord AB = $2\sqrt{16-\frac{n^2}{2}}$

$=\sqrt{64-2n^2}=\ell(\text{say})$

For n = 1, $\ell^2 = 62$

n = 2, $\ell^2 = 56$

n = 3, $\ell^2 = 46$

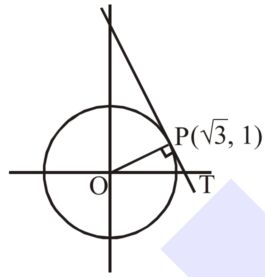
n = 4, $\ell^2 = 32$

n = 5, $\ell^2 = 14$

$\therefore$ Required sum $= 62+56+46+32+14 = 210$

13. Official Ans. by NTA (4)

Sol.



Given $x^2 + y^2 = 4$

equation of tangent

$\Rightarrow \sqrt{3}x+y=4$ ...(1)

Equation of normal

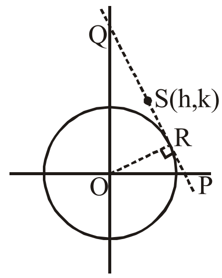
$x-\sqrt{3}y=0$ ...(2)

Coordinate of $T\left(\frac{4}{\sqrt{3}},0\right)$

$\therefore$ Area of triangle = $=\frac{2}{\sqrt{3}}$

14. Official Ans. by NTA (3)

Sol.



Let the mid point be S(h,k)

$\therefore$ P(2h,0) and Q(0,2k)

equation of PQ : $\frac{x}{2h}+\frac{y}{2k}=1$

$\because$ PQ is tangent to circle at R(say)

$\therefore$ OR = 1 $\Rightarrow \left|\frac{-1}{\sqrt{\left(\frac{1}{2h}\right)^2+\left(\frac{1}{2k}\right)^2}}\right|=1$

$\Rightarrow \frac{1}{4h^2} + \frac{1}{4k^2} = 1$

$\Rightarrow x^2 + y^2 - 4x^2y^2 = 0$

Aliter :

tangent to circle

$x\cos\theta + y\sin\theta = 1$

P : $(\sec\theta, 0)$

Q : $(0, \text{cosec}\theta)$

$2h = \sec\theta \Rightarrow \cos\theta = \frac{1}{2h}$ & $\sin\theta = \frac{1}{2k}$

$$\frac{1}{(2x)^2} + \frac{1}{(2y)^2} = 1$$

15. Official Ans. by NTA (2)

Sol. Circle touches internally

$C_1(0, 0)$; $r_1 = 2$

$C_2 : (-3, -4)$; $r_2 = 7$

$C_1C_2 = |r_1 - r_2|$

$S_1 - S_2 = 0 \Rightarrow$ eqn. of common tangent

$6x + 8y - 20 = 0$

$3x + 4y = 10$

$(6, -2)$ satisfy it

16. Official Ans. by NTA (3)

Sol. Equation of common chord

$4kx + \frac{1}{2}y + k + \frac{1}{2} = 0$ ....(1)

and given line is $4x + 5y - k = 0$ .....(2)

On comparing (1) & (2), we get

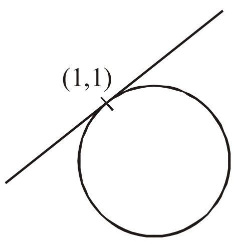
$$k = \frac{1}{10} = \frac{k + \frac{1}{2}}{-k}$$

$\Rightarrow$ No real value of k exist

17. Official Ans. by NTA (1)

ALLEN Ans. (3)

Sol.



Equation of circle can be written as
$(x-1)^2 + (y-1)^2 + \lambda(x-y) = 0$

It passes through $(1, -3)$

$16 + \lambda(4) = 0 \Rightarrow \lambda = -4$

So $(x-1)^2 + (y-1)^2 - 4(x-y) = 0$

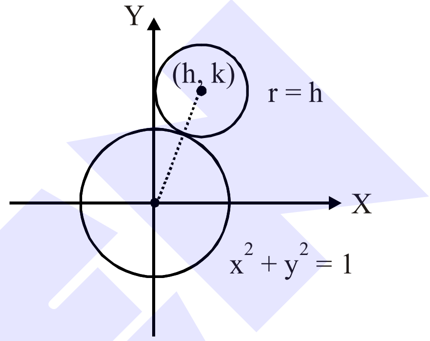
$\Rightarrow x^2 + y^2 - 6x + 2y + 2 = 0$

$\Rightarrow r = 2\sqrt{2}$

(correct key is 3)

18. Official Ans. by NTA (4)

Sol.



$\sqrt{h^2 + k^2} = |h| + 1$

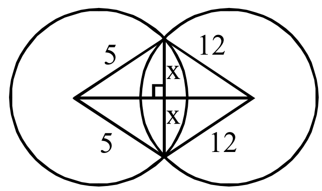
$\Rightarrow x^2 + y^2 = x^2 + 1 + 2x$

$\Rightarrow y^2 = 1 + 2x$

$\Rightarrow y = \sqrt{1 + 2x}$; $x \geq 0$.

19. Official Ans. by NTA (2)

Sol.



Let length of common chord = 2x

$\sqrt{25 - x^2} + \sqrt{144 - x^2} = 13$

after solving

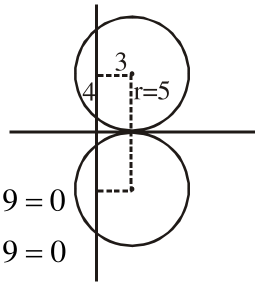
$$x = \frac{12 \times 5}{13}$$

$$2x = \frac{120}{13}$$

20. Official Ans. by NTA (1)

Sol. Equaiton of circles are

$$\begin{cases}(x-3)^2+(y-5)^2=25\\(x-3)^2+(y+5)^2=25\end{cases}$$



$$\Rightarrow \begin{cases}x^2+y^2-6x-10y+9=0\\x^2+y^2-6x+10y+9=0\end{cases}$$

PERMUTATION & COMBINATION

1. Ans. (2)

Required number of ways

= Total number of ways – When A and B are always included.

$= {}^5C_2 . {}^7C_3 - {}^5C_1 {}^5C_2 = 300$

2. Ans. (2)

a_1	a_2	a_3

Number of numbers = $5^3 - 1$

a_4	a_1	a_2	a_3

2 ways for a_4

Number of numbers = 2×5^3

Required number = $5^3 + 2 \times 5^3 - 1$

= 374

3. Ans. (4)

$$\sum_{r=2}^{13}(7r+2) = 7.\frac{2+13}{2}\times 6 + 2\times 12$$

= 7 × 90 + 24 = 654

$$\sum_{r=1}^{13}(7r+5) = 7\left(\frac{1+13}{2}\right)\times 13 + 5\times 13 = 702$$

Total = 654 + 702 = 1356

4. Ans. (1)

S = {1,2,3------100}

= Total non empty subsets-subsets with product of element is odd

$= 2^{100}-1-1[(2^{50}-1)]$

$= 2^{100} - 2^{50}$

$= 2^{50}(2^{50}-1)$

5. Ans (3)

Let m-men, 2-women

${}^mC_2 \times 2 = {}^mC_1\, {}^2C_1 . 2 + 84$

$m^2 - 5m - 84 = 0 \Rightarrow (m-12)(m+7) = 0$

m = 12

6. Ans. (1)

$2.{}^nC_5 = {}^nC_4 + {}^nC_6$

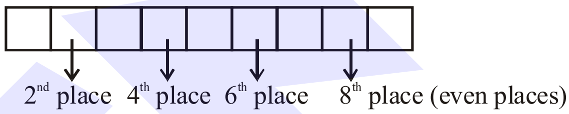
$$2.\frac{\lfloor n}{\lfloor 5 \lfloor n-5} = \frac{\lfloor n}{\lfloor 4 \lfloor n-4} + \frac{\lfloor n}{\lfloor 6 \lfloor n-6}$$

$$\frac{2}{5}.\frac{1}{n-5} = \frac{1}{(n-4)(n-5)} + \frac{1}{30}$$

n = 14 satisfying equation.

7. Official Ans. by NTA (4)

Sol.



Number of such numbers

$$= {}^4C_3 \times \frac{3!}{2!}\times\frac{6!}{2!4!} = 180$$

8. Official Ans. by NTA (4)

Sol. (1) The number of four-digit numbers Starting with 5 is equal to $6^3 = 216$

(2) Starting with 44 and 55 is equal to $36 \times 2 = 72$

(3) Starting with 433,434 and 435 is equal to $6 \times 3 = 18$

(3) Remaining numbers are 4322,4323,4324,4325is equal to 4 so total numbers are 216 + 72 + 18 + 4 = 310

9. Official Ans. by NTA (1)

Sol. Since there are 8 males and 5 females. Out of these 13, if we select 11 persons, then there will be at least 6 males and atleast 3 females in the selection.

$$m = n = \binom{13}{11} = \binom{13}{2} = \frac{13\times 12}{2} = 78$$

10. Official Ans. by NTA (1)

Sol. $\frac{n(n+1)}{2} + 99 = (n-2)^2$

$n^2 + n + 198 = 2(n^2 + 4 - 4n)$

$n^2 - 9n - 190 = 0$

$n^2 - 19n + 10 - 190 = 0$

$n(n - 19) + 10(n - 19) = 0$

$n = 19$

11. Official Ans. by NTA (2)

Sol. Sum of given digits 0, 1, 2, 5, 7, 9 is 24.

Let the six digit number be abcdef and to be divisible by 11

so $|(a + c + e) - (b + d + f)|$ is multiple of 11.

Hence only possibility is $a + c + e = 12 = b + d + f$

Case-I $\{a, c, e\} = \{9, 2, 1\}$ & $\{b, d, f\} = \{7, 5, 0\}$

So, Number of numbers $= 3! \times 3! = 36$

Case-II $\{a,c,e\} = \{7,5,0\}$ and $\{b,d,f\} = \{9,2,1\}$

So, Number of numbers $2 \times 2! \times 3! = 24$

Total = 60

12. Official Ans. by NTA (3)

Sol. Total cases = number of diagonals

$= {}^{20}C_2 - 20 = 170$

13. Official Ans. by NTA (1)

Sol. ${}^5C_1 \cdot {}^nC_2 + {}^5C_2 \cdot {}^nC_1 = 1750$

$n^2 + 3n = 700$

$\therefore\ n = 25$

14. Official Ans. by NTA (1)

Sol.

10 Identical	21Distinct	10 Object
0	10	${}^{21}C_{10} \times 1$
1	9	${}^{21}C_9 \times 1$
⋮	⋮	⋮
10	0	${}^{21}C_0 \times 1$

${}^{21}C_0 + ... + {}^{21}C_{10} + {}^{21}C_1 + ... + {}^{21}C_0 = 2^{21}$

$({}^{21}C_0 + ... + {}^{21}C_{10}) = 2^{20}$

BINOMIAL THEOREM

1. Ans. (4)

$$\frac{2^{403}}{15} = \frac{2^3.(2^4)^{100}}{15} = \frac{8}{15}(15+1)^{100}$$

$$= \frac{8}{15}(15\lambda + 1) = 8\lambda + \frac{8}{15}$$

$\because$ 8λ is integer

$\Rightarrow$ fractional part of $\frac{2^{403}}{15}$ is $\frac{8}{15} \Rightarrow k = 8$

2. Ans. (2)

$(1 - t^6)^3 (1 - t)^{-3}$

$(1 - t^{18} - 3t^6 + 3t^{12}) (1 - t)^{-3}$

$\Rightarrow$ cofficient of t^4 in $(1 - t)^{-3}$ is

${}^{3+4-1}C_4 = {}^6C_2 = 15$

3. Ans. (3)

$$\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$$

$$\Rightarrow \sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{21}C_i}\right)^3 = \frac{k}{21}$$

$$\Rightarrow \sum_{i=1}^{20}\left(\frac{i}{21}\right)^3 = \frac{k}{21}$$

$$\Rightarrow \frac{1}{(21)^3}\left[\frac{20(21)}{2}\right]^2 = \frac{k}{21}$$

$\Rightarrow 100 = k$

4. Ans. (4)

$(1 + x^{\log_2 x})^5$

$T_3 = {}^5C_2.(x^{\log_2 x})^2 = 2560$

$\Rightarrow 10.x^{2\log_2 x} = 2560$

$\Rightarrow x^{2\log_2 x} = 256$

$\Rightarrow 2(\log_2 x)^2 = \log_2 256$

$\Rightarrow 2(\log_2 x)^2 = 8$

$\Rightarrow (\log_2 x)^2 = 4 \quad \Rightarrow \log_2 x = 2$ or -2

$x = 4$ or $\frac{1}{4}$

5. Ans. (2)

$$x^2\left({}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{\lambda}{x^2}\right)^r\right)$$

$$x^2\left[{}^{10}C_r(x)^{\frac{10-r}{2}}(\lambda)^r(x)^{-2r}\right]$$

$$x^2\left[{}^{10}C_r\ \lambda^r\ x^{\frac{10-5r}{2}}\right]$$

$\therefore\ r = 2$

Hence, ${}^{10}C_2\ \lambda^2 = 720$

$\lambda^2 = 16$

$\lambda = \pm 4$

Option (2)

6. Ans. (3)

$$\sum_{r=0}^{25} {}^{50}C_r \cdot {}^{50-r}C_{25-r}$$

$$=\sum_{r=0}^{25}\frac{50!}{r!\,(50-r)!}\times\frac{(50-r)!}{(25)!\,(25-r)!}$$

$$=\sum_{r=0}^{25}\frac{50!}{25!\,25!}\times\frac{25!}{(25-r)!\,(r!)}$$

$$= {}^{50}C_{25}\sum_{r=0}^{25}{}^{25}C_r = \left(2^{25}\right)\,{}^{50}C_{25}$$

$\therefore$ K = 2^{25}

Option (3)

7. Ans. (3)

$$T_5 = {}^8C_4\frac{x^{12}}{81}\times\frac{81}{x^4} = 5670$$

$\Rightarrow 70x^8 = 5670$

$\Rightarrow x = \pm\sqrt{3}$

8. Ans. (1)

Given sum = coefficient of x^r in the expansion of $(1 + x)^{20}(1 + x)^{20}$,

which is equal to ${}^{40}C_r$

It is maximum when r = 20

9. Ans. (4)

$(10 + x)^{50} + (10 - x)^{50}$

$\Rightarrow a_2 = 2.{}^{50}C_2 10^{48},\ a_0 = 2.10^{50}$

$$\frac{a_2}{a_0} = \frac{{}^{50}C_2}{10^2} = 12.25$$

10. Ans. (1)

${}^{101}C_1 + {}^{101}C_2S_1 + ... + {}^{101}C_{101}S_{100} = \alpha T_{100}$

${}^{101}C_1 + {}^{101}C_2(1 + q) + {}^{101}C_3(1 + q + q^2) + ...$ $+ {}^{101}C_{101}(1 + q + ... + q^{100})$

$$= 2\alpha\frac{\left(1-\left(\frac{1+q}{2}\right)^{101}\right)}{(1-q)}$$

$\Rightarrow {}^{101}C_1(1 - q) + {}^{101}C_2(1 - q^2) + ... + {}^{101}C_{101}(1 - q^{101})$

$$= 2\alpha\left(1-\left(\frac{1+q}{2}\right)^{101}\right)$$

$\Rightarrow (2^{101} - 1) - ((1 + q)^{101} - 1)$

$$= 2\alpha\left(1-\left(\frac{1+q}{2}\right)^{101}\right)$$

$$\Rightarrow 2^{101}\left(1-\left(\frac{1+q}{2}\right)^{101}\right) = 2\alpha\left(1-\left(\frac{1+q}{2}\right)^{101}\right)$$

$\Rightarrow \alpha = 2^{100}$

11. Ans. (4)

$$\frac{T_5}{T_5^1} = \frac{{}^{10}C_4(2^{1/3})^{10-4}\left(\frac{1}{2(3)^{1/3}}\right)^4}{{}^{10}C_4\left(\frac{1}{2(3^{1/3})}\right)^{10-4}(2^{1/3})^4} = 4.(36)^{1/3}$$

12. Ans. (4)

General term $T_{r+1} = {}^{60}C_r\ 7^{\frac{60-r}{5}}\ 3^{\frac{r}{10}}$

$\therefore$ for rational term, r = 0, 10, 20, 30, 40, 50, 60

$\Rightarrow$ no of rational terms = 7

$\therefore$ number of irrational terms = 54

Also, $-27\times{}^{15}C_3 + 9a\times{}^{15}C_2 - 3b\times{}^{15}C_1 = 0$

$\Rightarrow 9 \times {}^{15}C_2\ a - 45\ b - 27 \times {}^{15}C_3 = 0$

$\Rightarrow 21a - b - 273 = 0$ ...(ii)

(i) + (ii)

$-24\ a + 672 = 0$

$\Rightarrow a = 28$

So, b = 315

13. Official Ans. by NTA (2)

Sol. $2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + ... +62.{}^{20}C_{20}$

$$=\sum_{r=0}^{20}(3r+2)\,{}^{20}C_r$$

$$=3\sum_{r=0}^{20}r.\,{}^{20}C_r + 2\sum_{r=0}^{20}{}^{20}C_r$$

$$=3\sum_{r=0}^{20}r\left(\frac{20}{r}\right){}^{19}C_{r-1} + 2.2^{20}$$

$= 60.2^{19} + 2.2^{20} = 2^{25}$

14. Official Ans. by NTA (4)

Sol. $\left(x+\sqrt{x^3-1}\right)^6 + \left(x-\sqrt{x^3-1}\right)^6$

$= 2[{}^6C_0x^6 + {}^6C_2x^4(x^3 - 1) + {}^6C_4x^2(x^3 - 1)^2 + {}^6C_6(x^3 - 1)^3]$

$= 2[{}^6C_0x^6 + {}^6C_2x^7 - {}^6C_2x^4 + {}^6C_4x^8 + {}^6C_4x^2 - 2{}^6C_4x^5 + (x^9 - 1 - 3x^6 + 3x^3)]$

$\Rightarrow$ Sum of coefficient of even powers of x = 2[1 – 15 + 15 + 15 – 1 – 3] = 24

15. Official Ans. by NTA (4)

Sol. $200 = {}^6C_3\left(x^{\frac{1}{x+\log_{10}x}}\right)^{\frac{3}{2}} \times x^{\frac{1}{4}}$

$\Rightarrow 10 = x^{\frac{3}{2(1+\log_{10}x)}+\frac{1}{4}}$

$\Rightarrow 1 = \left(\frac{3}{2(1+t)}+\frac{1}{4}\right)t$

where $t = \log_{10}x$

$\Rightarrow t^2+3t-4=0$

$\Rightarrow t = 1, -4$

$\Rightarrow x = 10, 10^{-4}$

$\Rightarrow x = 10$ (As $x > 1$)

16. Official Ans. by NTA (2)

Sol. $T_4 = T_{3+1} = \binom{6}{3}\left(\frac{2}{x}\right)^3.\left(x^{\log_8 x}\right)^3$

$20\times 8^7 = \frac{160}{x^3}.x^{3\log_8 x}$

$8^6 = x^{\log_2 x} - 3$

$2^{18} = x^{\log_2 x - 3}$

$\Rightarrow 18 = (\log_2 x - 3)(\log_2 x)$

Let $\log_2 x = t$

$\Rightarrow t^2-3t-18=0$

$\Rightarrow (t-6)(t+3)=0$

$\Rightarrow t = 6, -3$

$\log_2 x = 6 \Rightarrow x = 2^6 = 8^2$

$\log_2 x = -3 \Rightarrow x = 2^{-3} = 8^{-1}$

17. Official Ans. by NTA (4)

Sol. $\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{15}$

$\frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{2}{15}$

$\frac{r}{n-r+1} = \frac{2}{15}$

$15r = 2n - 2r + 2$

$\boxed{17r = 2n+2}$

$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15}{70}$

$\frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{3}{14}$

$\frac{r+1}{n-r} = \frac{3}{14}$

$14r + 14 = 3n - 3r$

$3n - 17r = 14$

$2n - 17r = -2$

$n = 16$

$17r = 34,\ r = 2$

${}^{16}C_1,\ {}^{16}C_2,\ {}^{16}C_3$

$\frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16+120+560}{3}$

$\frac{680+16}{3} = \frac{696}{3} = 232$

18. Official Ans. by NTA (1)

Sol. Coefiicient of $x^2 = {}^{15}C_2 \times 9 - 3a\left({}^{15}C_1\right) + b = 0$

$\Rightarrow -45a + b + {}^{15}C_2 \times 9 = 0$ ....(i)

Also, $-27\times{}^{15}C_3 + 9a\times{}^{15}C_2 - 3b\times{}^{15}C_1 = 0$

$\Rightarrow 9 \times {}^{15}C_2\, a - 45\, b - 27 \times {}^{15}C_3 = 0$

$\Rightarrow 21a - b - 273 = 0$ ...(ii)

(i) + (ii)

$-24a + 672 = 0$

$\Rightarrow a = 28$

So, $b = 315$

19. Official Ans. by NTA (2)

Sol. $T_r = \sum_{r=0}^{n} {}^nC_r x^{2n-2r}.x^{-3r}$

$2n - 5r = 1 \Rightarrow 2n = 5r + 1$

for r = 15. n = 38

smallest value of n is 38.

20. Official Ans. by NTA (2)

Sol. $(1 + x)(1 - x)^{10}(1 + x + x^2)^9$

$(1 - x^2)(1 - x^3)^9$

${}^9C_6 = 84$

21. Official Ans. by NTA (1)

Sol. $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ... + {}^nC_nx^n$

Diff. w.r.t. x

$\Rightarrow n(1+x)^{n-1} = {}^nC_1 + {}^nC_2(2x) + ...+ {}^nC_n\, n(x)^{n-1}$

Multiply by x both side

$\Rightarrow nx(1+x)^{n-1} = {}^nC_1\, x + {}^nC_2(2x^2) + ...+ {}^nC_n(n\, x^n)$

Diff w.r.t. x

$\Rightarrow n\,[(1+x)^{n-1} + (n-1)x\,(1+x)^{n-2}]$

$= {}^nC_1 + {}^nC_2\, 2^2x + ...\, {}^nC_n\,(n^2)x^{n-1}$

Put x = 1 and n = 20

$\Rightarrow {}^{20}C_1 + 2^2\, {}^{20}C_2 + 3^2\, {}^{20}C_3 + ... + (20)^2\, {}^{20}C_{20}$

$= 20 \times 2^{18}[2 + 19] = 420(2^{18}) = A(2^{\beta})$

22. Official Ans. by NTA (4)

Sol. $\frac{1}{60}\left(2x^2 - \frac{3}{x^2}\right)^6 - \frac{1}{81}\cdot x^8\left(2x^2 - \frac{3}{x^2}\right)^6$

its general term

$\frac{1}{60}\,{}^6C_r 2^{6-r}(-3)^r x^{12-r} - \frac{1}{81}\,{}^6C_r 2^{6-r}(-3)^r 12^{20-4r}$

for term independent of x, r for Ist expression is 3 and r for second expression is 5

$\therefore$ term independent of x = – 36

SET

1. Ans. (4)

Let n(A) = number of students opted Mathematics = 70,

n(B) = number of students opted Physics = 46,

n(C)=number of students opted Chemistry= 28,

$n(A \cap B) = 23$,

$n(B \cap C) = 9$,

$n(A \cap C) = 14$,

$n(A \cap B \cap C) = 4$,

Now $n(A \cup B \cup C)$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$= 70 + 46 + 28 - 23 - 9 - 14 + 4 = 102$

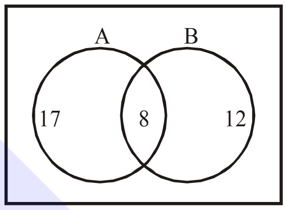
So number of students not opted for any course

$= \text{Total} - n(A \cup B \cup C)$

$= 140 - 102 = 38$

2. Official Ans. by NTA (3)

Sol. Let population = 100



$n(A) = 25$

$n(B) = 20$

$n(A \cap B) = 8$

$n(A \cap \bar{B}) = 17$

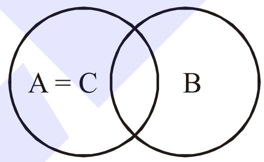
$n(\bar{A} \cap B) = 12$

$\frac{30}{100}\times 17 + \frac{40}{100}\times 12 + \frac{50}{100}\times 8$

$5.1 + 4.8 + 4 = 13.9$

3. Official Ans. by NTA (1)

Sol.



for $A = C,\ A - C = \phi$

$\Rightarrow \phi \subseteq B$

But $A \not\subseteq B$

$\Rightarrow$ option 1 is **NOT** true

Let $x \in (C\, x \in (C \cup A) \cap (C \cup B)$

$\Rightarrow x \in (C \cup A)$ and $x \in (C \cup B)$

$\Rightarrow (x \in C \text{ or } x \in A)$ and $(x \in C \text{ or } x \in B)$

$\Rightarrow x \in C \text{ or } x \in (A \cap B)$

$\Rightarrow x \in C \text{ or } x \in C \quad (\text{as } A \cup B \subseteq C)$

$\Rightarrow x \in C$

$\Rightarrow (C \cup A) \cap (C \cup B) \subseteq C \qquad (1)$

Now $x \in C \Rightarrow x \in (C \cup A)$ and $x \in (C \cup B)$

$\Rightarrow x \in (C \cup A) \cap (C \cup B)$

$\Rightarrow C \subseteq (C \cup A) \cap (C \cup B) \qquad (2)$

$\Rightarrow$ from (1) and (2)

$C = (C \cup A) \cap (C \cup B)$

$\Rightarrow$ option 2 is true

Let $x \in A$ and $x \notin B$

$\Rightarrow x \in (A - B)$

$\Rightarrow x \in C$ (as $A - B \subseteq C$)

Let $x \in A$ and $x \in B$

$\Rightarrow x \in (A \cap B)$

$\Rightarrow x \in C$ (as $A \cap B \subseteq C$)

Hence $x \in A \Rightarrow x \in C$

$\Rightarrow A \subseteq C$

$\Rightarrow$ Option 3 is true

as $C \supseteq (A \cap B)$

$\Rightarrow B \cap C \supseteq (A \cap B)$

as $A \cap B \neq \phi$

$\Rightarrow B \cap C \neq \phi$

$\Rightarrow$ Option 4 is true.

RELATION

1. Ans (3)

$A = \{x \in z : 2^{(x+2)(x^2-5x+6)} = 1\}$

$2^{(x+2)(x^2-5x+6)} = 2^0 \Rightarrow x = -2, 2, 3$

$A = \{-2, 2, 3\}$

$B = \{x \in Z : -3 < 2x - 1 < 9\}$

$B = \{0, 1, 2, 3, 4\}$

$A \times B$ has is 15 elements so number of subsets of $A \times B$ is 2^{15}.

7. Official Ans. by NTA (1)

Sol. $f(x) = \log_e\left(\frac{1-x}{1+x}\right), |x| < 1$

$$f\left(\frac{2x}{1+x^2}\right) = \ell n\left(\frac{1-\frac{2x}{1+2x^2}}{1+\frac{2x}{1+x^2}}\right)$$

$$= \ell n\left(\frac{(x-1)^2}{(x+1)^2}\right) = 2\ell n\left|\frac{1-x}{1+x}\right| = 2f(x)$$

8. Official Ans. by NTA (1)

Sol. $f(x) = a^x, a > 0$

$$f(x) = \frac{a^x + a^{-x} + a^x - a^{-x}}{2}$$

$$\Rightarrow f_1(x) = \frac{a^x + a^{-x}}{2}$$

$$f_2(x) = \frac{a^x - a^{-x}}{2}$$

$$\Rightarrow f_1(x+y) + f_1(x-y)$$

$$= \frac{a^{x+y} + a^{-x-y}}{2} + \frac{a^{x-y} + a^{-x+y}}{2}$$

$$= \frac{(a^x + a^{-x})}{2}(a^y + a^{-y})$$

$$= f_1(x) \times 2f_1(y)$$

$$= 2f_1(x)\, f_1(y)$$

9. Official Ans. by NTA (2)

Sol. From the given functional equation :

$f(x) = 2^x \quad \forall x \in N$

$2^{a+1} + 2^{a+2} + ... + 2^{a+10} = 16(2^{10} - 1)$

$2^a (2 + 2^2 + ... + 2^{10}) = 16(2^{10} - 1)$

$$2^a.\frac{2.(2^{10}-1)}{1} = 16(2^{10}-1)$$

$2^{a+1} = 16 = 2^4$

$a = 3$

10. Official Ans. by NTA (1)

Sol. $y = \frac{x^2}{1-x^2}$

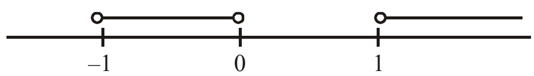
Range of y : R –[–1,0)

for surjective funciton, A must be same as above range.

11. Official Ans. by NTA (3)

Sol. $4 - x^2 \neq 0$; $x^3 - x > 0$

$x = \pm 2 \qquad x(x-1)(x+1) > 0$



$\therefore D_f \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

12. Official Ans. by NTA (3)

Sol. $g(S) = [-2, 2]$

So, $f(g(S)) = [0, 4] = S$

And $f(S) = [0, 16] \Rightarrow f(g(S) \neq f(S)$

Also, $g(f(S)) = [-4, 4] \neq g(S)$

So, $g(f(S) \neq S$

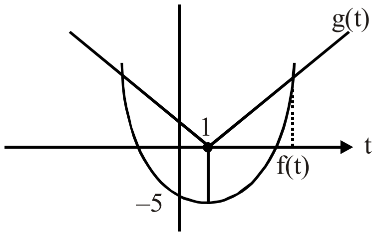
13. Official Ans. by NTA (4)

Sol. Let $2^x = t$

$5 + |t-1| = t^2 - 2t$

$\Rightarrow \underbrace{|t-1|}_{g(t)} = \underbrace{(t^2 - 2t - 5)}_{f(t)}$

From the graph



$\because t > 0$

So, number of real root is 1.

14. Official Ans. by NTA (3)

Sol. $f(x) = \sqrt{x}$, $g(x) = \tan x$, $h(x) = \dfrac{1-x^2}{1+x^2}$

$fog(x) = \sqrt{\tan x}$

$hofog(x) = h(\sqrt{\tan x}) = \dfrac{1-\tan x}{1+\tan x}$

$= -\tan\left(\dfrac{\pi}{4} - x\right)$

$\phi(x) = \tan\left(\dfrac{\pi}{4} - x\right)$

$\phi\left(\dfrac{\pi}{3}\right) = \tan\left(\dfrac{\pi}{4} - \dfrac{\pi}{3}\right) = \tan\left(-\dfrac{\pi}{12}\right) = -\tan\dfrac{\pi}{12}$

$= \tan\left(\pi - \dfrac{\pi}{12}\right) = \tan\dfrac{11\pi}{12}$

15. Official Ans. by NTA (2)

Sol. $\underbrace{\left[-\dfrac{1}{3}\right] + \left[-\dfrac{1}{3} - \dfrac{1}{100}\right] + \ldots + \left[-\dfrac{1}{3} - \dfrac{66}{100}\right]}_{(-1)67}$

$+ \underbrace{\left[-\dfrac{1}{3} - \dfrac{67}{100}\right] + \ldots + \left[-\dfrac{1}{3} - \dfrac{99}{100}\right]}_{-2(33)} = -133$

FUNCTION

1. Ans. (1)

Given $f_1(x) = \dfrac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \dfrac{1}{1-x}$

$(f_2 \circ J \circ f_1)(x) = f_3(x)$

$f_2 \circ (J(f_1(x))) = f_3(x)$

$f_2 \circ \left(J\left(\dfrac{1}{x}\right)\right) = \dfrac{1}{1-x}$

$1 - J\left(\dfrac{1}{x}\right) = \dfrac{1}{1-x}$

$J\left(\dfrac{1}{x}\right) = 1 - \dfrac{1}{1-x} = \dfrac{-x}{1-x} = \dfrac{x}{x-1}$

Now $x \to \dfrac{1}{x}$, $J(x) = \dfrac{\frac{1}{x}}{\frac{1}{x} - 1} = \dfrac{1}{1-x} = f_3(x)$

2. Ans. (1)

$f(x) = 2\left(1 + \dfrac{1}{x-1}\right)$

$f'(x) = -\dfrac{2}{(x-1)^2}$

$\Rightarrow f$ is one-one but not onto

3. Ans. (4)

$f(x) = \begin{cases} \dfrac{n+1}{2} & n \text{ is odd} \\ n/2 & n \text{ is even} \end{cases}$

$g(x) = n - (-1)^n \begin{cases} n+1 \; ; & n \text{ is odd} \\ n-1 \; ; & n \text{ is even} \end{cases}$

$f(g(n)) = \begin{cases} \dfrac{n}{2}; & n \text{ is even} \\ \dfrac{n+1}{2}; & n \text{ is odd} \end{cases}$

$\therefore$ many one but onto

Option (4)

4. Ans. (2)

$f(0) = 0$ & $f(x)$ is odd.

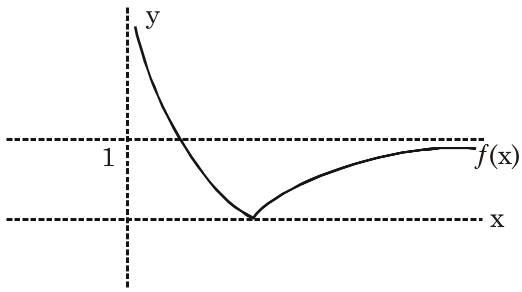
Further, if $x > 0$ then

$f(x) = \dfrac{1}{x + \frac{1}{x}} \in \left(0, \dfrac{1}{2}\right]$

Hence, $f(x) \in \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

5. Ans. (Bonus)

$f(x) = \left|1 - \dfrac{1}{|x|}\right| = \dfrac{\left||x|-1\right|}{x} = \begin{cases} \dfrac{1-x}{x} & 0 < x \le 1 \\ \dfrac{x-1}{x} & x \ge 1 \end{cases}$



$\Rightarrow f(x)$ is not injective

but range of function is $[0,\infty)$

Remark : If co-domain is $[0,\infty)$, then $f(x)$ will be surjective

6. Ans. (1)

$f(k) = 3m$ (3,6,9,12,15,18)

for k = 4,8,12,16,20   6.5.4.3.2 ways

For rest numbers 15! ways

Total ways = 6!(15!)

7. Official Ans. by NTA (1)

Sol. $f(x) = \log_e\left(\dfrac{1-x}{1+x}\right), |x|<1$

$$f\left(\frac{2x}{1+x^2}\right) = \ell n\left(\frac{1-\dfrac{2x}{1+2x^2}}{1+\dfrac{2x}{1+x^2}}\right)$$

$$= \ell n\left(\frac{(x-1)^2}{(x+1)^2}\right) = 2\ell n\left|\frac{1-x}{1+x}\right| = 2f(x)$$

8. Official Ans. by NTA (1)

Sol. $f(x) = a^x, a > 0$

$$f(x) = \frac{a^x + a^{-x} + a^x - a^{-x}}{2}$$

$$\Rightarrow f_1(x) = \frac{a^x + a^{-x}}{2}$$

$$f_2(x) = \frac{a^x - a^{-x}}{2}$$

$$\Rightarrow f_1(x+y) + f_1(x-y)$$

$$= \frac{a^{x+y} + a^{-x-y}}{2} + \frac{a^{x-y} + a^{-x+y}}{2}$$

$$= \frac{\left(a^x + a^{-x}\right)}{2}\left(a^y + a^{-y}\right)$$

$$= f_1(x) \times 2f_1(y)$$

$$= 2f_1(x)\, f_1(y)$$

9. Official Ans. by NTA (2)

Sol. From the given functional equation :

$f(x) = 2^x \quad \forall\, x \in N$

$2^{a+1} + 2^{a+2} + ... + 2^{a+10} = 16(2^{10} - 1)$

$2^a (2 + 2^2 + ... + 2^{10}) = 16(2^{10} - 1)$

$$2^a.\frac{2.\left(2^{10}-1\right)}{1} = 16\left(2^{10}-1\right)$$

$2^{a+1} = 16 = 2^4$

a = 3

10. Official Ans. by NTA (1)

Sol. $y = \dfrac{x^2}{1-x^2}$

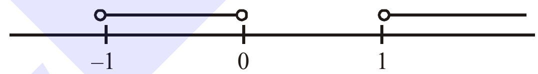
Range of y : R –[–1,0)

for surjective funciton, A must be same as above range.

11. Official Ans. by NTA (3)

Sol. $4 - x^2 \neq 0$; $x^3 - x > 0$

$x = \pm 2$ $x(x-1)(x+1) > 0$



$\therefore D_f \in (-1, 0) \cup (1, 2) \cup (2, \infty)$

12. Official Ans. by NTA (3)

Sol. g(S) = [–2, 2]

So, f(g(S)) = [0, 4] = S

And f(S) = [0, 16] $\Rightarrow$ f(g(S) $\neq$ f(S)

Also, g(f(S)) = [–4, 4] $\neq$ g (S)

So, g(f(S) $\neq$ S

13. Official Ans. by NTA (4)

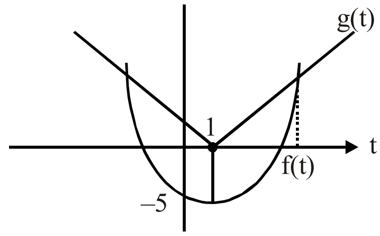
Sol. Let $2^x = t$

$5 + |t-1| = t^2 - 2t$

$\Rightarrow |t-1| = (t^2 - 2t - 5)$

  g(t)   f(t)

From the graph



$\because t > 0$

So, number of real root is 1.

14. Official Ans. by NTA (3)

Sol. $f(x) = \sqrt{x}$, $g(x) = \tan x$, $h(x) = \dfrac{1-x^2}{1+x^2}$

$fog\ (x) = \sqrt{\tan x}$

$hofog\ (x) = h(\sqrt{\tan x}) = \dfrac{1-\tan x}{1+\tan x}$

$= -\tan\left(\dfrac{\pi}{4} - x\right)$

$\phi(x) = \tan\left(\dfrac{\pi}{4} - x\right)$

$\phi\left(\dfrac{\pi}{3}\right) = \tan\left(\dfrac{\pi}{4} - \dfrac{\pi}{3}\right) = \tan\left(-\dfrac{\pi}{12}\right) = -\tan\dfrac{\pi}{12}$

$= \tan\left(\pi - \dfrac{\pi}{12}\right) = \tan\dfrac{11\pi}{12}$

15. Official Ans. by NTA (2)

Sol. $\underbrace{\left[-\dfrac{1}{3}\right] + \left[-\dfrac{1}{3} - \dfrac{1}{100}\right] + \ldots + \left[-\dfrac{1}{3} - \dfrac{66}{100}\right]}_{(-1)67}$

$+ \underbrace{\left[-\dfrac{1}{3} - \dfrac{67}{100}\right] + \ldots + \left[-\dfrac{1}{3} - \dfrac{99}{100}\right]}_{-2(33)} = -133$

INVERSE TRIGONOMETRIC FUNCTION

1. Ans. (1)

$\cos^{-1}\left(\dfrac{2}{3x}\right) + \cos^{-1}\left(\dfrac{3}{4x}\right) = \dfrac{\pi}{2} \quad \left(x > \dfrac{3}{4}\right)$

$\cos^{-1}\left(\dfrac{3}{4x}\right) = \dfrac{\pi}{2} - \cos^{-1}\left(\dfrac{2}{3x}\right)$

$\cos^{-1}\left(\dfrac{3}{4x}\right) = \sin^{-1}\left(\dfrac{2}{3x}\right)$

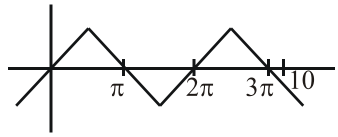
$\cos\left(\cos^{-1}\left(\dfrac{3}{4x}\right)\right) = \cos\left(\sin^{-1}\dfrac{2}{3x}\right)$

$\dfrac{3}{4x} = \dfrac{\sqrt{9x^2-4}}{3x}$

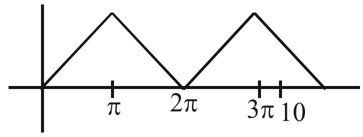
$\dfrac{81}{16} + 4 = 9x^2$

$x^2 = \dfrac{145}{16 \times 9} \Rightarrow x = \dfrac{\sqrt{145}}{12}$

2. Ans. (1)



$x = \sin^{-1}(\sin 10) = 3\pi - 10$



$y = \cos^{-1}(\cos 10) = 4\pi - 10$

$y - x = \pi$

3. Ans. (3)

$\cot\left(\sum_{n=1}^{19} \cot^{-1}(1 + n(n+1))\right)$

$\cot\left(\sum_{n=1}^{19} \cot^{-1}(n^2 + n + 1)\right) = \cot\left(\sum_{n=1}^{19} \tan^{-1}\dfrac{1}{1+n(n+1)}\right)$

$\sum_{n=1}^{19} (\tan^{-1}(n+1) - \tan^{-1} n)$

$\cot(\tan^{-1}20 - \tan^{-1}1) = \dfrac{\cot A \cot\beta + 1}{\cot\beta - \cot A}$

(Where tanA=20, tanB=1) $\dfrac{1\left(\dfrac{1}{20}\right)+1}{1-\dfrac{1}{20}} = \dfrac{21}{19}$

$\therefore$ Option (3)

4. Ans. (3)

$\cot^{-1}x > 5$ (reject), $\cot^{-1}x < 2$

$\therefore\ x > \cot 2$

$\therefore\ x \in (\cot 2, \infty)$

5. Ans. (4)

$\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$

$\Rightarrow \dfrac{5x}{1-6x^2} = 1$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$x = -1$ or $x = \dfrac{1}{6}$

$x = \dfrac{1}{6} \quad \because x > 0$

6. Official Ans. by NTA (1)

Sol. $\cos\alpha = \frac{3}{5}, \tan\beta = \frac{1}{3}$

$\Rightarrow \tan\alpha = \frac{4}{3}$

$\Rightarrow \tan(\alpha-\beta) = \frac{\frac{4}{3}-\frac{1}{3}}{1+\frac{4}{3}.\frac{1}{3}} = \frac{9}{13}$

$\Rightarrow \sin(\alpha-\beta) = \frac{9}{5\sqrt{10}}$

$\Rightarrow \alpha-\beta = \sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

7. Official Ans. by NTA (3)

Sol. $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$

$\cos(\cos^{-1}x - \cos^{-1}\frac{y}{2}) = \cos\alpha$

$\Rightarrow x\times\frac{y}{2}+\sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}} = \cos\alpha$

$\Rightarrow \left(\cos\alpha-\frac{xy}{2}\right)^2 = (1-x^2)\left(1-\frac{y^2}{4}\right)$

$x^2+\frac{y^2}{4} - xy\cos\alpha = 1-\cos^2\alpha = \sin^2\alpha$

8. Official Ans. by NTA (3)

Sol. $\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$

$\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right)$

$=\sin^{1}\left(\frac{33}{65}\right)=\cos^{-1}\left(\frac{56}{65}\right)=\frac{\pi}{2}-\sin^{1}\left(\frac{56}{65}\right)$

LIMIT

1. Ans. (1)

$\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

$= \lim_{y\to 0}\frac{1+\sqrt{1+y^4}-2}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$

$= \lim_{y\to 0}\frac{\left(\sqrt{1+y^4}-1\right)\left(\sqrt{1+y^4}+1\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$

$= \lim_{y\to 0}\frac{1+y^4-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$

$= \lim_{y\to 0}\frac{1}{\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)} = \frac{1}{4\sqrt{2}}$

2. Ans. (1)

$\lim_{x\to 0^-}\frac{x([x]+|x|)\sin[x]}{|x|}$

$x\to 0^-$

$[x]=-1 \Rightarrow \lim_{x\to 0^-}\frac{x(-x-1)\sin(-1)}{-x} = -\sin 1$

$|x| = -x$

3. Ans. (4)

$\lim_{x\to 1^+}\frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$

$=\lim_{x\to 1^+}\frac{(1-x)+\sin(x-1)}{(x-1)(-1)}\sin\left(\frac{\pi}{2}(-1)\right)$

$=\lim_{x\to 1^+}\left(1-\frac{\sin(x-1)}{(x-1)}\right)(-1) = (1-1)(-1) = 0$

4. Ans. (4)

R.H.L. $= \lim_{x\to 0^+}\frac{\tan(\pi\sin^2x)+(|x|-\sin(x[x]))^2}{x^2}$

(as $x\to 0^+ \Rightarrow [x]=0$)

$= \lim_{x\to 0^+}\frac{\tan(\pi\sin^2x)+x^2}{x^2}$

$= \lim_{x\to 0^+}\frac{\tan(\pi\sin^2x)}{(\pi\sin^2x)}+1 = \pi+1$

L.H.L. $= \lim_{x\to 0^-}\frac{\tan(\pi\sin^2x)+(-x+\sin x)^2}{x^2}$

(as $x \to 0^- \Rightarrow [x] = -1$)

$$\lim_{x\to 0+} \frac{\tan\left(\pi\sin^2 x\right)}{\pi\sin^2 x} \cdot \frac{\pi\sin^2 x}{x^2} + \left(-1+\frac{\sin x}{x}\right)^2 \Rightarrow \pi$$

R.H.L. ≠ L.H.L.

5. Ans. (4)

$$\lim_{x\to 0} \frac{x\tan^2 2x}{\tan 4x\sin^2 x} = \lim_{x\to 0} \frac{x\left(\frac{\tan^2 2x}{4x^2}\right)4x^2}{\left(\frac{\tan 4x}{4x}\right)4x\left(\frac{\sin^2 x}{x^2}\right)x^2} = 1$$

6. Ans. (3)

$$\lim_{x\to\pi/4} \frac{\cot^3 x - \tan x}{\cos\left(x+\frac{\pi}{4}\right)}$$

$$\lim_{x\to\pi/4} \frac{(1-\tan^4 x)}{\cos(x+\pi/4)}$$

$$2\lim_{x\to\pi/4} \frac{(1-\tan^2 x)}{\cos(x+\pi/4)}$$

$$R\lim_{x\to\pi/4} \frac{\cos^2 x - \sin^2 x}{\frac{\cos x - \sin x}{\sqrt{2}}} \frac{1}{\cos^2 x}$$

$$4\sqrt{2}\lim_{x\to\pi/4}(\cos x + \sin x) = 8$$

7. Ans. (3)

$$\lim_{x\to 1^-} \frac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}} \times \frac{\sqrt{\pi}+\sqrt{2\sin^{-1}x}}{\sqrt{\pi}+\sqrt{2\sin^{-1}x}}$$

$$\lim_{x\to 1^-} \frac{2\left(\frac{\pi}{2}-\sin^{-1}x\right)}{\sqrt{1-x}\cdot\left(\sqrt{\pi}+\sqrt{2\sin^{-1}x}\right)}$$

$$\lim_{x\to 1^-} \frac{2\cos^{-1}x}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{\pi}}$$

Put $x = \cos\theta$

$$\lim_{\theta\to 0^+} \frac{2\theta}{\sqrt{2}\sin\left(\frac{\theta}{2}\right)} \cdot \frac{1}{2\sqrt{\pi}} = \sqrt{\frac{2}{\pi}}$$

8. Official Ans. by NTA (2)

Sol. $$\lim_{x\to 0} \frac{\left(\frac{\sin^2 x}{x^2}\right)\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{\left(\frac{1-\cos x}{x^2}\right)}$$

$$= \frac{(1)^2.\left(2\sqrt{2}\right)}{\frac{1}{2}} = 4\sqrt{2}$$

9. Official Ans. by NTA (4)

Sol. $$\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}} \quad (1^\infty \text{ form})$$

$$\Rightarrow e^{\lim_{x\to 0}\frac{f(3+x)-f(2-x)-f(3)+f(2)}{x(1+f(2-x)-f(2))}}$$

using L'Hopital

$$\Rightarrow e^{\lim_{x\to 0}\frac{f'(3+x)+f'(2-x)}{-xf'(2-x)+(1+f(2-x)-f(2))}}$$

$$\Rightarrow e^{\frac{f'(3)+f'(2)}{1}} = 1$$

10. Official Ans. by NTA (4)

Sol. $f(x) = [x] - \left[\frac{x}{4}\right]$

$$\lim_{x\to 4+} f(x) = \lim_{x\to 4+}\left(\left([x]-\left[\frac{x}{4}\right]\right)\right) = 4-1 = 3$$

$$\lim_{x\to 4-} f(x) = \lim_{x\to 4-}\left([x]-\frac{x}{4}\right) = 3-0 = 3$$

f(x) = 3

∴ continuous at x = 4

11. Official Ans. by NTA (4)

Sol. $$\lim_{x\to 1}\frac{x^4-1}{x-1} = \lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$$

$$\Rightarrow \lim_{x\to 1}(x+1)(x^2+1) = \frac{k^2+k^2+k^2}{2k}$$

$\Rightarrow$ k = 8/3

12. Official Ans. by NTA (1)

Sol. $\lim_{x\to 1}\frac{x^2-ax+b}{x-1}=5$

$1 - a + b = 0$ ...(i)

$2 - a = 5$ ...(ii)

$\Rightarrow a + b = -7.$

13. Official Ans. by NTA (2)

Sol. Rationalize

$$\lim_{x\to 0}\frac{(x+2\sin x)\left(\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x-x+1}\right)}{x^2+2\sin x+1-\sin^2 x+x-1}$$

$$\lim_{x\to 0}\frac{(x+2\sin x)(2)}{x^2+2\sin x-\sin^2 x+x}$$

$\frac{0}{0}$ form using L' hospital

$$\Rightarrow \lim_{x\to 0}\frac{(1+2\cos x)\times 2}{2x+2\cos x-2\sin x\cos x+1}\Rightarrow\frac{2\times 3}{(2+1)}=2$$

14. Official Ans. by NTA (1)

Sol. Maxima of f(x) occured at x = 2 i.e. $\alpha = 2$

Minima of g(x) occured at x = –1 i.e. $\beta = -1$

$$\therefore \lim_{x\to 2}\frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}=\frac{1}{2}$$

CONTINUITY

1. Ans. (4)

$$f(x)=\begin{cases}5 & \text{if } x\le 1\\ a+bx & \text{if } 1<x<3\\ b+5x & \text{if } 3\le x<5\\ 30 & \text{if } x\ge 5\end{cases}$$

$f(1)=5,\ f(1^-)=5,\ f(1^+)=a+b$

$f(3^-) = a + 3b,\ f(3) = b + 15,\ f(3^+) = b + 15$

$f(5^-) = b + 25$; $f(5) = 30$ $f(5^+) = 30$

from above we concluded that f is not continuous for any values of a and b.

2. Official Ans. by NTA (4)

Sol.
$$f(x)=\begin{cases}-(x+1) & , -1\le x<0\\ x & , 0\le x<1\\ 2x & , 1\le x<2\\ x+2 & , 2\le x<3\\ x+3 & , x=3\end{cases}$$

function discontinuous at $x = 0,1,3$

3. Official Ans. by NTA (1)

Sol. $\therefore$ function should be continuous at $x=\frac{\pi}{4}$

$$\therefore \lim_{x\to\frac{\pi}{4}} f(x)=f\left(\frac{\pi}{4}\right)$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\cos x-1}{\cot x-1}=k$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{-\sqrt{2}\sin x}{-\text{cosec}^2 x}=k \quad (\text{Using L'H}\hat{o}\text{pital rule})$$

$$\lim_{x\to\frac{\pi}{4}}\sqrt{2}\sin^3 x=k$$

$$\Rightarrow k=\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)^3=\frac{1}{2}$$

4. Official Ans. by NTA (1)

Sol. f(x) →
- $a|\pi - x| + 1; x \ge 5$
- $b|\pi - x| + 3; x > 5$

$a|\pi - 5| + 1 = b|5 - \pi| + 3$

$a(5 - \pi) + 1 = b(5 - \pi) + 3$

$(a - b)(5 - \pi) = 2$

$$a-b=\frac{2}{5-\pi}$$

5. Official Ans. by NTA (4)

Sol. $\text{RHL}=\lim_{x\to 0^+}\frac{\sqrt{x+x^2}-\sqrt{x}}{x^{\frac{3}{2}}}=\lim_{x\to 0^+}\frac{\sqrt{1+x}-1}{x}=\frac{1}{2}$

$$\text{LHL}=\lim_{x\to 0}\frac{\sin(p+1)x+\sin x}{x}=(p+1)+1=p+2$$

for continuity LHL = RHL = f(0)

$$\Rightarrow (p,q)=\left(\frac{-3}{2},\frac{1}{2}\right)$$

DIFFERENTIABILITY

1. Ans. (4)

$|f(x) - f(y)| \le 2|x - y|^{3/2}$

divide both sides by $|x - y|$

$$\left|\frac{f(x)-f(y)}{x-y}\right| \le 2.|x-y|^{1/2}$$

apply limit $x \to y$

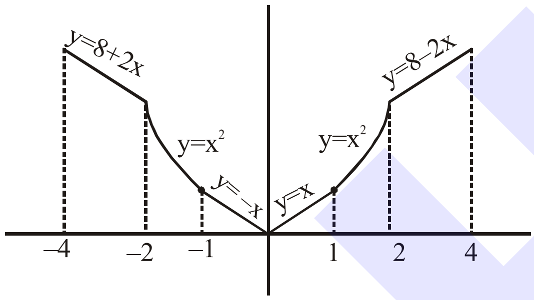
$|f'(y)| \le 0 \Rightarrow f'(y) = 0$

$\Rightarrow f(y) = c \Rightarrow f(x) = 1$

$$\int_0^1 1.dx = 1$$

2. Ans. (3)

$$f(x) = \begin{cases} 8+2x, & -4 \le x < -2 \\ x^2, & -2 \le x \le -1 \\ |x|, & -1 < x < 1 \\ x^2, & 1 \le x \le 2 \\ 8-2x, & 2 < x \le 4 \end{cases}$$



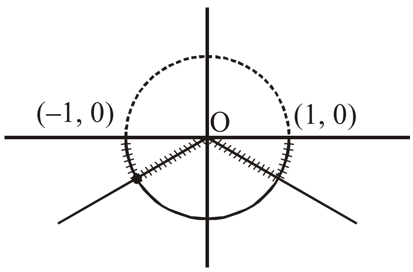
$f(x)$ is not differentiable at $x = \{-2,-1,0,1,2\}$

$\Rightarrow S = \{-2, -1, 0, 1, 2\}$

3. Ans. (1)

$f : (-1, 1) \to R$

$$f(x) = \max\left\{-|x|, -\sqrt{1-x^2}\right\}$$



Non-derivable at 3 points in (–1, 1)

Option (1)

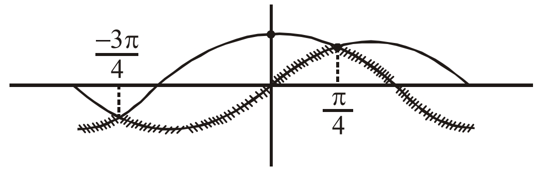
4. Ans. (3)

$f(x) = \sin|x| - |x| + 2(x - \pi)\cos x$

$\because \sin|x| - |x|$ is differentiable function at x=0

$\therefore k = \phi$

5. Ans. (1)



6. Ans. (4)

$$|f(x)| = \begin{cases} 1 & , & -2 \le x < 0 \\ 1-x^2 & , & 0 \le x < 1 \\ x^2-1 & , & 1 \le x \le 2 \end{cases}$$

and $f(|x|) = x^2 - 1,\ x \in [-2, 2]$

$$\text{Hence } g(x) = \begin{cases} x^2 & , & x \in [-2,0) \\ 0 & , & x \in [0,1) \\ 2(x^2-1) & , & x \in [1,2] \end{cases}$$

It is not differentiable at x = 1

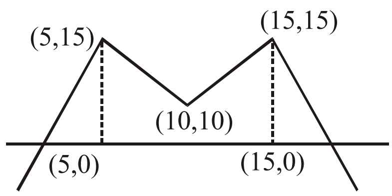
7. Official Ans. by NTA (3)

Sol. $f(x) = 15 - |x - 10|,\ x \in R$

$f(f(x)) = 15 - |f(x) - 10|$

$= 15 - |15 - |x - 10| - 10|$

$= 15 - |5 - |x - 10||$



x = 5, 10, 15 are points of non differentiability

Aliter :

At x = 10 $f(x)$ is non differentiable

also, when $15 - |x - 10| = 10$

$\Rightarrow x = 5, 15$

$\therefore$ non differentiability points are {5, 10, 15}

8. Official Ans. by NTA (1)

Sol. $$g'(c) = \lim_{h\to0}\frac{|f(c+h)|-|f(c)|}{h}$$

$$= \lim_{h\to0}\frac{|f(c+h)|}{h} = \lim_{h\to0}\frac{|f(c+h)-f(c)|}{h}$$

$$= \lim_{h\to 0}\left|\frac{f(c+h)-f(c)}{h}\right|\frac{|h|}{h}$$

$$= \lim_{h\to 0}|f'(c)|\frac{|h|}{h}=0,\ \text{ if } f'(c) = 0$$

i.e., g(x) is differentiable at x = c, if f '(c) = 0

METHOD OF DIFFERENTIATION

1. Ans. (4)

$$\frac{dx}{dt}=3\sec^2 t$$

$$\frac{dy}{dt}=3\sec t\tan t$$

$$\frac{dy}{dx}=\frac{\tan t}{\sec t}=\sin t$$

$$\frac{d^2y}{dx^2}=\cos t\frac{dt}{dx}$$

$$=\frac{\cos t}{3\sec^2 t}=\frac{\cos^3 t}{3}\quad=\frac{1}{3.2\sqrt{2}}=\frac{1}{6\sqrt{2}}$$

2. Ans. (2)

$f(x) = x^3 + x^2f'(1) + xf''(2) + f'''(3)$

$\Rightarrow f'(x) = 3x^2 + 2xf'(1) + f''(x)$ .....(1)

$\Rightarrow f''(x) = 6x + 2f'(1)$ .....(2)

$\Rightarrow f'''(x) = 6$ .....(3)

put x = 1 in equation (1) :

$f'(1) = 3 + 2f'(1) + f''(2)$ .....(4)

put x = 2 in equation (2) :

$f''(2) = 12 + 2f'(1)$ .....(5)

from equation (4) & (5) :

$-3 - f'(1) = 12 + 2f'(1)$

$\Rightarrow 3f'(1) = -15$

$\Rightarrow f'(1) = -5 \quad \Rightarrow \quad f''(2) = 2$ ....(2)

put x = 3 in equation (3) :

$f'''(3) = 6$

$\therefore f(x) = x^3 - 5x^2 + 2x + 6$

$f(2) = 8 - 20 + 4 + 6 = -2$

3. Ans. (3)

Differentiating with respect to x,

$$x.\frac{1}{\ell nx}.\frac{1}{x}+\ell n(\ell nx)-2x+2y.\frac{dy}{dx}=0$$

at x = e we get

$$1-2e+2y\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{2e-1}{2y}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2e-1}{2\sqrt{4+e^2}} \quad \text{as } y(e)=\sqrt{4+e^2}$$

4. Ans. (4)

$(2x)^{2y} = 4e^{2x-2y}$

$2y\ell n2x = \ell n4 + 2x - 2y$

$$y=\frac{x+\ell n2}{1+\ell n2x}$$

$$y'=\frac{(1+\ell n2x)-(x+\ell n2)\frac{1}{x}}{(1+\ell n2x)^2}$$

$$y'(1+\ell n2x)^2=\left[\frac{x\ell n2x-\ell n2}{x}\right]$$

5. Ans (1)

$$\frac{f'(x)}{f(x)}=1\ \forall\ x \in R$$

Intergrate & use f(1) = 2

$f(x) = 2e^{x-1} \Rightarrow f'(x) = 2e^{x-1}$

$h(x) = f(f(x)) \Rightarrow h'(x) = f'(f(x))\, f'(x)$

$h'(1) = f'(f(1))\, f'(1)$

$= f'(2)\, f'(1)$

$= 2e \cdot 2 = 4e$

6. Official Ans. by NTA (4)

Sol. Consider $\cot^{-1}\left(\dfrac{\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x}{\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\sin x}\right)$

$$=\cot^{-1}\left(\frac{\sin\left(x+\frac{\pi}{3}\right)}{\cos\left(x+\frac{\pi}{3}\right)}\right)$$

$$= \cot^{-1}\left(\tan\left(x+\frac{\pi}{3}\right)\right) = \frac{\pi}{2} - \tan^{-1}\left(\tan\left(x+\frac{\pi}{3}\right)\right)$$

$$\begin{cases} \frac{\pi}{2}-\left(x+\frac{\pi}{3}\right)=\left(\frac{\pi}{6}-x\right); & 0<x<\frac{\pi}{6} \\ \frac{\pi}{2}-\left(\left(x-\frac{\pi}{3}\right)-\pi\right)=\left(\frac{7\pi}{6}-x\right); & \frac{\pi}{6}<x<\frac{\pi}{2} \end{cases}$$

$$\therefore \quad 2y = \begin{cases} \left(\frac{\pi}{6}-x\right)^2; & 0<x<\frac{\pi}{6} \\ \left(\frac{7\pi}{6}-x\right)^2; & \frac{\pi}{6}<x<\frac{\pi}{2} \end{cases}$$

$$\therefore \quad 2\frac{dy}{dx} = \begin{cases} 2\left(\frac{\pi}{6}-x\right).(-1); & 0<x<\frac{\pi}{6} \\ 2\left(\frac{7\pi}{6}-x\right).(-1); & \frac{\pi}{6}<x<\frac{\pi}{2} \end{cases}$$

7. Official Ans. by NTA (2)

Sol. $y = f(f(f(x))) + (f(x))^2$

$$\frac{dy}{dx} = f'(f(f(x)))f'(f(x))f'(x) + 2f(x)f'(x)$$

$= f'(1)f'(1)f'(1) + 2f(1)f'(1)$

$= 3 \times 5 \times 3 + 2 \times 1 \times 3$

$= 27 + 6$

$= 33$

8. Official Ans. by NTA (4)

Sol. $fog(x) = (-x) \Rightarrow (fg(\alpha)) = -\alpha = b$

$(fg(x))' = -1 \Rightarrow (fg(\alpha))' = -1 = a$

9. Official Ans. by NTA (1)

Sol. $e^y = xy = e$

differentiate w.r.t. x

$$e^y\frac{dy}{dx} + x\frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx}(x+e^y) = -y, \quad \left.\frac{dy}{dx}\right|_{(0,1)} = -\frac{1}{e}$$

again differentiate w.r.t. x

$$e^y \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot e^y \cdot \frac{dy}{dx} + x\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = 0$$

$$(x+e^y)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 \cdot e^y + 2\frac{dy}{dx} = 0$$

$$e\frac{d^2y}{dx^2} + \frac{1}{e^2}e + 2\left(-\frac{1}{e}\right) = 0$$

$$\therefore \quad \frac{d^2y}{dx^2} = \frac{1}{e^2}$$

10. Official Ans. by NTA (4)

Sol. $f(x) = \tan^{-1}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$

$$= \tan^{-1}\left(\frac{\tan x - 1}{\tan x + 1}\right) = \tan^{-1}\left(\tan\left(x - \frac{\pi}{4}\right)\right)$$

$$\because \quad x - \frac{\pi}{4} \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$$

$$\therefore \quad f(x) = x - \frac{\pi}{4}$$

$\Rightarrow$ its derivative w.r.t. $\frac{x}{2}$ is $\frac{1}{1/2} = 2$

INDEFINITE INTEGRATION

1. (1 or 3)

Put $(x^2 - 1) = 1$

$\Rightarrow 2xdx = dt$

$$\therefore \quad I = \frac{1}{2}\int\sqrt{\frac{1-\cos t}{1+\cos t}}\,dt$$

$$= \frac{1}{2}\int \tan\left(\frac{t}{2}\right)dt$$

$$= \ln\left|\sec\frac{t}{2}\right| + c$$

$$I = \ln\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$$

2. Ans. (4)

$$\int \frac{5x^8 + 7x^6}{\left(x^2 + 1 + 2x^7\right)^2}dx$$

$$= \int \frac{5x^{-6} + 7x^{-8}}{\left(\frac{1}{x^7} + \frac{1}{x^5} + 2\right)^2}dx = \frac{1}{2 + \frac{1}{x^5} + \frac{1}{x^7}} + C$$

As $f(0) = 0$, $f(x)=\frac{x^7}{2x^7+x^2+1}$

$f(1)=\frac{1}{4}$

3. Ans. (3)

$\int\frac{(\sin^n\theta-\sin\theta)^{1/n}\cos\theta}{\sin^{n+1}\theta}d\theta$

$=\int\frac{\sin\theta\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{1/n}}{\sin^{n+1}\theta}d\theta$

Put $1-\frac{1}{\sin^{n-1}\theta}=t$

So $\frac{(n-1)}{\sin^n\theta}\cos\theta d\theta=dt$

Now $\frac{1}{n-1}\int(t)^{1/n}dt$

$=\frac{1}{(n-1)}\frac{(t)^{\frac{1}{n}+1}}{\frac{1}{n}+1}+C$

$=\frac{1}{(n-1)}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{1}{n}+1}+C$

4. Ans. (1)

$\int x^5\cdot e^{-4x^3}dx=\frac{1}{48}e^{-4x^3}f(x)+c$

Put $x^3 = t$

$3x^2\,dx = dt$

$\int x^3\cdot e^{-4x^3}\cdot x^2dx$

$\frac{1}{3}\int t\cdot e^{-4t}\,dt$

$\frac{1}{3}\left[t\cdot\frac{e^{-4t}}{-4}-\int\frac{e^{-4t}}{-4}dt\right]$

$-\frac{e^{-4t}}{48}[4t+1]+c$

$\frac{-e^{-4x^3}}{48}[4x^3+1]+c$

$\therefore f(x) = -1 - 4x^3$

Option (1)

(From the given options (1) is most suitable)

5. Ans. (2)

$\int\frac{\sqrt{1-x^2}}{x^4}dx=A(x)\left(\sqrt{1-x^2}\right)^m+C$

$\int\frac{|x|\sqrt{\frac{1}{x^2}-1}}{x^4}dx$,

Put $\frac{1}{x^2}-1=t \Rightarrow \frac{dt}{dx}=\frac{-2}{x^3}$

Case-1 $x \ge 0$

$-\frac{1}{2}\int\sqrt{t}\,dt \Rightarrow -\frac{t^{3/2}}{3}+C$

$\Rightarrow -\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{3/2} \Rightarrow \frac{\left(\sqrt{1-x^2}\right)^3}{-3x^2}+C$

$A(x)=-\frac{1}{3x^3}$ and m = 3

$(A(x))^m=\left(-\frac{1}{3x^3}\right)^3=-\frac{1}{27x^9}$

Case-II $x \le 0$

We get $\frac{\left(\sqrt{1-x^2}\right)^3}{-3x^3}+C$

$A(x)=\frac{1}{-3x^3}$, m = 3

$(A(x))^m=\frac{-1}{27x^9}$

6. Ans. (1)

$\sqrt{2x-1}=t\Rightarrow 2x-1=t^2\Rightarrow 2dx=2t.dt$

$\int\frac{x+1}{\sqrt{2x-1}}dx=\int\frac{\frac{t^2+1}{2}+1}{t}t\,dt=\int\frac{t^2+3}{2}dt$

$=\frac{1}{2}\left(\frac{t^3}{3}+3t\right)=\frac{t}{6}\left(t^2+9\right)+c$

$=\sqrt{2x-1}\left(\frac{2x-1+9}{6}\right)+c=\sqrt{2x-1}\left(\frac{x+4}{3}\right)+c$

$\Rightarrow f(x)=\frac{x+4}{3}$

7. Ans. (2)

$I=\int\cos(\ell n\,x)dx$

$I=\cos(\ln x).x+\int\sin(\ell n\,x)\,dx$

$\cos(\ell n\,x)x+[\sin(\ell n\,x).x-\int\cos(\ell n\,x)dx]$

$I=\frac{x}{2}[\sin(\ell n\,x)+\cos(\ell n\,x)]+C$

8. Ans (2)

$$\int \frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx$$

$$\int \frac{\left(\frac{3}{x^3}+\frac{2}{x^5}\right)dx}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}$$

Let $\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right) = t$

$$-\frac{1}{2}\int\frac{dt}{t^4}=\frac{1}{6t^3}+C \Rightarrow \frac{x^{12}}{6(2x^4+3x^2+1)^3}+C$$

9. Official Ans. by NTA (3)

Sol. $$\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx=\int\frac{2\sin\frac{5x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}dx$$

$$=\int\frac{\sin 3x+\sin 2x}{\sin x}dx$$

$$=\int\frac{3\sin x-4\sin^3 x-2\sin x\cos x}{\sin x}dx$$

$$=\int(3-4\sin^2 x+2\cos x)dx$$

$$=\int\left(3-2\left(1-\cos 2x\right)+2\cos x\right)dx$$

$$=\int\left(1+2\cos 2x+2\cos x\right)dx$$

$= x + \sin 2x + 2\sin x + c$

10. Official Ans. by NTA (4)

Sol. $$\int\frac{dx}{x^3\left(1+x^6\right)^{2/3}}=xf(x)(1+x^6)^{1/3}+c$$

$$\int\frac{dx}{x^7\left(\frac{1}{x^6}+1\right)^{2/3}}=xf(x)(1+x^6)^{1/3}+c$$

Let $t=\frac{1}{x^6}+1$

$$dt=\frac{-6}{x^7}dx$$

$$-\frac{1}{6}\int\frac{dt}{t^{2/3}}=-\frac{1}{2}t^{1/3}$$

$$=-\frac{1}{2}\left(\frac{1}{x^6}+1\right)^{1/3}=-\frac{1}{2}\frac{\left(1+x^6\right)^{1/3}}{x^2}$$

$$\therefore\ f(x)=-\frac{1}{2x^3}$$

11. Official Ans. by NTA (4)

Sol. $$I=\int\frac{dx}{\left(\sin x\right)^{4/3}.\left(\cos x\right)^{2/3}}$$

$$I=\int\frac{dx}{\left(\frac{\sin x}{\cos x}\right)^{4/3}.\cos^2 x}$$

$$\Rightarrow I=\int\frac{\sec^2 x}{\left(\tan x\right)^{4/3}}dx$$

put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$$\therefore\ I=\int\frac{dt}{t^{4/3}} \Rightarrow I=\frac{-3}{t^{1/3}}+c$$

$$\Rightarrow I=\frac{-3}{\left(\tan x\right)^{1/3}}+c$$

12. Official Ans. by NTA (4)

Sol. $$\int\frac{dx}{\left(\left(x-1\right)^2+9\right)^2}=\frac{1}{27}\int\cos^2\theta\, d\theta$$

(Put $x-1= 3\tan\theta$)

$$=\frac{1}{54}\int\left(1+\cos 2\theta\right)d\theta=\frac{1}{54}\left(\theta+\frac{\sin 2\theta}{2}\right)+C$$

$$=\frac{1}{54}\left(\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{3\left(x-1\right)}{x^2-2x+10}\right)+C$$

13. Official Ans. by NTA (1)

Sol. Let $x^2 = t$ $\qquad$ $2xdx = dt$

$$\Rightarrow \frac{1}{2}\int t^2.e^{-t}dt=\frac{1}{2}\left[-t^2.e^{-t}+\int 2t.e^{-t}.dt\right]$$

$$=\frac{1}{2}\left(-t^2.e^{-t}\right)+\left(-t.e^{-t}+\int 1.e^{-t}.dt\right)$$

$= -\frac{t^2e^{-t}}{2} - te^{-t} - e^{-t} = \left(-\frac{t^2}{2} - t - 1\right)e^{-t}$

$= \left(-\frac{x^4}{2} - x^2 - 1\right)e^{-x^2} + C$

$g(x) = -1 - x^2 - \frac{x^4}{2} + ke^{x^2}$

for k = 0

$g(-1) = -1 - 1 - \frac{1}{2} = -\frac{5}{2}$

14. Official Ans. by NTA (1)

Sol. $\int \frac{2x^3 - 1}{x^4 + x}dx$

$\int \frac{2x - \frac{1}{x^2}}{x^2 + \frac{1}{x}}dx$

$x^2 + \frac{1}{x} = t$

$\left(2x - \frac{1}{x^2}\right)dx = dt$

$\int \frac{dt}{t} = \ell n(t) + C$

$= \ell n\left(x^2 + \frac{1}{x}\right) + C$

15. Official Ans. by NTA (3)

Sol. $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha}dx = \int \frac{\sin(x+\alpha)}{\sin(x-\alpha)}dx$

Let x – α = t

$\Rightarrow \int \frac{\sin(t + 2\alpha)}{\sin t}dt = \int \cos 2\alpha dt + \int \cot(t)\sin 2\alpha dt$

$= t.\cos 2\alpha + \ell n|\sin t|.\sin 2\alpha + C$

$= (x - \alpha)\cos 2\alpha + \ln|\sin(x - \alpha)|.\sin 2\alpha + C$

16. Official Ans. by NTA (4)

Sol. $\int e^{\sec x}(\sec x \tan x\, f(x) + (\sec x \tan x + \sec^2 x)\,dx$

$= e^{\sec x} f(x) + C$

Diff. both sides w.r.t. 'x'

$e^{\sec x}(\sec x \tan x\, f(x) + (\sec x \tan x + \sec^2 x))$

$= e^{\sec x} \cdot \sec x \tan x\, f(x) + e^{\sec x} f'(x)$

$f'(x) = \sec^2 x + \tan x \sec x$

$\Rightarrow f(x) = \tan x + \sec x + c$

DEFINITE INTEGRATION

1. Ans. (4)

$\int_0^{\pi} |\cos x|^3\,dx = \int_0^{\pi/2} \cos^3 x\,dx - \int_{\pi/2}^{\pi} \cos^3 x\,dx$

$= \int_0^{\pi/2}\left(\frac{\cos 3x + 3\cos x}{4}\right)dx - \int_{\pi/2}^{\pi}\left(\frac{\cos 3x + 3\cos x}{4}\right)dx$

$= \frac{1}{4}\left[\left(\frac{\sin 3x}{3} + 3\sin x\right)_0^{\pi/2} - \left(\frac{\sin 3x}{3} + 3\sin x\right)_{\pi/2}^{\pi}\right]$

$= \frac{1}{4}\left[\left(\frac{-1}{3} + 3\right) - (0+0) - \left\{(0+0) - \left(\frac{-1}{3} + 3\right)\right\}\right]$

$= \frac{4}{3}$

2. Ans. (1)

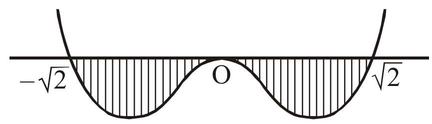
$\frac{1}{\sqrt{2k}}\int_0^{\pi/3} \frac{\tan\theta}{\sqrt{\sec\theta}}d\theta = \frac{1}{\sqrt{2k}}\int_0^{\pi/3}\frac{\sin\theta}{\sqrt{\cos\theta}}d\theta$

$= -\frac{1}{\sqrt{2k}}2\sqrt{\cos\theta}\Big|_0^{\pi/3} = -\frac{\sqrt{2}}{\sqrt{k}}\left(\frac{1}{\sqrt{2}} - 1\right)$

given it is $1 - \frac{1}{\sqrt{2}} \Rightarrow k = 2$

3. Ans. (2)

Let $f(x) = x^2(x^2 - 2)$



As long as $f(x)$ lie below the x-axis, definite integral will remain negative,

so correct value of (a, b) is $(-\sqrt{2}, \sqrt{2})$ for minimum of I

4. Ans. (4)

$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{[x] + [\sin x] + 4}$

$= \int_{-\frac{\pi}{2}}^{-1}\frac{dx}{-2-1+4} + \int_{-1}^{0}\frac{dx}{-1-1+4} + \int_0^1 \frac{dx}{0+0+4} + \int_1^{\frac{\pi}{2}}\frac{dx}{1+0+4}$

$$\int_{-\frac{\pi}{2}}^{-1}\frac{dx}{1}+\int_{-1}^{0}\frac{dx}{2}+\int_{0}^{1}\frac{dx}{4}+\int_{1}^{\frac{\pi}{2}}\frac{dx}{5}$$

$$\left(-1+\frac{\pi}{2}\right)+\frac{1}{2}(0+1)+\frac{1}{4}+\frac{1}{5}\left(\frac{\pi}{2}-1\right)$$

$$-1+\frac{1}{2}+\frac{1}{4}-\frac{1}{5}+\frac{\pi}{2}+\frac{\pi}{10}$$

$$\frac{-20+10+5-4}{20}+\frac{6\pi}{10}$$

$$\frac{-9}{20}+\frac{3\pi}{5}$$

Option (4)

5. Ans. (2)

$$\int_0^x f(t)\,dt = x^2+\int_x^1 t^2 f(t)\,dt \qquad f'\left(\frac{1}{2}\right)=?$$

Differentiate w.r.t. 'x'

$f(x) = 2x + 0 - x^2 f(x)$

$$f(x)=\frac{2x}{1+x^2}\ \Rightarrow\ f'(x)=\frac{(1+x^2)2-2x(2x)}{(1+x^2)^2}$$

$$f'(x)=\frac{2x^2-4x^2+2}{(1+x^2)^2}$$

$$f'\left(\frac{1}{2}\right)=\frac{2-2\left(\frac{1}{4}\right)}{\left(1+\frac{1}{4}\right)^2}=\frac{\left(\frac{3}{2}\right)}{\frac{25}{16}}=\frac{48}{50}=\frac{24}{25}$$

Option (2)

6. Ans. (4)

$$I=\int_{-2}^{2}\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$$

$$I=\int_0^2\left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}+\frac{\sin^2(-x)}{\left[-\frac{x}{\pi}\right]+\frac{1}{2}}\right)dx$$

$$\left(\left[\frac{x}{\pi}\right]+\left[-\frac{x}{\pi}\right]=-1 \text{ as } \quad x\neq n\pi\right)$$

$$I=\int_0^2\left(\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}+\frac{\sin^2 x}{-1-\left[\frac{x}{\pi}\right]+\frac{1}{2}}\right)dx=0$$

7. Ans. (1)

$$I=\int_{\pi/6}^{\pi/4}\frac{dx}{\sin 2x\left(\tan^5 x+\cot^5 x\right)}$$

$$I=\frac{1}{2}\int_{\pi/6}^{\pi/4}\frac{\tan^4 x\sec^2 x\,dx}{\left(1+\tan^{10}x\right)} \quad \text{Put } \tan^5 x = t$$

$$I=\frac{1}{10}\int_{\left(\frac{1}{\sqrt{3}}\right)^5}^{1}\frac{dt}{1+t^2}=\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\frac{1}{9\sqrt{3}}\right)$$

8. Ans. (2)

$$I=\int_0^a f(x)g(x)dx$$

$$I=\int_0^a f(a-x)g(a-x)dx$$

$$I=\int_0^a f(x)(4-g(x)dx$$

$$I=4\int_0^a f(x)dx-I$$

$$\Rightarrow\ I=2\int_0^a f(x)dx$$

9. Ans. (4)

$$\int_1^e\left(\frac{x}{e}\right)^{2x}\log_e x.dx-\int_1^e\left(\frac{e}{x}\right)^x\log_e x.dx$$

$$\text{Let } \left(\frac{x}{e}\right)^{2x}=t,\ \left(\frac{e}{x}\right)^x=v$$

$$=\frac{1}{2}\int_{\left(\frac{1}{e}\right)^2}^{1}dt\ +\ \int_e^1 dv$$

$$=\frac{1}{2}\left(1-\frac{1}{e^2}\right)+(1-e)=\frac{3}{2}-\frac{1}{2e^2}-e$$

10. Ans. (2)

$$\lim_{x\to\infty}\sum_{r=1}^{2n}\frac{n}{n^2+r^2}$$

$$\lim_{x\to\infty}\sum_{r=1}^{2n}\frac{1}{n\left(1+\frac{r^2}{n^2}\right)}=\int_0^2\frac{dx}{1+x^2}=\tan^{-1}2$$

11. Official Ans. by NTA (4)

Sol. $g(f(x)) = \ell n(f(x)) = \ell n\left(\frac{2 - x.\cos x}{2 + x.\cos x}\right)$

$\therefore \quad I = \int_{-\pi/4}^{\pi/4} \ell n\left(\frac{2 - x.\cos x}{2 + x.\cos x}\right)dx$

$= \int_{0}^{\pi/4}\left(\ell n\left(\frac{2 - x.\cos x}{2 + x.\cos x}\right) + \ell n\left(\frac{2 + x.\cos x}{2 - x.\cos x}\right)\right)dx$

$= \int_{0}^{\pi/2} (0)dx = 0 = \log_e(1)$

12. Official Ans. by NTA (1)

Sol. $f(x) = \int_0^x g(t)dt$

$f(-x) = \int_0^{-x} g(t)dt$

put $t = -u$

$= -\int_0^x g(-u)du$

$= -\int_0^x g(u)d(u) = -f(x)$

$\Rightarrow f(-x) = -f(x)$

$\Rightarrow f(x)$ is an odd function

Also $f(5 + x) = g(x)$

$f(5 - x) = g(-x) = g(x) = f(5 + x)$

$\Rightarrow f(5 - x) = f(5 + x)$

Now

$I = \int_0^x f(t)dt$

$t = u + 5$

$I = \int_{-5}^{x-5} f(u+5)du$

$= \int_{-5}^{x-5} g(u)du$

$= \int_{-5}^{x-5} f'(u)du$

$= f(x - 5) - f(-5)$

$= -f(5 - x) + f(5)$

$= f(5) - f(5+x)$

$= \int_{5+x}^{5} f'(t)dt = \int_{5+x}^{5} g(t)dt$

13. Official Ans. by NTA (3)

Sol. $I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$

$\Rightarrow I = \int_0^{\pi/4} \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} dx$

$= \int_0^{\pi/4} (1 - \sin x \cos x)\, dx$

$= \left(x - \frac{\sin^2 x}{2}\right)_0^{\pi/4}$

$= \frac{\pi}{4} - \frac{1}{4}$

$= \frac{\pi - 1}{4}$

14. Official Ans. by NTA (3)

Sol. $\lim_{x\to 2} \frac{\int_6^{f(x)} 2t\, dt}{x - 2}$

L Hopital Rule

$\lim_{x\to 2} \frac{2f(x)f'(x)}{1} = 2f(2) = f'(2) = 12f'(2)$

15. Official Ans. by NTA (3)

Sol. $I = \int_0^{2\pi} \left[\sin 2x(1 + \cos 3x)\right]dx$

$I = \int_0^{\pi}\left(\left[\sin 2x + \sin 2x \cos 3x\right] + \left[-\sin 2x - \sin 2x \cos 3x\right]\right)dx$

$= \int_0^{\pi} -dx = -\pi$

16. Official Ans. by NTA (3)

Sol. $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left(\frac{n + r}{n}\right)^{1/3}$

$= \int_0^1 (1 + x)^{1/3} dx = \frac{3}{4}\left(2^{4/3} - 1\right)$

17. Official Ans. by NTA (1)

Sol. $I = \int \frac{1}{\cos^{2/3} x \sin^{1/3} x.\sin x} dx$

$= \int \frac{\tan^{2/3} x}{\tan^2 x}.\sec^2 x.dx$

$= \int \frac{\sec^2 x}{\tan^{4/3} x}.dx$ $\quad \{\tan x = t, \sec^2 x dx = dt\}$

$= \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{-1/3} = -3\left(t^{-1/3}\right)$

$\Rightarrow I = -3\tan(x)^{-1/3}$

$\Rightarrow I = \frac{3}{(\tan x)^{1/3}}\Bigg|_{\pi/6}^{\pi/3} = -3\left(\frac{1}{\left(\sqrt{3}\right)^{1/3}} - \left(\sqrt{3}\right)^{1/3}\right)$

$= 3\left(3^{1/3} - \frac{1}{3^{1/6}}\right) = 3^{7/6} - 3^{5/6}$

18. Official Ans. by NTA (1)

Sol. $\int_0^{\pi/2} \frac{\cot x dx}{\cot x + \operatorname{cosec} x}$

$\int_0^{\pi/2} \frac{\cos x}{\cos x + 1} = \int \frac{2\cos^2 \frac{x}{2} - 1}{2\cos^2 \frac{x}{2}}$

$\int_0^{\pi/2} \left(1 - \frac{1}{2}\sec^2 \frac{x}{2}\right) dx$

$\left[x - \tan \frac{x}{2}\right]_0^{\frac{\pi}{2}}$

$\frac{1}{2}[\pi - 2]$ $\qquad m = \frac{1}{2}, \quad n = -2$

$mn = -1$

19. Official Ans. by NTA (1)

Sol. $I = \int_0^1 x \tan\left(\frac{1}{1 + x^2(x^2 - 1)}\right) dx$

$I = \int_0^1 x\left(\tan^{-1} x^2 - \tan^{-1}(x^2 - 1)\right) dx$

$x^2 = t \Rightarrow 2x dx = dt$

$I = \frac{1}{2}\int_0^1 \left(\tan^{-1} t - \tan^{-1}(t-1)\right) dx$

$= \frac{1}{2}\int_0^1 \tan^{-1} t\, dt - \frac{1}{2}\int_0^1 \tan^{-1}(t-1)\, dt$

$= \frac{1}{2}\int_0^1 \tan^{-1} t\, dt - \frac{1}{2}\int_0^1 \tan^{-1} dt = \int_0^1 \tan^{-1} dt$

$\tan^{-1} t = \theta \Rightarrow t = \tan\theta$

$dt = \sec^2\theta d\theta$

$\int_0^{\pi/4} \theta \cdot \sec^2\theta\, d\theta$

$I = (\theta.\tan\theta)\big|_0^{\pi/4} - \int_0^{\pi/4} \tan\theta\, d\theta$

$= \left(\frac{\pi}{4} - 0\right) - \ln(\sec\theta)\Big|_0^{\pi/4}$

$= \frac{\pi}{4} - \left(\ell n\sqrt{2} - 0\right)$

$= \frac{\pi}{4} - \frac{1}{2}\ell n 2$

20. Official Ans. by NTA (4)

Sol. $\lim_{x\to 2} g(x) = \lim_{x\to 2} \frac{\int_6^{f(x)} 4t^3 dt}{x - 2}$

$= \lim_{x\to 2} \frac{4.f^3(x) \cdot f'(x)}{1}$

$= 4f^3(2)\, f'(2) = 18$

21. Official Ans. by NTA (4)

Sol. $\int_\alpha^{\alpha+1} \frac{(x+\alpha+1) - (x+\alpha)}{(x+\alpha)(x+\alpha+1)} dx = \left(\ell n|x+\alpha| - \ell n|x+\alpha+1|\right)_\alpha^{\alpha+1}$

$= \ell n\left|\frac{2\alpha+1}{2\alpha+2} \times \frac{2\alpha+1}{2\alpha}\right| = \ell n\frac{9}{8}$

$\Rightarrow \alpha = -2, 1$

TANGENT & NORMAL

1. Ans. (4)

Point of intersection is P(2,6).

Also, $m_1 = \left(\frac{dy}{dx}\right)_{P(2,6)} = -2x = -4$

$m_2 = \left(\frac{dy}{dx}\right)_{P(2,6)} = 2x = 4$

$\therefore |\tan\theta| = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{8}{15}$

2. Ans (4)

$y = x^2 - 5x + 5$

$\frac{dy}{dx} = 2x - 5 = 2 \Rightarrow x = \frac{7}{2}$

at $x = \frac{7}{2}$, $y = \frac{-1}{4}$

Equation of tangent at $\left(\frac{7}{2}, \frac{-1}{4}\right)$ is $2x - y - \frac{29}{4} = 0$

Now check options

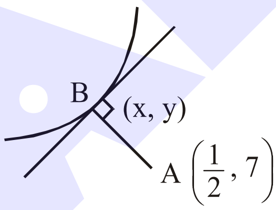
$x = \frac{1}{8}$, $y = -7$

3. Ans. (3)

$y - x^{3/2} = 7\ (x \ge 0)$

$\frac{dy}{dx} = \frac{3}{2}x^{1/2}$

$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{7-y}{\frac{1}{2}-x}\right) = -1$



$\left(\frac{3}{2}\sqrt{x}\right)\left(\frac{-x^{3/2}}{\frac{1}{2}-x}\right) = -1$

$\frac{3}{2}\cdot x^2 = \frac{1}{2} - x$

$3x^2 = 1 - 2x$

$3x^2 + 2x - 1 = 0$

$3x^2 + 3x - x - 1 = 0$

$(x + 1)(3x - 1) = 0$

$\therefore\ x = -1$ (rejected)

$x = \frac{1}{3}$

$y = 7 + x^{3/2} = 7 + \left(\frac{1}{3}\right)^{3/2}$

$\ell_{AB} = \sqrt{\left(\frac{1}{2} - \frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3} = \sqrt{\frac{1}{36} + \frac{1}{27}}$

$= \sqrt{\frac{3+4}{9 \times 12}}$

$= \sqrt{\frac{7}{108}} = \frac{1}{6}\sqrt{\frac{7}{3}}$

Option (3)

4. Official Ans. by NTA (3)

Sol. $f(1) = 1 - 1 - 2 = -2$

$f(-1) = -1 - 1 + 2 = 0$

$m = \frac{f(1) - f(-1)}{1+1} = \frac{-2-0}{2} = -1$

$\frac{dy}{dx} = 3x^2 - 2x - 2$

$3x^2 - 2x - 2 = -1$

$\Rightarrow 3x^2 - 2x - 1 = 0$

$\Rightarrow (x - 1)(3x + 1) = 0$

$\Rightarrow x = 1, -\frac{1}{3}$

5. Official Ans. by NTA (2)

Sol. $y = x^3 + ax - b$

(1, –5) lies on the curve

$\Rightarrow -5 = 1 + a - b \Rightarrow a - b = -6$ ... (i)

Also, $y' = 3x^2 + a$

$y'_{(1,-5)} = 3 + a$ (slope of tangent)

$\because$ this tangent is $\perp$ to $-x + y + 4 = 0$

$\Rightarrow (3 + a)(1) = -1$

$\Rightarrow a = -4$ ....(ii)

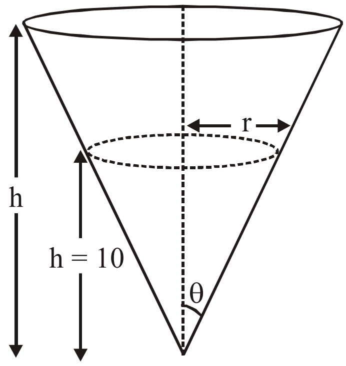
By (i) and (ii) : $a = -4$, $b = 2$

$\therefore\ y = x^3 - 4x - 2.$

(2,–2) lies on this curve.

6. Official Ans. by NTA (2)

Sol. $\tan\theta = \frac{1}{2} = \frac{r}{h}$



$r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h$

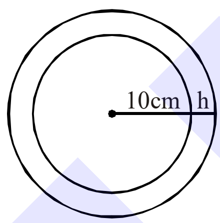
$V = \frac{1}{3}\pi . \frac{h^3}{4}$

$\frac{dV}{dt} = \frac{\pi}{12}(3h)^2\left(\frac{dh}{dt}\right)$

$5 = \frac{\pi}{4}.(100)\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{5\pi}$

7. Official Ans. by NTA (3)

Sol. $V = \frac{4}{3}\pi\left((10+h)^3 - 10^3\right)$



$\frac{dV}{dt} = 4\pi(10+h)^2\frac{dh}{dt}$

$-50 = 4\pi\ (10 + 5)^2\frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = -\frac{1}{18}\frac{cm}{min}$

8. Official Ans. by NTA (1)

Sol. $\left.\frac{dy}{dx}\right|_{(\alpha,\beta)} = \frac{-\alpha^2-3}{\left(\alpha^2-3\right)^2}$

Given that :

$\frac{-\alpha^2-3}{\left(\alpha^2-3\right)^2} = -\frac{1}{3}$

$\Rightarrow \alpha = 0, \pm 3$ $\quad (\alpha \neq 0)$

$\Rightarrow \beta = \pm\ \frac{1}{2}.$ $\quad (\beta \neq 0)$

$|6\alpha + 2\beta| = 19$

MONOTONICITY

1. Ans. (4)

$f(x) = \frac{x}{\sqrt{a^2+x^2}} - \frac{d-x}{\sqrt{b^2+(d-x)^2}}$

$f'(x) = \frac{a^2}{\left(a^2+x^2\right)^{3/2}} + \frac{b^2}{\left(b^2+(d-x)^2\right)^{3/2}} > 0\ \forall x \in R$

$f(x)$ is an increasing function.

2. Ans (3)

$f'(x) = 3x^2 - 6(a-2)x + 3a$

$f'(x) \geq 0\ \forall\ x \in (0, 1]$

$f'(x) \leq 0\ \forall\ x \in [1, 5)$

$\Rightarrow f'(x) = 0$ at $x = 1 \Rightarrow a = 5$

$f(x) - 14 = (x-1)^2\ (x-7)$

$\frac{f(x)-14}{(x-1)^2} = x - 7$

3. Official Ans. by NTA (2)

Sol. $\phi(x) = f(x) + f(2-x)$

$\phi'(x) = f'(x) - f'(2-x)$ ......(1)

Since $f''(x) > 0$

$\Rightarrow$ $f'(x)$ is increasing $\forall\ x \in (0, 2)$

Case-I : When $x > 2 - x \quad \Rightarrow \quad x > 1$

$\Rightarrow \quad \phi'(x) > 0\ \forall\ x \in (1, 2)$

$\therefore \quad \phi(x)$ is increasing on $(1, 2)$

Case-II : When $x < 2 - x \quad \Rightarrow \quad x < 1$

$\Rightarrow \quad \phi'(x) < 0\ \forall\ x \in (0,1)$

$\therefore \quad \phi(x)$ is decreasing on $(0, 1)$

4. Official Ans. by NTA (2)

Sol. $h(x) = f(g(x))$

$\Rightarrow h'(x) = f'(g(x)).\ g'(x)$ and $f'(x) = e^x - 1$

$\Rightarrow h'(x) = (e^{g(x)} - 1)\ g'(x)$

$\Rightarrow h'(x) = \left(e^{x^2-x} - 1\right)\ (2x-1) \geq 0$

Case-I $e^{x^2-x} \geq 1$ and $2x - 1 \geq 0$

$\Rightarrow x \in [1, \infty)$ ......(1)

Case-II $e^{x^2-x} \leq 1$ and $2x - 1 \leq 0$

$\Rightarrow x \in \left[0, \frac{1}{2}\right]$.....(2)

Hence, $x \in \left[0, \frac{1}{2}\right] \cup [1, \infty)$

5. Official Ans. by NTA (2)

Sol. $f(x) = x\sqrt{kx - x^2}$

$f'(x) = \dfrac{3kx - 4x^2}{2\sqrt{kx - x^2}}$

For ↑ $f'(x) \geq 0$ | $3kx - 4x^2 \geq 0$

$kx - x^2 \geq 0$ | $4x^2 - 3kx \leq 0$

$x^2 - kx \leq 0$ | $4x(x - \frac{3k}{4}) \leq 0$

$x(x-k) \leq 0$ so $x \in [0, 3]$

$3 - \dfrac{3k}{4} \leq 0$

+ve $\boxed{x \geq 3}$ $\boxed{k \geq 4}$

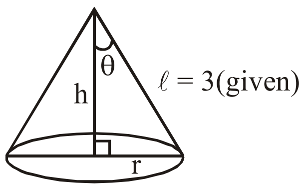
minimum value of k is $\boxed{m = 4}$

$f(x) = x\sqrt{kx - x^2}$

$= 3\sqrt{4 \times 3 - 3^2} = 3\sqrt{3}, M = 3\sqrt{3}$

MAXIMA & MINIMA

1. Ans. (3)



$\therefore h = 3\cos\theta$

$r = 3\sin\theta$

Now,

$V = \dfrac{1}{3}\pi r^2 h = \dfrac{\pi}{3}(9\sin^2\theta).(3\cos\theta)$

$\therefore \dfrac{dV}{d\theta} = 0 \Rightarrow \sin\theta = \sqrt{\dfrac{2}{3}}$

Also, $\left.\dfrac{d^2V}{d\theta^2}\right]_{\sin\theta = \sqrt{\frac{2}{3}}}$ = negative

$\Rightarrow$ Volume is maximum,

when $\sin\theta = \sqrt{\dfrac{2}{3}}$

$\therefore V_{max}\left(\sin\theta = \sqrt{\dfrac{2}{3}}\right) = 2\sqrt{3}\pi$ (in cu. m)

2. Ans. (1)

Let points $\left(\dfrac{3}{2}, 0\right)$, (t^2, t), $t > 0$

Distance $= \sqrt{t^2 + \left(t^2 - \dfrac{3}{2}\right)^2}$

$= \sqrt{t^4 - 2t^2 + \dfrac{9}{4}} = \sqrt{(t^2-1)^2 + \dfrac{5}{4}}$

So minimum distance is $\sqrt{\dfrac{5}{4}} = \dfrac{\sqrt{5}}{2}$

3. Ans. (1)

$S = \{x \in R, x^2 + 30 - 11x \leq 0\}$

$= \{x \in R, 5 \leq x \leq 6\}$

Now $f(x) = 3x^3 - 18x^2 + 27x - 40$

$\Rightarrow f'(x) = 9(x-1)(x-3)$,

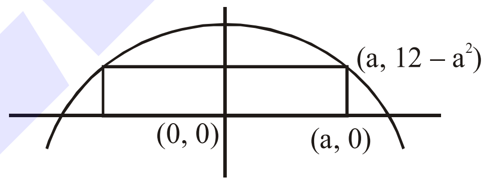
which is positive in [5, 6]

$\Rightarrow f(x)$ increasing in [5, 6]

Hence maximum value $= f(6) = 122$

4. Ans. (3)

$f(a) = 2a(12 - a)^2$



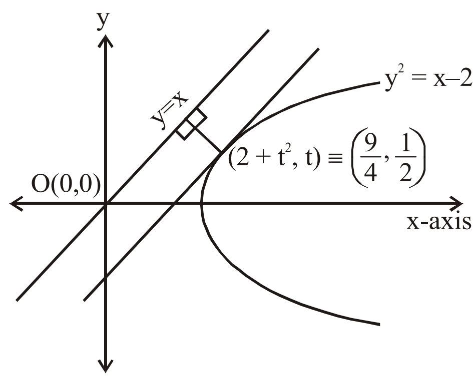
$f'(a) = 2(12 - 3a^2)$

maximum at a = 2

maximum area = f(2) = 32

5. Official Ans. by NTA (1)

Sol.



we have, $2y.\dfrac{dy}{dx} = 1 \Rightarrow \left.\dfrac{dy}{dx}\right]_{P(2+t^2, t)} = \dfrac{1}{2t} = 1$

$\Rightarrow t = \dfrac{1}{2}$

$\therefore \quad P\left(\frac{9}{4}, \frac{1}{2}\right)$

So, shortest distance

$$= \frac{\left|\frac{9}{4} - \frac{2}{4}\right|}{\sqrt{2}} = \frac{7}{4\sqrt{2}}$$

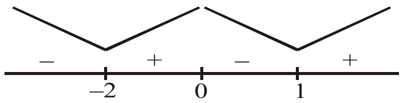
6. Official Ans. by NTA (1)

Sol. $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$

$f'(x) = 36x^3 + 36x^2 - 72x$

$= 36x(x^2 + x - 2)$

$= 36x(x - 1)(x + 2)$

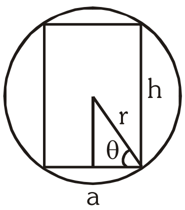


Points of minima = $\{-2, 1\} = S_1$

Point of maxima = $\{0\} = S_2$

7. Official Ans. by NTA (1)

Sol.



$h = 2r\sin\theta$

$a = 2r\cos\theta$

$v = \pi(r\cos\theta)^2(2r\sin\theta)$

$v = 2\pi r^3\cos^2\theta\sin\theta$

$$\frac{dv}{d\theta} = \pi r^3\left(-2\cos\theta\sin^2\theta + \cos^3\theta\right) = 0$$

or $\tan\theta = \frac{1}{\sqrt{2}}$

$\because \; h = 2 \times 3 \times \frac{1}{\sqrt{3}} = 2\sqrt{3}$

8. Official Ans. by NTA (2)

Sol. $f'(x) = \lambda(x + 1)(x - 0)(x - 1) = \lambda(x^3 - x)$

$$\Rightarrow f(x) = \lambda\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + \mu$$

Now $f(x) = f(0)$

$$\Rightarrow \lambda\left(\frac{x^4}{4} - \frac{x^2}{2}\right) + \mu = \mu$$

$\Rightarrow x = 0, 0, \pm\sqrt{2}$

Two irrational and one rational number

9. Official Ans. by NTA (2)

Sol. Let a is first term and d is common difference

then, $a + 5d = 2$ (given) ...(1)

$f(d) = (2 - 5d)(2 - 2d)(2 - d)$

$f'(d) = 0 \quad \Rightarrow d = \frac{2}{3}, \frac{8}{5}$

$f''(d) < 0$ at $d = 8/5$

$\Rightarrow d = \frac{8}{5}$

DIFFERENTIAL EQUATION

1. Ans. (3)

$$\frac{dy}{dx} + \left(\frac{2}{x}\right)y = x$$

$\Rightarrow$ I.F. $= x^2$

$\therefore yx^2 = \frac{x^4}{4} + \frac{3}{4}$ (As, $y(1) = 1$)

$\therefore y\left(x = \frac{1}{2}\right) = \frac{49}{16}$

2. Ans. (2)

$f(xy) = f(x).f(y)$

$f(0) = 1$ as $f(0) \neq 0$

$\Rightarrow f(x) = 1$

$$\frac{dy}{dx} = f(x) = 1$$

$\Rightarrow y = x + c$

At, $x = 0$, $y = 1 \Rightarrow c = 1$

$y = x + 1$

$$\Rightarrow y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right) = \frac{1}{4} + 1 + \frac{3}{4} + 1 = 3$$

3. Ans. (1)

$$\frac{dy}{dx} + 3\sec^2 x.y = \sec^2 x$$

I.F. $= e^{3\int \sec^2 x dx} = e^{3\tan x}$

or $y.e^{3\tan x} = \int \sec^2 x.e^{3\tan x} dx$

or $y.e^{3\tan x} = \frac{1}{3}e^{3\tan x} + C$ ....(1)

Given

$$y\left(\frac{\pi}{4}\right)=\frac{4}{3}$$

$$\therefore\ \frac{4}{3}.e^{3}=\frac{1}{3}e^{3}+C$$

$\therefore$ $C = e^3$

Now put $x = -\frac{\pi}{4}$ in equation (1)

$$\therefore\ y.e^{-3}=\frac{1}{3}e^{-3}+e^{3}$$

$$\therefore\ y=\frac{1}{3}+e^{6}$$

$$\therefore\ y\left(-\frac{\pi}{4}\right)=\frac{1}{3}+e^{6}$$

4. Ans. (1)

$$f'(x)=7-\frac{3}{4}\frac{f(x)}{x} \qquad (x>0)$$

Given $f(1) \neq 4$ $\quad \lim\limits_{x\to0^{+}} xf\left(\frac{1}{x}\right) = ?$

$$\frac{dy}{dx}+\frac{3}{4}\frac{y}{x}=7 \text{ (This is LDE)}$$

$$\text{IF} = e^{\int\frac{3}{4x}dx}=e^{\frac{3}{4}\ln|x|}=x^{\frac{3}{4}}$$

$$y.x^{\frac{3}{4}}=\int 7.x^{\frac{3}{4}}dx$$

$$y.x^{\frac{3}{4}}=7.\frac{x^{\frac{7}{4}}}{\frac{7}{4}}+C$$

$$f(x)= 4x+C.x^{-\frac{3}{4}}$$

$$f\left(\frac{1}{x}\right)=\frac{4}{x}+C.x^{\frac{3}{4}}$$

$$\lim_{x\to0^{+}} xf\left(\frac{1}{x}\right)=\lim_{x\to0^{+}}\left(4+C.x^{\frac{7}{4}}\right)=4$$

$\therefore$ Option (1)

5. Ans. (2)

$(x^2 - y^2)\, dx + 2xy\, dy = 0$

$$\frac{dy}{dx}=\frac{y^{2}-x^{2}}{2xy}$$

Put $y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

Solving we get,

$$\int\frac{2v}{v^{2}+1}dv=\int-\frac{dx}{x}$$

$\ln(v^2 + 1) = -\ln x + C$

$(y^2 + x^2) = Cx$

$1 + 1 = C \Rightarrow C = 2$

$$\boxed{y^{2}+x^{2}=2x}$$

$\therefore$ Option (2)

6. Ans. (2)

$$\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}$$

$$\text{I.F.}=e^{\int\left(\frac{2x+1}{x}\right)dx}=e^{\int\left(2+\frac{1}{x}\right)dx}=e^{2x+\ell nx}=e^{2x}.x$$

So, $y(xe^{2x})=\int e^{-2x}.xe^{2x}+C$

$\Rightarrow\ xye^{2x}=\int x\ dx+C$

$\Rightarrow 2xye^{2x} = x^2 + 2C$

It passess through $\left(1, \frac{1}{2}e^{-2}\right)$ we get C = 0

$$y=\frac{xe^{-2x}}{2}$$

$$\Rightarrow\frac{dy}{dx}=\frac{1}{2}e^{-2x}(-2x+1)$$

$\Rightarrow f(x)$ is decreasing in $\left(\frac{1}{2}, 1\right)$

$$y(\log_{e}2)=\frac{(\log_{e}2)e^{-2(\log_{e}2)}}{2}$$

$$=\frac{1}{8}\log_{e}2$$

7. Ans. (4)

$$x - y = t \Rightarrow \frac{dy}{dx}=1-\frac{dt}{dx}$$

$$\Rightarrow 1-\frac{dt}{dx}=t^{2}\Rightarrow\int\frac{dt}{1-t^{2}}=\int 1dx$$

$$\Rightarrow\frac{1}{2}\ell n\left(\frac{1+t}{1-t}\right)=x+\lambda$$

$$\Rightarrow\frac{1}{2}\ell n\left(\frac{1+x-y}{1-x+y}\right)=x+\lambda \quad \text{given } y(1) = 1$$

$$\Rightarrow\frac{1}{2}\ell n(1)=1+\lambda\Rightarrow\lambda=-1$$

$$\Rightarrow \ell n\left(\frac{1+x-y}{1-x+y}\right)=2(x-1)$$

$$\Rightarrow -\ell n\left(\frac{1-x+y}{1+x-y}\right)=2(x-1)$$

8. Ans. (2)

$$\frac{dy}{dx}=\frac{y}{x}=\ell n x$$

$$e^{\int \frac{1}{x}dx}=x$$

$$xy=\int x\ell nx+C$$

$$\ell n x\frac{x^2}{2}-\int \frac{1}{x}\cdot\frac{x^2}{2}$$

$xy=\frac{x}{2}\ell n x-\frac{x^2}{4}$ +C, for 2y(2)= $2\ell n2 - 1$

$$\Rightarrow C=0$$

$$y=\frac{x}{2}\ell n x-\frac{x}{4}$$

$$y(e)=\frac{e}{4}$$

9. Ans. (2)

$\frac{dy}{dx}=\frac{x^2-2y}{x}$ (Given)

$$\frac{dy}{dx}+2\frac{y}{x}=x$$

$$I.F = e^{\int \frac{2}{x}dx} = x^2$$

$$\therefore\ y.x^2 = \int x.x^2dx + C = \frac{x^4}{y}+C$$

hence bpasses through (1, –2) $\Rightarrow$ $C = -\frac{9}{4}$

$$\therefore\ yx^2 = \frac{x^4}{4}-\frac{9}{4}$$

Now check option(s) , Which is satisly by option (ii)

10. Official Ans. by NTA (2)

Sol. $\frac{dy}{dx}+\left(\frac{2x}{x^2+1}\right)y=\frac{1}{(x^2+1)^2}$

(Linear differential equation)

$\therefore$ I.F. = $e^{\ell n(x^2+1)}=(x^2+1)$

So, general solution is $y.(x^2 + 1) = \tan^{-1}x + c$

As y(0) = 0 $\Rightarrow$ c = 0

$$\therefore\ y(x)=\frac{\tan^{-1}x}{x^2+1}$$

As, $\sqrt{a}.\ y(1)=\frac{\pi}{32}$

$$\Rightarrow\ \sqrt{a}=\frac{1}{4}\Rightarrow a=\frac{1}{16}$$

11. Official Ans. by NTA (1)

Sol. given $\frac{dy}{dx}=\frac{2y}{x^2}$

$$\Rightarrow \int\frac{dy}{2y}=\int\frac{dx}{x^2}$$

$$\Rightarrow \frac{1}{2}\ell ny=-\frac{1}{x}+c$$

passes through centre (1,1)

$$\Rightarrow c=1$$

$$\Rightarrow x\ell ny=2(x-1)$$

12. Official Ans. by NTA (4)

Sol. $x\frac{dy}{dx}+2y=x^2 : y(1)=1$

$\frac{dy}{dx}+\left(\frac{2}{x}\right)y=x$ (LDE in y)

$$IF = e^{\int \frac{2}{x}dx}=e^{2\ell nx}=x^2$$

$$y.(x^2)=\int x.x^2dx=\frac{x^4}{4}+C$$

y(1) =1

$$1=\frac{1}{4}+C\Rightarrow C=1-\frac{1}{4}=\frac{3}{4}$$

$$yx^2=\frac{x^4}{4}+\frac{3}{4}$$

$$y=\frac{x^2}{4}+\frac{3}{4x^2}$$

13. Official Ans. by NTA (3)

Sol. $\frac{dy}{dx} - y\tan x = 6x\sec x$

$y\left(\frac{\pi}{3}\right) = 0 ; y\left(\frac{\pi}{6}\right) = 7$

$e^{\int pdx} = e^{-\int \tan x dx} = e^{\ell n \cos x} = \cos x$

$y . \cos x = \int 6x \sec x \cos x \, dx$

$y.\cos x = \frac{6x^2}{2} + C$

$y = 3x^2\sec x + C \sec x$

$0 = 3\cdot\frac{\pi^2}{9}\cdot(2) + C(2)$

$2C = \frac{-2\pi^2}{3} \Rightarrow \boxed{C = -\frac{\pi^2}{3}}$

$y(\pi/6) = 3\cdot\frac{\pi^2}{36}\cdot\left(\frac{2}{\sqrt{3}}\right) + \left(\frac{2}{\sqrt{3}}\right)\cdot\left(-\frac{\pi^2}{3}\right)$

$\Rightarrow \; y = -\frac{\pi^2}{2\sqrt{3}}$

14. Official Ans. by NTA (3)

Sol. $\frac{dy}{dx}$ = (tan x – y) $\sec^2 x$

Now, put tanx = t $\Rightarrow \frac{dt}{dx} = \sec^2 x$

So $\frac{dy}{dt} + y = t$

On solving, we get $ye^t = e^t (t - 1) + c$

$\Rightarrow y = (\tan x - 1) + ce^{-\tan x}$

$\Rightarrow y(0) = 0 \Rightarrow c = 1$

$\Rightarrow y = \tan x - 1 + e^{-\tan x}$

So $y\left(-\frac{\pi}{4}\right) = e - 2$

15. Official Ans. by NTA (2)

Sol. $\frac{dy}{dx} + y(\tan x) = 2x + x^2 \tan x$

I.F $= e^{\int \tan x \, dx} = e^{\ln .\sec x} = \sec x$

$\therefore \; y \, . \, \sec x = \int (2x + x^2 \tan x)\sec x.dx$

$= \int 2x \sec x \, dx + \int x^2 (\sec x.\tan x) dx$

$y \sec x = x^2 \sec x + \lambda$

$\Rightarrow y = x^2 + \lambda\cos x$

$y(0) = 0 + \lambda = 1 \qquad \Rightarrow \lambda = 1$

$y = x^2 + \cos x$

$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$

$y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{16} + \frac{1}{\sqrt{2}}$

$y'(x) = 2x - \sin x$

$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} - \frac{1}{\sqrt{2}}$

$y'\left(\frac{-\pi}{4}\right) = \frac{-\pi}{2} + \frac{1}{\sqrt{2}}$

$y'\left(\frac{\pi}{4}\right) - y'\left(\frac{-\pi}{4}\right) = \pi - \sqrt{2}$

16. Official Ans. by NTA (4)

Sol. $y^2dx + xdy = \frac{dy}{y}$

$\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

IF $= e^{\int \frac{1}{y^2} dy} = e^{-\frac{1}{y}}$

$e^{-\frac{1}{y}} \cdot x = \int e^{-\frac{1}{y}} \cdot \frac{1}{y^3} dy + C$

$xe^{-\frac{1}{y}} = e^{-\frac{1}{y}} + \frac{e^{-\frac{1}{y}}}{y} + C$

$C = -\frac{1}{e}$

$x = \frac{3}{2} - \frac{1}{\sqrt{e}}$ when y = 2

17. Official Ans. by NTA (1)

Sol. $xy\frac{dy}{dx} - y^2 + x^3 = 0$

put $y^2 = k \Rightarrow y\frac{dy}{dx} = \frac{1}{2}\frac{dk}{dx}$

$\therefore$ given differential equation becomes

$$\frac{dk}{dx} + k\left(-\frac{2}{x}\right) = -2x^2$$

I.F. = $e^{\int -\frac{2}{x}dx} = \frac{1}{x^2}$

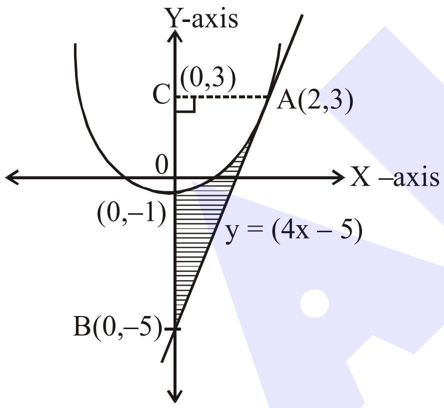
$\therefore$ solution is $k \cdot \frac{1}{x^2} = \int -2x^2 \cdot \frac{1}{x^2}dx + \lambda$

$y^2 + 2x^3 = \lambda x^2$

take $\lambda = -c$ (integration constant)

AREA UNDER THE CURVE

1. Ans. (3)



Equation of tangent at (2,3) on

$y = x^2 - 1$, is $y = (4x - 5)$ ....(i)

$\therefore$ Required shaded area

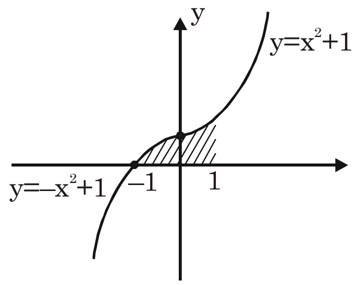
$= \text{ar}(\Delta ABC) - \int_{-1}^{3}\sqrt{y+1}\,dy$

$= \frac{1}{2}.(8).(2) - \frac{2}{3}\left((y+1)^{3/2}\right)_{-1}^{3}$

$= 8 - \frac{16}{3} = \frac{8}{3}$ (square units)

2. Ans. (3)

The graph is a follows



$$\int_{-1}^{0}\left(-x^2+1\right)dx + \int_{0}^{1}\left(x^2+1\right)dx = 2$$

3. Ans. (1)

Area bounded by $y^2 = 4ax$ & $x^2 = 4by$,

a, b $\neq$ 0 is $\left|\frac{16ab}{3}\right|$

by using formula : $4a = \frac{1}{k} = 4b, k > 0$

Area $= \left|\frac{16.\frac{1}{4k}.\frac{1}{4k}}{3}\right| = 1$

$\Rightarrow k^2 = \frac{1}{3}$

$\Rightarrow k = \frac{1}{\sqrt{3}}$

4. Ans. (1)

$y = xe^{x^2}$

$\left.\frac{dy}{dx}\right|_{(1,e)} = \left.\left(e \cdot e^{x^2} \cdot 2x + e^{x^2}\right)\right|_{(1,e)} = 2 \cdot e + e = 3e$

T : $y - e = 3e(x - 1)$

$y = 3ex - 3e + e$

$y = (3e)x - 2e$

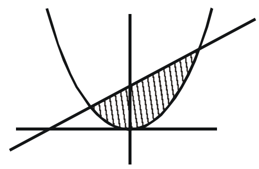
$\left(\frac{4}{3}, 2e\right)$ lies on it

Option (1)

5. Ans. (2)

$x = 4y - 2$ & $x^2 = 4y$

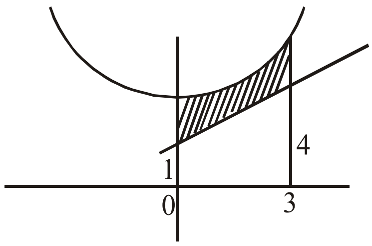
$\Rightarrow x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$



$x = 2, -1$

So, $\int_{-1}^{2}\left(\frac{x+2}{4} - \frac{x^2}{4}\right)dx = \frac{9}{8}$

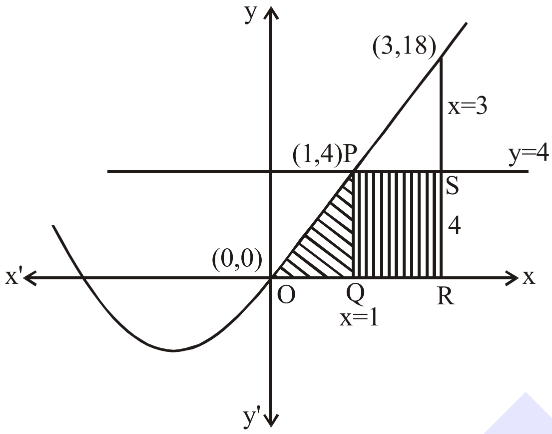
6. Ans. (2)



Req. area $= \int_0^3 (x^2+2)dx - \frac{1}{2}.5.3 = 9 + 6 - \frac{15}{2} = \frac{15}{2}$

7. Official Ans. by NTA (2)

Sol.



Required Area

$= \int_0^1 (x^2+3x)dx + \text{Area of rectangle PQRS}$

$= \frac{11}{6} + 8 = \frac{59}{6}$

8. Official Ans. by NTA (2)

Sol. $S(\alpha) = \{(x,y) : y^2 \le x, 0 \le x \le \alpha\}$

$$A(\alpha) = 2\int_0^{\alpha} \sqrt{x}dx = 2\alpha^{\frac{3}{2}}$$

$$A(4) = 2 \times 4^{3/2} = 16$$

$$A(\lambda) = 2 \times \lambda^{3/2}$$

$$\frac{A(\lambda)}{A(4)} = \frac{2}{5} \Rightarrow \lambda = 4.\left(\frac{4}{25}\right)^{1/3}$$

9. Official Ans. by NTA (2)

Sol. $x^2 \le y \le x + 2$

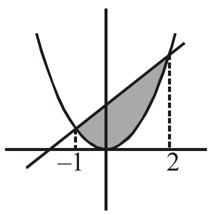
$x^2 = y \; ; \; y = x + 2$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x - 2)(x - 1) = 0$

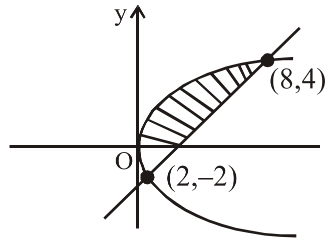
$x = 2, -1$



Area $= \int_{-1}^{2} (x+2) - x^2 \, dx = \frac{9}{2}$

10. Official Ans. by NTA (2)

Sol.



$y^2 = 2x$

$x - y - 4 = 0$

$(x - 4)^2 = 2x$

$x^2 + 16 - 8x - 2x = 0$

$x^2 - 10x + 16 = 0$

$x = 8, 2$

$y = 4, -2$

$$A = \int_{-2}^{4} \left(y + 4 - \frac{y^2}{2} \right) dy$$

$$= \frac{y^2}{2}\Big|_{-2}^{4} + 4y\Big|_{-2}^{4} - \frac{y^3}{6}\Big|_{-2}^{4}$$

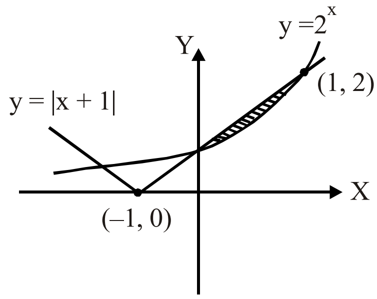
$$= (8-2) + 4(6) - \frac{1}{6}(64+8)$$

$= 6 + 24 - 12 = 18$

11. Official Ans. by NTA (1)

Sol. Required Area

$$\int_0^1 \left((x+1) - 2^x\right)dx$$



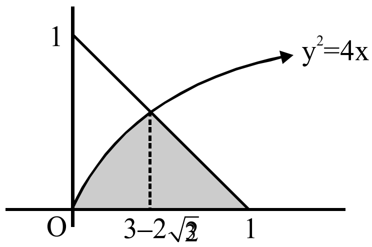
$$= \left(\frac{x^2}{2} + x - \frac{2^x}{\ln 2}\right)_0^1$$

$$= \left(\frac{1}{2} + 1 - \frac{2}{\ln 2}\right) - \left(0 + 0 - \frac{1}{\ln 2}\right)$$

$$= \frac{3}{2} - \frac{1}{\ln 2}$$

12. Official Ans. by NTA (3)

Sol. $\{(x, y) : y^2 \le 4x,\ x + y \le 1,\ x \ge 0,\ y \ge 0\}$



$$A\int_0^{3-2\sqrt{2}} 2\sqrt{x}dx + \frac{1}{2}\left(1-\left(3-2\sqrt{2}\right)\right)\left(1-(3-2\sqrt{2})\right)$$

$$= \frac{2[x^{3/2}]_0^{3-2\sqrt{2}}}{3/2} + \frac{1}{2}\left(2\sqrt{2}-2\right)\left(2\sqrt{2}-2\right)$$

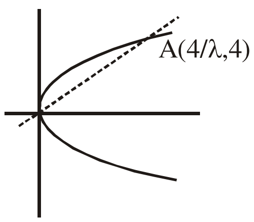
$$= \frac{8\sqrt{2}}{3} + \left(-\frac{10}{3}\right)$$

$a = \frac{8}{3}$, $b = -\frac{10}{3}$

$a - b = 6$

13. Official Ans. by NTA (1)

Sol.



$$\text{Area } = \frac{1}{9} = \int_0^{\frac{4}{\lambda}}\left(\sqrt{4\lambda x} - \lambda x\right)dx$$

$\Rightarrow \lambda = 24$

MATRIX

1. Ans. (1)

Here, $AA^T = I$

$$\Rightarrow A^{-1} = A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\text{Also, } A^{-n} = \begin{bmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{bmatrix}$$

$$\therefore A^{-50} = \begin{bmatrix} \cos(50)\theta & \sin(50)\theta \\ -\sin(50)\theta & \cos(50)\theta \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

2. Ans. (3)

$$|A| = e^{-t}\begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$= e^{-t}[5\cos^2 t + 5\sin^2 t]\ \forall\ t \in R$

$= 5e^{-t} \ne 0\ \forall\ t \in R$

3. Ans. (4)

$$A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix} (b > 0)$$

$|A| = 2(2b^2 + 2 - b^2) - b(2b - b) + 1\ (b^2 - b^2 - 1)$

$|A| = 2(b^2 + 2) - b^2 - 1$

$|A| = b^2 + 3$

$$\frac{|A|}{b} = b + \frac{3}{b} \Rightarrow \frac{b+\frac{3}{b}}{2} \ge \sqrt{3}$$

$$b + \frac{3}{b} \ge 2\sqrt{3}$$

Option (4)

4. Ans. (1)

A is orthogonal matrix

$\Rightarrow 0^2 + p^2 + p^2 = 1 \Rightarrow |p| = \frac{1}{\sqrt{2}}$

5. Ans. (2)

$|A|^2.|B| = 8$ and $\frac{|A|}{|B|} = 8 \Rightarrow |A| = 4$ and $|B| = \frac{1}{2}$

$\therefore\ \det(BA^{-1}.B^T) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

6. Ans. (4)

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3 & 1 & 0 \\ 9+9+9 & 3+3 & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 3+3+3 & 1 & 0 \\ 6.9 & 3+3+3 & 1 \end{bmatrix}$$

$$P^n = \begin{bmatrix} 1 & 0 & 0 \\ 3n & 1 & 0 \\ \frac{n(n+1)}{2}3^2 & 3n & 1 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 1 & 0 & 0 \\ 5.3 & 1 & 0 \\ 15.9 & 5.3 & 1 \end{bmatrix}$$

$Q = P^5 + I_3$

$$Q = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$$

$$\frac{q_{21}+q_{31}}{q_{32}} = \frac{15+135}{15} = 10$$

Aliter

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$$

$P = I + X$

$$X = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{pmatrix}$$

$$X^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{pmatrix}$$

$X^3 = 0$

$P^5 = I + 5X + 10X^2$

$Q = P^5 + I = 2I + 5X + 10X^2$

$$Q = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 15 & 0 & 0 \\ 15 & 15 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 90 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow Q = \begin{pmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{pmatrix}$$

7. Ans (2)

$$|A| = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$$

$= 2(1 + \sin^2\theta)$

$$\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \Rightarrow \frac{1}{\sqrt{2}} < \sin\theta < \frac{1}{\sqrt{2}}$$

$$\Rightarrow 0 \le \sin^2\theta < \frac{1}{2}$$

$\therefore |A| \in [2, 3)$

8. Official Ans. by NTA (4)

Sol. put $b = \frac{2+c}{2}$ in determinant of A

$$|A| = \frac{c^3 - 6c^2 + 12c - 8}{4} \in [2,16]$$

$\Rightarrow (c-2)^3 \in [8, 64]$

$\Rightarrow c \in [4, 6]$

9. Official Ans. by NTA (4)

Sol. $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 3\alpha & -\sin 3\alpha \\ \sin 3\alpha & \cos 3\alpha \end{bmatrix}$$

Similarly

$$A^{32} = \begin{bmatrix} \cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$\Rightarrow \cos 32\alpha = 0$ & $\sin 32\alpha = 1$

$$\Rightarrow 32\alpha = (4n+1)\frac{\pi}{2}, n \in I$$

$$\alpha = (4n+1)\frac{\pi}{64}, n \in I$$

$$\alpha = \frac{\pi}{64} \text{ for } n = 0$$

10. Official Ans. by NTA (1)

Sol. $\begin{bmatrix}1&1\\0&1\end{bmatrix}\begin{bmatrix}1&2\\0&1\end{bmatrix}\begin{bmatrix}1&2\\0&1\end{bmatrix}\begin{bmatrix}1&3\\0&1\end{bmatrix}.....\begin{bmatrix}1&n-1\\0&1\end{bmatrix}=\begin{bmatrix}1&78\\0&1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1&1+2+3+.....+n-1\\0&1\end{bmatrix}=\begin{bmatrix}1&78\\0&1\end{bmatrix}$

$\Rightarrow \frac{n(n-2)}{2}=78 \Rightarrow n = 13, -12\text{(reject)}$

$\therefore$ We have to find inverse of $\begin{bmatrix}1&13\\0&1\end{bmatrix}$

$\therefore \begin{bmatrix}1&-13\\0&1\end{bmatrix}$

11. Official Ans. by NTA (4)

Sol. $A^TA = 3I_3$

$\begin{pmatrix}0&2x&2x\\2y&y&-y\\1&-1&1\end{pmatrix}\begin{pmatrix}0&2y&1\\2x&y&-1\\2x&-y&1\end{pmatrix}=\begin{pmatrix}3&0&0\\0&3&0\\0&0&3\end{pmatrix}$

$\begin{pmatrix}8x^2&0&0\\0&6y^2&0\\0&0&3\end{pmatrix}=\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}$

$8x^2 = 3$

$6y^2 = 3$

$x^2 = 3/8$

$y^2 = 1/2$

$x=\pm\sqrt{\frac{3}{8}};y=\pm\sqrt{\frac{1}{2}}$

12. Official Ans. by NTA (3)

Sol. $|B| = 5(-5) -2\alpha(-\alpha) -2\alpha$

$= 2\alpha^2 - 2\alpha -25$

$1 + |A| = 0$

$\alpha^2 - \alpha - 12 = 0$

Sum = 1

13. Official Ans. by NTA (3)

Sol. $A = A', B = -B'$

$A + B = \begin{bmatrix}2&3\\5&-1\end{bmatrix}$ ...(1)

$A' + B' = \begin{bmatrix}2&5\\3&-1\end{bmatrix}$

$A - B = \begin{bmatrix}2&5\\3&-1\end{bmatrix}$ ...(2)

After adding Eq. (1) & (2)

$A=\begin{bmatrix}2&4\\4&-1\end{bmatrix},\ B=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$

$AB = \begin{bmatrix}4&-2\\-1&-4\end{bmatrix}$

VECTORS

1. Ans. (1)

$\vec{a}\times\vec{c}=-\vec{b}$

$(\vec{a}\times\vec{c})\times\vec{a}=-\vec{b}\times\vec{a}$

$\Rightarrow (\vec{a}\times\vec{c})\times\vec{a}=\vec{a}\times\vec{b}$

$\Rightarrow (\vec{a}.\vec{a})\vec{c}-(\vec{c}.\vec{a})\vec{a}=\vec{a}\times\vec{b}$

$\Rightarrow 2\vec{c}-4\vec{a}=\vec{a}\times\vec{b}$

Now $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&0\\1&1&1\end{vmatrix}=-\hat{i}-\hat{j}+2\hat{k}$

So, $2\vec{c}=4\hat{i}-4\hat{j}-\hat{i}-\hat{j}+2\hat{k}$

$=3\hat{i}-5\hat{j}+2\hat{k}$

$\Rightarrow \vec{c}=\frac{3}{2}\hat{i}-\frac{5}{2}\hat{j}+\hat{k}$

$|\vec{c}|=\sqrt{\frac{9}{4}+\frac{25}{4}+1}=\sqrt{\frac{38}{4}}=\sqrt{\frac{19}{2}}$

$|\vec{c}|^2=\frac{19}{2}$

2. Ans. (4)

Projection of $\vec{b}$ on $\vec{a}=\frac{\vec{a}.\vec{b}}{|\vec{a}|}=|\vec{a}|$

$\Rightarrow b_1 + b_2 = 2$ ...(1)

and $(\vec{a}+\vec{b})\perp\vec{c}\Rightarrow(\vec{a}+\vec{b}).\vec{c}=0$

$\Rightarrow 5b_1 + b_2 = -10$ ...(2)

from (1) and (2) $\Rightarrow b_1 = -3$ and $b_2 = 5$

then $|\vec{b}|=\sqrt{b_1^2+b_2^2+2}=6$

3. Ans. (2)

$4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}=4\hat{i}+2\lambda_1\hat{j}+6\hat{k}$

$\Rightarrow 3 - \lambda_2 = 2\lambda_1 \Rightarrow 2\lambda_1 + \lambda_2 = 3$ ....(1)

Given $\vec{a}.\vec{c} = 0$

$\Rightarrow 6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0$

$\Rightarrow 2\lambda_1 + \lambda_3 = -1$ .....(2)

Now $(\lambda_1, \lambda_2, \lambda_3) = (\lambda_1, 3 - 2\lambda_1, -1 - 2\lambda_1)$

Now check the options, option (2) is correct

4. Ans. (4)

$\vec{\alpha} = (\lambda - 2)\vec{\alpha} + \vec{b}$

$\vec{\beta} = (4\lambda - 2)\vec{\alpha} + 3\vec{b}$

$\frac{\lambda - 2}{4\lambda - 2} = \frac{1}{3}$

$3\lambda - 6 = 4\lambda - 2$

$\boxed{\lambda = -4}$

$\therefore$ Option (4)

5. Ans. (3)

$\left[\vec{a} \quad \vec{b} \quad \vec{c}\right] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & \lambda^2 - 1 \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$

$\Rightarrow \lambda^2(\lambda - 2) - 9(\lambda - 2) = 0$

$\Rightarrow (\lambda - 3)(\lambda + 3)(\lambda - 2) = 0$

$\Rightarrow \lambda = 2, 3, -3$

So, $\lambda = 2$ (as $\vec{a}$ is parallel to $\vec{c}$ for $\lambda = \pm 3$)

$$\text{Hence } \vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix} = -10\hat{i} + 5\hat{j}$$

6. Ans. (2)

Angle bisector is $x - y = 0$

$$\Rightarrow \frac{|\beta - (1 - \beta)|}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

$\Rightarrow |2\beta - 1| = 3$

$\Rightarrow \beta = 2 \text{ or } -1$

7. Ans. (3)

$$\begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0$$

$\mu(\mu^2 - 1) - 1(\mu - 1) + 1(1 - \mu) = 0$

$\mu^3 - \mu - \mu + 1 + 1\ \mu = 0$

$\mu^3 - 3\mu + 2 = 0$

$\mu^3 - 1 - 3(\mu - 1) = 0$

$\mu = 1, \mu^2 + \mu - 2 = 0$

$\mu = 1, \mu = -2$

sum of distinct solutions $= -1$

8. Ans. (2)

$(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b}).\vec{c} = \frac{1}{2}\vec{b}$

$\because$ $\vec{b}$ & $\vec{c}$ are linearly independent

$\therefore$ $\vec{a}.\vec{c} = \frac{1}{2}$ & $\vec{a}.\vec{b} = 0$

(All given vectors are unit vectors)

$\therefore$ $\vec{a} \wedge \vec{c} = 60°$ & $\vec{a} \wedge \vec{b} = 90°$

$\therefore$ $|\alpha - \beta| = 30°$

9. Official Ans. by NTA (2)

Sol. Vector perpendicular to plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ & $\hat{i} + 2\hat{j} + 3\hat{k}$ is parallel to vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i} - 2\hat{j} + \hat{k}$$

$\therefore$ Required magnitude of projection

$$= \frac{|(2\hat{i} + 3\hat{j} + \hat{k}).(\hat{i} - 2\hat{j} + \hat{k})|}{|\hat{i} - 2\hat{j} + \hat{k}|}$$

$$= \frac{|2 - 6 + 1|}{|\sqrt{6}|} = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

10. Official Ans. by NTA (4)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix}$$

$= (2 + x)\hat{i} + (x - 3)\hat{j} - 5k$

$|\vec{a} \times \vec{b}| = \sqrt{4 + x^2 + 4x + x^2 + 9 - 6x + 25}$

$= \sqrt{2x^2 - 2x + 38}$

$\Rightarrow |\vec{a} \times \vec{b}| \geq \sqrt{\frac{75}{2}}$

$\Rightarrow |\vec{a} \times \vec{b}| \geq 5\sqrt{\frac{3}{2}}$

11. Official Ans. by NTA (3)

Sol. $\vec{\alpha} = 3\hat{i} + \hat{j}$

$\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$

$\vec{\beta}_1 = \lambda\left(3\hat{i} + \hat{j}\right), \vec{\beta}_2 = \lambda\left(3\hat{i} + \hat{j}\right) - 2\hat{i} + j - 3\hat{k}$

$\vec{\beta}_2.\vec{\alpha} = 0$

$(3\lambda - 2).3 + (\lambda + 1) = 0$

$9\lambda - 6 + \lambda + 1 = 0$

$\lambda = \frac{1}{2}$

$\Rightarrow \vec{\beta}_1 = \frac{3}{2}\hat{i} + \frac{1}{2}\hat{j}$

$\Rightarrow \vec{\beta}_2 = -\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

Now $\vec{\beta}_1 \times \vec{\beta}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{3}{2} & -3 \end{vmatrix}$

$= \hat{i}\left(-\frac{3}{2} - 0\right) - \hat{j}\left(-\frac{9}{2} - 0\right) + \hat{k}\left(\frac{9}{4} + \frac{1}{4}\right)$

$= -\frac{3}{2}\hat{i} + \frac{9}{2}\hat{j} + \frac{5}{2}\hat{k}$

$= \frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$

Aliter :

$\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2 \Rightarrow \vec{\beta}.\hat{\alpha} = \vec{\beta}_1.\hat{\alpha} = \left|\vec{\beta}_1\right|$

$\Rightarrow \vec{\beta}_1 = \left(\vec{\beta}.\hat{\alpha}\right)\hat{\alpha}$

$\Rightarrow \vec{\beta}_2 = \left(\vec{\beta}.\hat{\alpha}\right)\hat{\alpha} - \vec{\beta}$

$\Rightarrow \vec{\beta}_1 \times \vec{\beta}_2 = -\left(\vec{\beta}.\hat{\alpha}\right)\hat{\alpha} \times \vec{\beta}$

$= \frac{-5}{10}\left(3\hat{i} + \hat{j}\right) \times \left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

$= \frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$

12. Official Ans. by NTA (3)

Sol. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

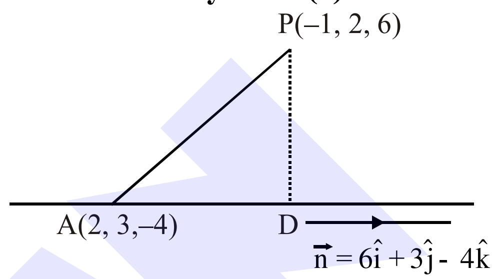
$\frac{1}{4} + \frac{1}{2} + \cos^2\gamma = 1$

$\cos^2\gamma = 1 - \frac{3}{4} = \frac{1}{4}$

$\cos^2\gamma = \pm\frac{1}{2} \Rightarrow \gamma = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$

13. Official Ans. by NTA (1)

Sol.



AD = $\left|\frac{\overrightarrow{AP}.\vec{n}}{|\vec{n}|}\right| = \sqrt{61}$

$\Rightarrow \quad PD = \sqrt{AP^2 - AD^2} = \sqrt{110 - 61} = 7$

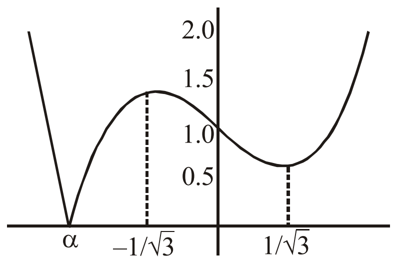
14. Official Ans. by NTA (3)

ALLEN Ans. Bonus

Sol. Volume of parallelopiped $= \left\|\begin{matrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{matrix}\right\|$

$f(\lambda) = |\lambda^3 - \lambda + 1|$

Its graph as follows



where $\alpha \approx -1.32$

$\because$ Question is asking minimum value of volume of parallelopiped & corresponding value of λ; the minimum value is zero, $\because$ cubic always has atleast one real root.

Hence answer to the question must be root of cubic $\lambda^3 - \lambda + 1 = 0$. None of the options satisfies the cubic.

Hence Question must be Bonus.

In JEE (Screening) 2003 same Question was asked and answer was given to be none of these, where the options were :

(A) –3 (B) 3

(C) $\frac{1}{\sqrt{3}}$ (D) none of these

15. Official Ans. by NTA (3)

Sol. $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = 2(\vec{b}\times\vec{a})$

$$= 2\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -2 \\ 3 & 2 & 2 \end{vmatrix} = 2(8\hat{i}-8\hat{j}+4\hat{k})$$

Required vector = $\pm 12\frac{(2\hat{i}-2\hat{j}-\hat{k})}{3}$

$= \pm 4(2\hat{i}-2\hat{j}-\hat{k})$

16. Official Ans. by NTA (4)

Sol. $\begin{vmatrix} \alpha & 1 & 3 \\ 2 & 1 & -4 \\ \alpha & -2 & 3 \end{vmatrix} = 0$

$\Rightarrow 3\alpha^2 + 18 = 0$

$\Rightarrow \alpha \in \phi$

3D

1. Ans. (3)

Equation of plane

$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$

$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z - 1 + 4\lambda = 0$

dr's of normal of the plane are

$1 + 2\lambda,\ 1 + 3\lambda,\ 1 - \lambda$

Since plane is parallel to y - axis, $1 + 3\lambda = 0$

$\Rightarrow \lambda = -1/3$

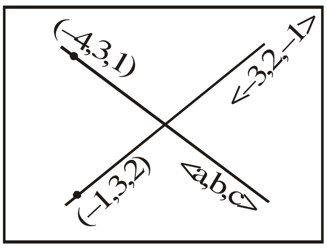
So the equation of plane is

$x + 4z - 7 = 0$

Point (3, 2, 1) satisfies this equation

Hence Answer is (3)

2. Ans. (2)



Normal vector of plane containing two intersecting lines is parallel to vector.

$$(\vec{V}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 1 \\ -3 & 2 & -1 \end{vmatrix}$$

$= -2\hat{i} + 6\hat{k}$

$\therefore$ Required line is parallel to vector

$$(\vec{V}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 0 & 6 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$$

$\Rightarrow$ Required equation of line is

$$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$

3. Ans. (2)

Vector along the normal to the plane containing the lines

$\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$

is $(8\hat{i} - \hat{j} - 10\hat{k})$

vector perpendicular to the vectors $2\hat{i} + 3\hat{j} + 4\hat{k}$

and $8\hat{i} - \hat{j} - 10\hat{k}$ is $26\hat{i} - 52\hat{j} + 26\hat{k}$

so, required plane is

$26x - 52y + 26z = 0$

$x - 2y + z = 0$

4. Ans. (2)

Line $x = ay + b,\ z = cy + d \Rightarrow \frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$

Line $x = a'z + b',\ y = c'z + d'$

$\Rightarrow \frac{x-b'}{a'} = \frac{y-d'}{c'} = \frac{z}{1}$

Given both the lines are perpendicular

$\Rightarrow aa' + c' + c = 0$

5. Ans. (4)

Let $\vec{n}$ be the normal vector to the plane passing through $(4, -1, 2)$ and parallel to the lines L_1 & L_2

then $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$

$\therefore\ \vec{n} = -7\hat{i} - 7\hat{j} + 7\hat{k}$

$\therefore$ Equation of plane is

$-1(x-4) - 1(y+1) + 1(z-2) = 0$

$\therefore\ x + y - z - 1 = 0$

Now check options

6. **Ans. (3)**

Let point A is

$(1-3\mu)\hat{i} + (\mu-1)\hat{j} + (2+5\mu)\hat{k}$

and point B is (3, 2, 6)

then $\overrightarrow{AB} = (2+3\mu)\hat{i} + (3-\mu)\hat{j} + (4-5\mu)\hat{k}$

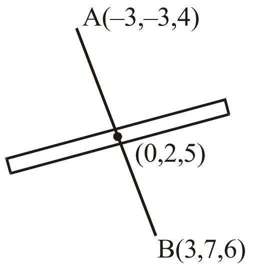
which is parallel to the plane $x - 4y + 3z = 1$

$\therefore\ 2 + 3\mu - 12 + 4\mu + 12 - 15\mu = 0$

$8\mu = 2$

$\mu = \frac{1}{4}$

7. **Ans. (2)**



$\vec{n} = 3\hat{i} + 5\hat{j} + \hat{k}$

$p : 3(x-0) + 5(y-2) + 1(z-5) = 0$

$3x + 5y + z = 15$

$\therefore$ Option (2)

8. **Ans. (3)**

General point on the given line is

$x = 2\lambda + 4$

$y = 2\lambda + 5$

$z = \lambda + 3$

Solving with plane,

$2\lambda + 4 + 2\lambda + 5 + \lambda + 3 = 2$

$5\lambda + 12 = 2$

$5\lambda = -10$

$\boxed{\lambda = -2}$

$\therefore$ Option (3)

9. **Ans. (2, 4)**

Let the equation of plane be

$a(x-0) + b(y+1) + c(z-0) = 0$

It passes through (0,0,1) then

$b + c = 0$ ...(1)

Now $\cos\frac{\pi}{4} = \frac{a(0)+b(1)+c(-1)}{\sqrt{2}\sqrt{a^2+b^2+c^2}}$

$\Rightarrow a^2 = -2bc$ and $b = -c$

we get $a^2 = 2c^2$

$\Rightarrow\ a = \pm\sqrt{2}\,c$

$\Rightarrow$ direction ratio (a, b, c) = $(\sqrt{2}, -1, 1)$ or $(\sqrt{2}, 1, -1)$

10. **Ans. (1)**

The normal vector of required plane

$= (2\hat{i} - \hat{j} + 3\hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$

$= -8\hat{i} + 8\hat{j} + 8\hat{k}$

So, direction ratio of normal is (–1, 1, 1)

So required plane is

$-(x-3) + (y+2) + (z-1) = 0$

$\Rightarrow -x + y + z + 4 = 0$

Which is satisfied by (2, 0, –2)

11. **Ans. (4)**

Normal vector of plane

$= \begin{vmatrix} i & j & k \\ 2 & -5 & 0 \\ 4 & -4 & 4 \end{vmatrix} = -4(5\hat{i} + 2\hat{j} - 3\hat{k})$

equation of plane is $5(x-7) + 2y - 3(z-6) = 0$

$5x + 2y - 3z = 17$

12. **Ans. (3)**

Point on L_1 $(\lambda + 3, 3\lambda - 1, -\lambda + 6)$

Point on L_2 $(7\mu - 5, -6\mu + 2, 4\mu + 3)$

$\Rightarrow \lambda + 3 = 7\mu - 5$ ...(i)

$3\lambda - 1 = -6\mu + 2$ ...(ii)

$\Rightarrow \lambda = -1,\ \mu = 1$

point R(2,–4,7)

Reflection is (2,–4,–7)

13. **Ans. (1)**

$\begin{vmatrix} i & j & k \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$

$\hat{i}(35-28) - \hat{j}(21.7) + \hat{k}(12-5)$

$7\hat{i} - 14\hat{j} + 7\hat{k}$

$\hat{i} - 2\hat{j} + \hat{k}$

$1(x + 2) - 2(y - 2) + 1\,(z+15) = 0$

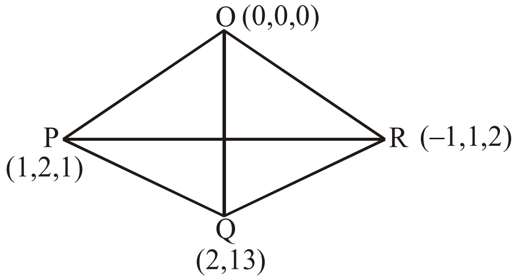
$x - 2y + z + 11 = 0$

$$\frac{11}{\sqrt{4+1+1}} = \frac{11}{\sqrt{6}}$$

14. Ans. (2)

$\overrightarrow{OP} \times \overrightarrow{OQ} = (\hat{i} + 2\hat{j} + \hat{k}) \times (2\hat{i} + \hat{j} + 3\hat{k})$

$5\hat{i} - \hat{j} - 3\hat{k}$



$\overrightarrow{PQ} \times \overrightarrow{PR} = (\hat{i} - \hat{j} + 2\hat{k}) \times (-2\hat{i} - \hat{j} + \hat{k})$

$\hat{i} - 5\hat{j} - 3\hat{k}$

$$\cos\theta = \frac{5+5+9}{\left(\sqrt{25+9+1}\right)^2} = \frac{19}{35}$$

15. Ans (3)

DR's of line are 2, 1, –2

normal vector of plane is $\hat{i} - 2\hat{j} - k\hat{k}$

$$\sin\alpha = \frac{(2\hat{i} + \hat{j} - 2\hat{k}).(\hat{i} - 2\hat{j} - k\hat{k})}{3\sqrt{1+4+k^2}}$$

$$\sin\alpha = \frac{2k}{3\sqrt{k^2+5}} \qquad ....(1)$$

$$\cos\alpha = \frac{2\sqrt{2}}{3} \qquad ....(2)$$

$(1)^2 + (2)^2 = 1 \Rightarrow k^2 = \frac{5}{3}$

16. Ans (2)

All four points are coplaner so

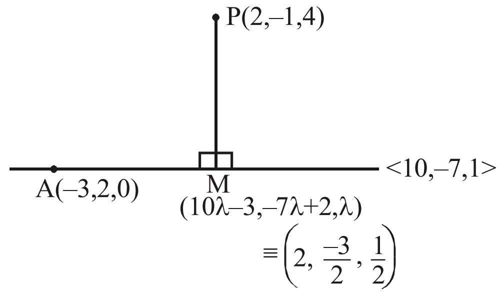
$$\begin{vmatrix} 1-\lambda^2 & 2 & 0 \\ 2 & -\lambda^2+1 & 0 \\ 2 & 2 & -\lambda^2-1 \end{vmatrix} = 0$$

$(\lambda^2 + 1)^2\,(3 - \lambda^2) = 0$

$\lambda = \pm\sqrt{3}$

17. Official Ans. by NTA (2)

Sol.



Now, $\overrightarrow{MP}.\left(10\hat{i} - 7\hat{j} + \hat{k}\right) = 0$

$\Rightarrow \quad \lambda = \frac{1}{2}$

$\therefore$ Length of perpendicular

$$(= PM) = \sqrt{0 + \frac{1}{4} + \frac{49}{4}}$$

$$= \sqrt{\frac{50}{4}} = \sqrt{\frac{25}{2}} = \frac{5}{\sqrt{2}},$$

which is greater than 3 but less than 4.

18. Official Ans. by NTA (2)

Sol. The required plane is

$(2x - y - 4) + \lambda(y + 2z - 4) = 0$

it passes through (1, 1, 0)

$\Rightarrow \quad (2 - 1 - 4) + \lambda(1 - 4) = 0$

$\Rightarrow \quad -3 - 3\lambda = 0 \quad \Rightarrow \quad \lambda = -1$

$\Rightarrow \quad x - y - z = 0$

19. Official Ans. by NTA (3)

Sol. Let the plane be

$(x + y + z - 1) + \lambda\,(2x + 3y + 4z - 5) = 0$

$\Rightarrow (2\lambda+1)x + (3\lambda+1)y + (4\lambda+1)z - (5\lambda+1) = 0$

$\perp$ to the plane $x - y + z = 0$

$\Rightarrow \lambda = -\frac{1}{3}$

$\Rightarrow$ the required plane is $x - z + 2 = 0$

20. Official Ans. by NTA (1)

Sol. $\frac{4}{2} = \frac{-y}{y+3} = \frac{10-z}{z-4}$

$\Rightarrow z = 6 \;\&\; y = -2$

$\Rightarrow R(4, -2, 6)$

dist. from origin $= \sqrt{16+4+36} = 2\sqrt{14}$

21. Official Ans. by NTA (2)

Sol. Let ax + by + cz = 1 be the equation of the plane

$\Rightarrow 0 - b + 0 = 1$

$\Rightarrow b = -1$

0 + 0 + c = 1

$\Rightarrow c = 1$

$$\cos\theta = \left|\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right|$$

$$\frac{1}{\sqrt{2}} = \frac{|0-1-1|}{\sqrt{(a^2+1+1)}\sqrt{0+1+1}}$$

$\Rightarrow a^2 + 2 = 4$

$\Rightarrow a = \pm\sqrt{2}$

$\Rightarrow \pm\sqrt{2}x - y + z = 1$

Now for –sign

$-\sqrt{2}.\sqrt{2} - 1 + 4 = 1$

option (2)

22. Official Ans. by NTA (1)

Sol. Any point on the given line can be

(1 + 2λ , –1 + 3λ, 2 + 4λ) ; $\lambda \in R$

Put in plane

1 + 2λ + (–2 + 6λ) + (6 + 12λ) = 15

20λ + 5 = 15

20λ = 10

$\lambda = \frac{1}{2}$

$\therefore$ Point $\left(2, \frac{1}{2}, 4\right)$

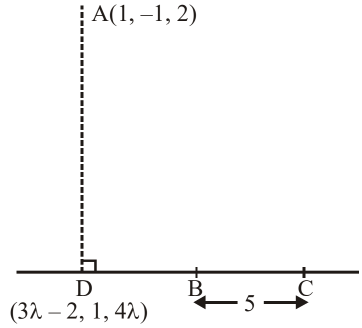
Distance from origin

$$= \sqrt{4 + \frac{1}{4} + 16} = \frac{\sqrt{16+1+64}}{2} = \frac{\sqrt{81}}{2}$$

$$= \frac{9}{2}$$

23. Official Ans. by NTA (2)

Sol.



$\overrightarrow{AD} \cdot (3\hat{i} + 4\hat{k}) = 0$

3(3λ – 3) + 0 + 4(4λ – 2) = 0

(9λ – 9) + (16λ – 8) = 0

$25\lambda = 17 \Rightarrow \lambda = \frac{17}{25}$

$$\therefore \overrightarrow{AD} = \left(\frac{51}{25} - 3\right)\hat{i} + 2\hat{j} + \left(\frac{68}{25} - 2\right)\hat{k}$$

$$= \frac{24}{25}\hat{i} + 2\hat{j} + \frac{18}{25}\hat{k}$$

$$|\overrightarrow{AD}| = \sqrt{\frac{576}{625} + 4 + \frac{324}{625}}$$

$$= \sqrt{\frac{900}{625} + 4} = \sqrt{\frac{3400}{625}}$$

$$= \sqrt{34} \cdot \frac{10}{25} = \frac{2}{5}\sqrt{34}$$

Area of $\Delta = \frac{1}{2} \times 5 \times \frac{2\sqrt{34}}{5} = \sqrt{34}$

24. Official Ans. by NTA (4)

Sol. λ(x + y + z – 6) + 2x + 3y + z + 5 = 0

(λ + 2)x + (λ + 3)y + (λ + 1)z + 5 – 6λ = 0

λ + 1 = 0 ⇒ λ = –1

P : x + 2y + 11 = 0

perpendicular distance $= \frac{11}{\sqrt{5}}$

25. Official Ans. by NTA (2)

Sol. x + 3y + λz – μ = p (x + y + z –5) + q (x + 2y + 2z – 6)

on comparing the coefficient;

p + q = 1 and p + 2q = 3

⇒ (p,q) = (–1,2)

Hence x + 3y + λz –μ = x + 3y + 3z –7

⇒ λ = 3, μ = 7

26. Official Ans. by NTA (3)

Sol. G is the centroid of Δ ABC

$G \equiv (2,4,2)$

$\overrightarrow{OG} = 2\hat{i} + 4\hat{j} + 2\hat{k}$

$\overrightarrow{OA} = 3\hat{i} - \hat{k}$

$\cos(\angle GOA) = \frac{\overrightarrow{OG}\cdot\overrightarrow{OA}}{|\overrightarrow{OG}||\overrightarrow{OA}|} = \frac{1}{\sqrt{15}}$

27. Official Ans. by NTA (1)

Sol. One of the point on line is P(0, 1, –1) and given point is Q(β, 0, β).

So, $\overrightarrow{PQ} = \beta\hat{i} - \hat{j} + (\beta+1)\hat{k}$

Hence, $\beta^2 + 1 + (\beta+1)^2 - \frac{(\beta-\beta-1)^2}{2} = \frac{3}{2}$

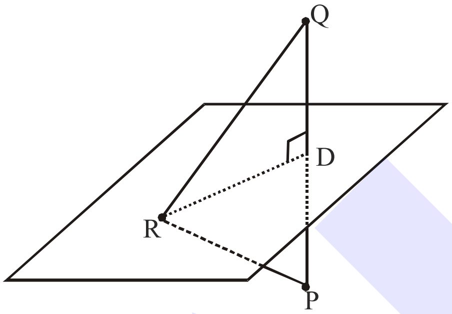
$\Rightarrow 2\beta^2 + 2\beta = 0$

$\Rightarrow \beta = 0, -1$

$\Rightarrow \beta = -1$ (as $\beta \neq 0$)

28. Official Ans. by NTA (4)

Sol. R lies on the plane.



$DQ = \frac{|1-12-2|}{\sqrt{9+1+16}} = \frac{13}{\sqrt{26}} = \sqrt{\frac{13}{2}}$

$\Rightarrow PQ = \sqrt{26}$

Now, $RQ = \sqrt{9+1} = \sqrt{10}$

$\Rightarrow RD = \sqrt{10 - \frac{13}{2}} = \sqrt{\frac{7}{2}}$

Hence, $ar(\Delta PQR) = \frac{1}{2}\times\sqrt{26}\times\sqrt{\frac{7}{2}} = \frac{\sqrt{91}}{2}$.

29. Official Ans. by NTA (1)

Sol. Let point P on the line is (2λ +1, –λ–1, λ) foot of perpendicular Q is given by

$\frac{x-2\lambda-1}{1} = \frac{y+\lambda+1}{1} = \frac{z-\lambda}{1} = \frac{-(2\lambda-3)}{3}$

$\because$ Q lies on x + y + z = 3 & x – y + z = 3

$\Rightarrow x + z = 3 \ \& \ y = 0$

$y = 0 \Rightarrow \lambda + 1 = \frac{-2\lambda+3}{3} \Rightarrow \lambda = 0$

$\Rightarrow$ Q is (2, 0, 1)

30. Official Ans. by NTA (3)

Sol. 4x – 2y + 4z + 6 = 0

$\frac{|\lambda-6|}{\sqrt{16+4+16}} = \left|\frac{\lambda-6}{6}\right| = \frac{1}{3}$

$|\lambda - 6| = 2$

$\lambda = 8, 4$

$\frac{|\mu-3|}{\sqrt{4+4+1}} = \frac{2}{3}$

$|\mu - 3| = 2$

$\mu = 5, 1$

$\therefore$ Maximum value of $(\mu + \lambda) = 13$.

31. Official Ans. by NTA (1)

Sol. $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1} = \lambda$

x = 3λ + 2, y = 2λ – 1, z = –λ + 1

Intersection with plane 2x + 3y – z + 13 = 0

2(3λ + 2) + 3(2λ – 1) – (–λ + 1) + 13 = 0

$13\lambda + 13 = 0 \quad \boxed{\lambda = -1}$

$\therefore$ P(–1, –3, 2)

Intersection with plane

3x + y + 4z = 16

3(3λ+2) + (2λ–1) + 4(–λ+1) = 16

λ = 1

Q(5, 1, 0)

$PQ = \sqrt{6^2+4^2+2^2} = \sqrt{56} = 2\sqrt{14}$

32. Official Ans. by NTA (1)

Sol. perpendicular vector to the plane

$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = -3\hat{i} + 3\hat{j} + 3\hat{k}$

Eq. of plane

– 3 (x – 1) + 3 (y – 1) + 3z = 0

$\Rightarrow$ x – y – z = 0

$d_{(2,1,4)} = \frac{|2-1-4|}{\sqrt{1^2+1^2+1^2}} = \sqrt{3}$

33. Official Ans. by NTA (2)

Sol. equation of bisector of angle

$$\frac{2x-y+2z-4}{3}=\pm\frac{x+2y+2z-2}{3}$$

(+) gives $x - 3y = 2$

(–) gives $3x + y + 4z = 6$

therefore option (ii) satisfy

PARABOLA

1. Ans. (3)

Let equation of tangent to the parabola $y^2 = 4x$ is

$$y = mx+\frac{1}{m},$$

$\Rightarrow m^2x-ym+1 = 0$ is tangent to $x^2 + y^2 - 6x = 0$

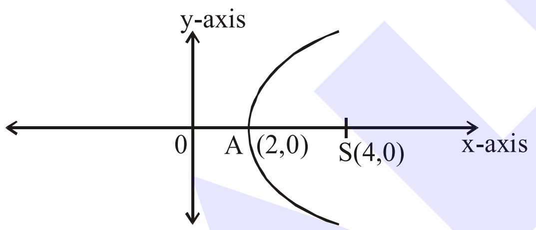
$$\Rightarrow \frac{|3m^2+1|}{\sqrt{m^4+m^2}}=3$$

$$m=\pm\frac{1}{\sqrt{3}}$$

$\Rightarrow$ tangent are $x+\sqrt{3}y+3=0$

and $x-\sqrt{3}y+3=0$

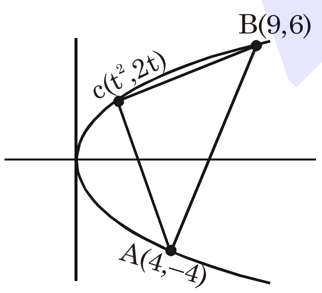
2. Ans. (3)



equation of parabola is $y^2 = 8(x - 2)$

(8, 6) does not lie on parabola.

3. Ans. (4)



Area $= 5|t^2 - t - 6| = 5\left|\left(t-\frac{1}{2}\right)^2-\frac{25}{4}\right|$

is maximum if $t=\frac{1}{2}$

4. Ans. (1,2,3,4)

Normal to these two curves are

$y = m(x - c) - 2bm - bm^3$,

$y = mx - 4am - 2am^3$

If they have a common normal

$(c + 2b)\ m + bm^3 = 4am + 2am^3$

Now $(4a - c - 2b)\ m = (b - 2a)m^3$

We get all options are correct for $m = 0$

(common normal x-axis)

Ans. (1), (2), (3), (4)

Remark :

If we consider question as

If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal other than x-axis, then which one of the following is a valid choice for the ordered triad (a, b, c) ?

When $m \neq 0$: $(4a - c - 2b) = (b - 2a)m^2$

$$m^2=\frac{c}{2a-b}-2>0\Rightarrow\frac{c}{2a-b}>2$$

Now according to options, option 4 is correct

5. Ans. (3)

$x^2 = 4y$

$x-\sqrt{2}y+4\sqrt{2}=0$

Solving together we get

$$x^2=4\left(\frac{x+4\sqrt{2}}{\sqrt{2}}\right)$$

$$\sqrt{2}x^2+4x+16\sqrt{2}$$

$$\sqrt{2}x^2-4x-16\sqrt{2}=0$$

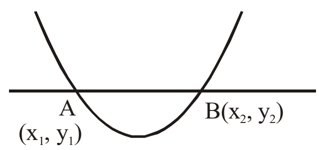
$$x_1+x_2=2\sqrt{2};\quad x_1x_2=\frac{-16\sqrt{2}}{\sqrt{2}}=-16$$

Similarly,

$$\left(\sqrt{2}y-4\sqrt{2}\right)^2=4y$$

$2y^2 + 32 - 16y = 4y$

$2y^2 - 20y + 32 = 0$ → $y_1 + y_2 = 10$, $y_1 y_2 = 16$



$$\ell_{AB}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{\left(2\sqrt{2}\right)^2+64+(10)^2-4(16)}$$

$$=\sqrt{8+64+100-64}$$

$= \sqrt{108} = 6\sqrt{3}$

Option (3)

6. Ans. (4)

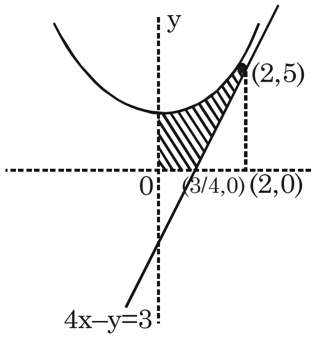
Vertex is $(a^2,0)$

$y^2 = -(x - a^2)$ and $x = 0 \Rightarrow (0, \pm 2a)$

Area of triangle is $= \frac{1}{2}.4a.\left(a^2\right) = 250$

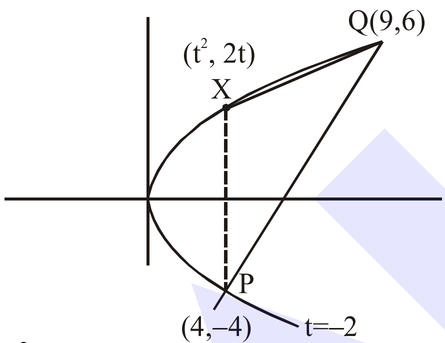
$\Rightarrow a^3 = 125$ or $a = 5$

7. Ans. (3)



Area $= \int_0^2 \left(x^2+1\right)dx - \frac{1}{2}\left(\frac{5}{4}\right)(5) = \frac{37}{24}$

8. Ans. (1)



$y^2 = 4x$

$2yy' = 4$

$y' = \frac{1}{t} = 2,\ t = \frac{1}{2}$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} \frac{1}{4} & 1 & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{vmatrix} = \frac{125}{4}$$

9. Ans (1)

$x^2 = 8y$

$\Rightarrow \frac{dy}{dx} = \frac{x}{4} = \tan\theta$

$\therefore x_1 = 4\tan\theta$

$y_1 = 2\tan^2\theta$

Equation of tangent :-

$y - 2\tan^2\theta = \tan\theta\,(x - 4\tan\theta)$

$\Rightarrow x = y\cot\theta + 2\tan\theta$

10. Official Ans. by NTA (3)

Sol. Given $y^2 = 4x$ ...(1)

and $x^2 + y^2 = 5$ ...(2)

by (1) and (2)

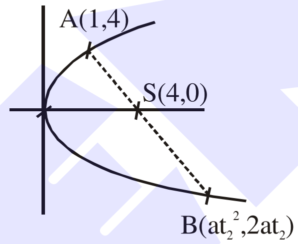
$\Rightarrow x = 1$ and $y = 2$

equation of tangent at (1,2) to $y^2 = 4x$

is $y = x + 1$

11. Official Ans. by NTA (1)

Sol.



$y^2 = 4ax = 16x \Rightarrow a = 4$

$A(1,4) \Rightarrow 2.4.t_1 = 4 \Rightarrow t_1 = \frac{1}{2}$

$\therefore$ length of focal chord $= a\left(t + \frac{1}{t}\right)^2$

$= 4\left(\frac{1}{2} + 2\right)^2 = 4.\frac{25}{4} = 25$

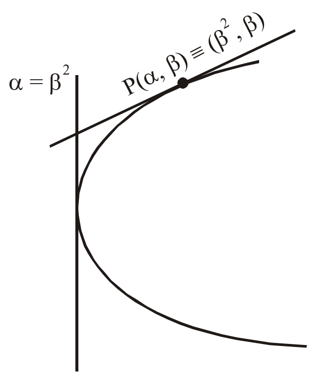
12. Official Ans. by NTA (3)

Sol. $T: y(\beta) = \frac{1}{2}(x + \beta^2)$

$2y\beta = x + \beta^2$

$y = \left(\frac{1}{2\beta}\right)x + \frac{\beta}{2}$

$m = \frac{1}{2\beta}; C = \frac{\beta}{2}$



$\frac{\beta}{2} = \pm\sqrt{\frac{1}{4\beta^2} + \frac{1}{2}}$

$\frac{\beta^2}{4} = \frac{1}{4\beta^2} + \frac{1}{2}$

$\frac{\beta^2}{4} = \frac{1+2\beta^2}{4\beta^2}$

$\Rightarrow \beta^4 - 2\beta^2 - 1 = 0$

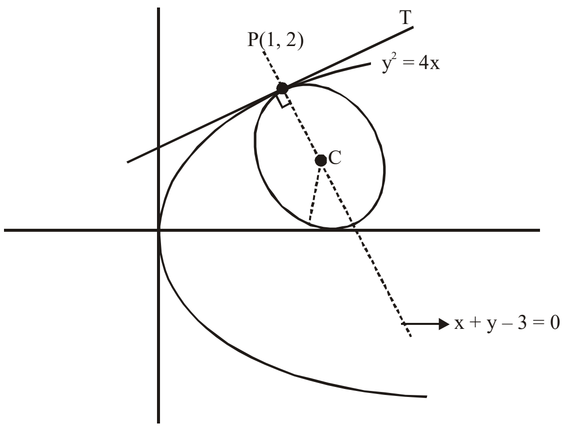
$(\beta^2 - 1)^2 = 2$

$\beta^2 - 1 = \sqrt{2}$

$\beta^2 = \sqrt{2} + 1$

13. Official Ans. by NTA (2)

Sol.



Equation of circle is

$(x - 1)^2 + (y - 2)^2 + \lambda(x - y + 1) = 0$

$\Rightarrow x^2 + y^2 + x(\lambda - 2) + y(-4 - \lambda) + (5 + \lambda) = 0$

As cirlce touches x axis then $g^2 - c = 0$

$\frac{(\lambda - 2)^2}{4} = (5 + \lambda)$

$\lambda^2 + 4 - 4\lambda = 20 + 4\lambda$

$\lambda^2 - 8\lambda - 16 = 0$

$\lambda = \frac{8 \pm \sqrt{128}}{2}$

$\lambda = 4 \pm 4\sqrt{2}$

Radius = $\left|\frac{(-4-\lambda)}{2}\right|$

Put λ and get least radius.

14. Official Ans. by NTA (3)

Sol. Tangent to $y^2 = 4\sqrt{2}\,x$ is $y = mx + \frac{\sqrt{2}}{m}$

it is also tangent to $x^2 + y^2 = 1$

$\Rightarrow \left|\frac{\sqrt{2}/m}{\sqrt{1+m^2}}\right| = 1 \quad \Rightarrow m = \pm 1$

$\Rightarrow$ Tagent will be $y = x + \sqrt{2}$ or $y = -x - \sqrt{2}$

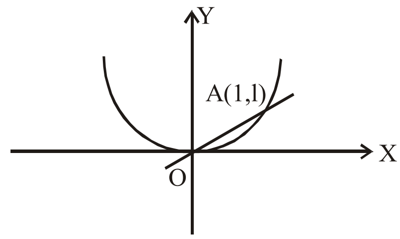
compare with $y = -ax + C$

$\Rightarrow a = \pm 1$ & $C = \pm\sqrt{2}$

15. Official Ans. by NTA (3)

Sol. Put $x - 2 = X$ & $y + 1 = Y$

$\therefore$ given curve becomes $Y = X^2$ and $Y = X$



tangent at origin is X-axis

and tangent at A(1,1) is Y + 1 = 2X

$\therefore$ there intersection is $\left(\frac{1}{2}, 0\right)$

$\therefore \; x - 2 = \frac{1}{2}$ & $y + 1 = 0$

therefore $x = \frac{5}{2}, y = -1$

16. Official Ans. by NTA (4)

Sol. tangent to the parabola $y^2 = 16x$ is $y = mx + \frac{4}{m}$

solve it by curve $xy = -4$

i.e. $mx^2 + \frac{4}{m}x + 4 = 0$

condition of common tangent is D = 0

$\therefore m^3 = 1$

$\Rightarrow m = 1$

$\therefore$ equation of common tangent is $y = x + 4$

ELLIPSE

1. Ans. (3)

Equation of general tangent on ellipse

$\frac{x}{a\sec\theta} + \frac{y}{b\operatorname{cosec}\theta} = 1$

$a = \sqrt{2}$, $b = 1$

$\Rightarrow \frac{x}{\sqrt{2}\sec\theta} + \frac{y}{\operatorname{cosec}\theta} = 1$

Let the midpoint be (h, k)

$h = \frac{\sqrt{2}\sec\theta}{2} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}h}$

and $k = \frac{\text{cosec}\theta}{2} \Rightarrow \sin\theta = \frac{1}{2k}$

$\because \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \frac{1}{2h^2} + \frac{1}{4k^2} = 1$

$\Rightarrow \frac{1}{2x^2} + \frac{1}{4y^2} = 1$

2. Ans. (2)

$\frac{2b^2}{a} = 8$ and $2ae = 2b$

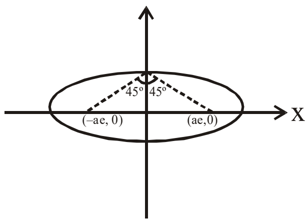
$\Rightarrow \frac{b}{a} = e$ and $1 - e^2 = e^2 \Rightarrow e = \frac{1}{\sqrt{2}}$

$\Rightarrow b = 4\sqrt{2}$ and $a = 8$

so equation of ellipse is $\frac{x^2}{64} + \frac{y^2}{32} = 1$

3. Ans. (3)

$m_{SB} \cdot m_{S'B} = -1$



$b^2 = a^2e^2$ .... (i)

$\frac{1}{2}S'B \cdot SB = 8$

$S'B \cdot SB = 16$

$a^2e^2 + b^2 = 16$ ..... (ii)

$b^2 = a^2(1 - e^2)$ ..... (iii)

using (i),(ii), (iii) $a = 4$

$b = 2\sqrt{2}$

$e = \frac{1}{\sqrt{2}}$

$\therefore \ell(\text{L.R}) = \frac{2b^2}{a} = 4$ Ans.3

4. Official Ans. by NTA (2)

Sol. $4a^2 + b^2 = 8$ ....(1)

also $\left.\frac{dy}{dx}\right)_{(1,2)} = -\frac{4x}{y} = -2$

$\Rightarrow -\frac{4a}{b} = \frac{1}{2}$

$b = -8a$

$\Rightarrow b^2 = 64a^2$

$68a^2 = 8$

$a^2 = \frac{2}{17}$

5. Official Ans. by NTA (3)

Sol. Let equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$2a - 2b = 10$ ...(1)

$ae = 5\sqrt{3}$ ...(2)

$\frac{2b^2}{a} = ?$

$b^2 = a^2(1 - e^2)$

$b^2 = a^2 - a^2e^2$

$b^2 = a^2 - 25 \times 3$

$\Rightarrow b = 5$ and $a = 10$

$\therefore$ length of L.R. $= \frac{2(25)}{10} = 5$

6. Official Ans. by NTA (1)

Sol. Tangent at $\left(3, -\frac{9}{2}\right)$

$\frac{3x}{a^2} - \frac{9y}{2b^2} = 1$

Comparing this with $x - 2y = 12$

$\frac{3}{a^2} = \frac{9}{4b^2} = \frac{1}{12}$

we get $a = 6$ and $b = 3\sqrt{3}$

$L(LR) = \frac{2b^2}{a} = 9$

7. Official Ans. by NTA (3)

Sol. $3x^2 + 5y^2 = 32$

$\left.\frac{dy}{dx}\right|_{(2,2)} = -\frac{3}{5}$

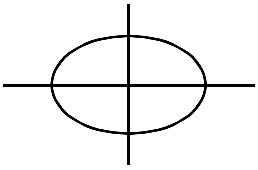
Tangent : $y - 2 = -\frac{3}{5}(x - 2) \Rightarrow Q\left(\frac{16}{3}, 0\right)$

Normal : $y - 2 = \frac{5}{3}(x-2) \Rightarrow R\left(\frac{4}{5},0\right)$

Area is $= \frac{1}{2}(QR)\times 2 = QR = \frac{68}{15}$.

8. Official Ans. by NTA (4)

Sol. $3x^2 + 4y^2 = 12$



$\frac{x^2}{4}+\frac{y^2}{3}=1$

$x = 2\cos\theta,\ y = \sqrt{3}\sin\theta$

Let $P(2\cos\theta, \sqrt{3}\sin\theta)$

Equation of normal is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

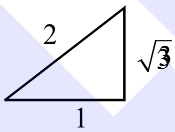
$2x\sin\theta-\sqrt{3}\cos\theta y=\sin\theta\cos\theta$

Slope $\frac{2}{\sqrt{3}}\tan\theta=-2 \quad \therefore \tan\theta=-\sqrt{3}$

Equation of tangent is

it passes through (4, 4)

$3x\cos\theta + 2\sqrt{3}\sin\theta\, y = 6$



$12\cos\theta + 8\sqrt{3}\sin\theta = 6$

$\cos\theta=-\frac{1}{2},\ \sin\theta=\frac{\sqrt{3}}{2} \quad \therefore \theta = 120°$

Hence point P is $(2\cos 120°, \sqrt{3}\sin 120°)$

$P\left(-1,\frac{3}{2}\right)$, Q (4, 4)

$PQ=\frac{5\sqrt{5}}{2}$

9. Official Ans. by NTA (4)

Sol. given that be = 2 and a = 2

(here a < b)

$\because a^2 = b^2(1 - e^2)$

$\therefore b^2 = 8$

$\therefore$ equation of ellipse $\frac{x^2}{4}+\frac{y^2}{8}=1$

HYPERBOLA

1. Ans. (2)

$e=\sqrt{1+\tan^2\theta}=\sec\theta$

As, $\sec\theta > 2 \Rightarrow \cos\theta < \frac{1}{2}$

$\Rightarrow \theta \in (60°, 90°)$

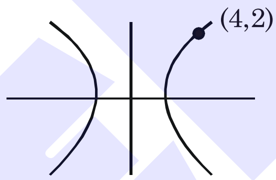
Now, $\ell(L{\cdot}R) = \frac{2b^2}{a}=2\frac{(1-\cos^2\theta)}{\cos\theta}$

$=2(\sec\theta - \cos\theta)$

Which is strictly increasing, so

$\ell(L.R) \in (3,\infty)$.

2. Ans. (1)



$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$2a = 4 \Rightarrow a = 2$

$\frac{x^2}{4}-\frac{y^2}{b^2}=1$

Passes through (4,2)

$4-\frac{4}{b^2}=1\Rightarrow b^2=\frac{4}{3}\Rightarrow e=\frac{2}{\sqrt{3}}$

3. Ans. (3)

Hyperbola $\frac{x^2}{5}-\frac{y^2}{4}=1$

slope of tangent = 1

equation of tangent $y=x\pm\sqrt{5-4}$

$\Rightarrow y = x \pm 1$

$\Rightarrow y = x + 1$ or $y = x - 1$

4. Ans. (4)

$\frac{y^2}{1+r}-\frac{x^2}{1-r}=1$

for $r > 1$, $\frac{y^2}{1+r}+\frac{x^2}{r-1}=1$

$e=\sqrt{1-\left(\frac{r-1}{r+1}\right)}$

$= \sqrt{\dfrac{(r+1)-(r-1)}{(r+1)}}$

$= \sqrt{\dfrac{2}{r+1}} = \sqrt{\dfrac{2}{r+1}}$

Option (4)

5. Ans. (1)

Let the equation of tangent to parabola

$y^2 = 4x$ be $y = mx + \dfrac{1}{m}$

It is also a tangent to hyperbola xy = 2

$\Rightarrow \quad x\left(mx + \dfrac{1}{m}\right) = 2$

$\Rightarrow \quad x^2 m + \dfrac{x}{m} - 2 = 0$

D = 0 $\Rightarrow$ $m = -\dfrac{1}{2}$

So tangent is 2y + x + 4 = 0

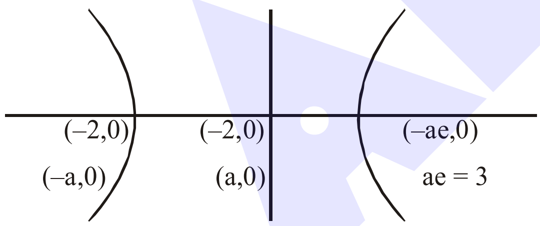
6. Ans. (4)

2b = 5 and 2ae = 13

$b^2 = a^2(e^2 - 1) \Rightarrow \dfrac{25}{4} = \dfrac{169}{4} - a^2$

$\Rightarrow a = 6 \Rightarrow e = \dfrac{13}{12}$

7. Ans. (3)



ae = 3, $e = \dfrac{3}{2}$, $b^2 = 4\left(\dfrac{9}{4} - 1\right)$, $b^2 = 5$

$\dfrac{x^2}{4} - \dfrac{y^2}{5} = 1$

8. Official Ans. by NTA (1)

Sol. Let us Suppose equation of hyperbola is

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$e = 2 \Rightarrow b^2 = 3a^2$

passing through (4,6) $\Rightarrow a^2 = 4$, $b^2 = 12$

$\Rightarrow$ equaiton of tangent

$x - \dfrac{y}{2} = 1$

$\Rightarrow 2x - y - 2 = 0$

9. Official Ans. by NTA (3)

Sol. $\dfrac{x^2}{24} - \dfrac{y^2}{18} = 1 \Rightarrow a = \sqrt{24}; b = \sqrt{18}$

Parametric normal :

$\sqrt{24}\cos\theta.x + \sqrt{18}.y\cot\theta = 42$

At x = 0 : $y = \dfrac{42}{\sqrt{18}}\tan\theta = 7\sqrt{3}$ (from given equation)

$\Rightarrow \tan\theta = \sqrt{\dfrac{3}{2}} \Rightarrow \sin\theta = \pm\sqrt{\dfrac{3}{5}}$

slope of parametric normal $= \dfrac{-\sqrt{24}\cos\theta}{\sqrt{18}\cot\theta} = m$

$\Rightarrow m = -\sqrt{\dfrac{4}{3}}\sin\theta = -\dfrac{2}{\sqrt{5}}$ or $\dfrac{2}{\sqrt{5}}$

10. Official Ans. by NTA (1)

Sol. Hyperbola is $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$\dfrac{a}{e} = \dfrac{4}{\sqrt{5}}$ and $\dfrac{16}{a^2} - \dfrac{12}{b^2} = 1$

$a^2 = \dfrac{16}{5}e^2$ ....(1) and $\dfrac{16}{a^2} - \dfrac{12}{a^2(e^2-1)} = 1$ ....(2)

From (1) & (2)

$16\left(\dfrac{5}{16e^2}\right) - \dfrac{12}{(e^2-1)}\left(\dfrac{5}{16e^2}\right) = 1$

$\Rightarrow 4e^4 - 24e^2 + 35 = 0$

11. Official Ans. by NTA (3)

Sol. $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

a = 3, b = 4 & e = $\sqrt{1 + \dfrac{16}{9}} = \dfrac{5}{3}$

corresponding focus will be (–ae, 0) i.e., (–5, 0).

12. Official Ans. by NTA (1)

Sol. Equation of tangents

$$y^2 = 12x \quad \Rightarrow \quad y = 2x + \frac{3}{m}$$

$$\frac{x^2}{1} - \frac{y^2}{8} = 1 \;\Rightarrow\; y = mx \pm \sqrt{m^2 - 8}$$

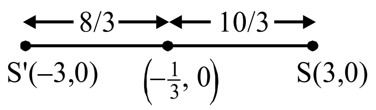
Since they are common tangent

$$\therefore \quad \frac{3}{m} = \pm\sqrt{m^2 - 8} \qquad \left| \quad \frac{x^2}{1} - \frac{y^2}{8} = 1 \right.$$

$m^4 - 8m^2 - 9 = 0$ | $e = 3$

$m = \pm 3$ | $ae = 3$

$$\therefore \; \left.\begin{matrix} y = 3x + 1 \\ y = -3x - 1 \end{matrix}\right\rangle P\left(-\frac{1}{3}, 0\right) \quad S = (3, 0),\; S' = (-3, 0)$$



COMPLEX NUMBER

1. Ans. (2)

Given $z = \dfrac{3 + 2i\sin\theta}{1 - 2i\sin\theta}$ is purely img

so real part becomes zero.

$$z = \left(\frac{3 + 2i\sin\theta}{1 - 2i\sin\theta}\right) \times \left(\frac{1 + 2i\sin\theta}{1 + 2i\sin\theta}\right)$$

$$z = \frac{(3 - 4\sin^2\theta) + i(8\sin\theta)}{i + 4\sin^2\theta}$$

Now $\operatorname{Re}(z) = 0$

$$\frac{3 - 4\sin^2\theta}{1 + 4\sin^2\theta} = 0$$

$$\sin^2\theta = \frac{3}{4}$$

$$\sin\theta = \pm\frac{\sqrt{3}}{2} \;\Rightarrow\; \theta = -\; -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$$

$$\because \theta \in \left(-\frac{\pi}{2}, \pi\right)$$

then sum of the elements in A is

$$-\frac{\pi}{3} + \frac{\pi}{3} + \frac{2\pi}{3} = \frac{2\pi}{3}$$

2. Ans. (1)

$z_0 = \omega$ or ω^2 (where ω is a non-real cube root of unity)

$z = 3 + 6i(\omega)^{81} - 3i(\omega)^{93}$

$z = 3 + 3i$

$$\Rightarrow \arg z = \frac{\pi}{4}$$

3. Ans. (Bonus)

$3|z_1| = 4|z_2|$

$$\Rightarrow \frac{|z_1|}{|z_2|} = \frac{4}{3}$$

$$\Rightarrow \frac{|3z_1|}{|2z_2|} = 2$$

Let $\dfrac{3z_1}{2z_2} = a = 2\cos\theta + 2i\sin\theta$

$$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1} = a + \frac{1}{a}$$

$$= \frac{5}{2}\cos\theta + \frac{3}{2}i\sin\theta$$

Now all options are incorrect

Remark :

There is a misprint in the problem actual problem should be :

"Let z_1 and z_2 be any non-zero complex number such that $3|z_1| = \mathbf{2}|z_2|$.

If $z = \dfrac{3z_1}{2z_2} + \dfrac{2z_2}{3z_1}$, then"

Given

$3|z_1| = 2|z_2|$

Now $\left|\dfrac{3z_1}{2z_2}\right| = 1$

Let $\dfrac{3z_1}{2z_2} = a = \cos\theta + i\sin\theta$

$$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$$

$$= a + \frac{1}{a} = 2\cos\theta$$

$\therefore \operatorname{Im}(z) = 0$

Now option (4) is correct.

4. Ans. (4)

$$z = \left(\frac{\sqrt{3}+i}{2}\right)^5 + \left(\frac{\sqrt{3}-i}{2}\right)^5$$

$$z = \left(e^{i\pi/6}\right)^5 + \left(e^{-i\pi/6}\right)^5$$

$$= e^{i5\pi/6} + e^{-i5\pi/6}$$

$$= \cos\frac{5\pi}{6} + i\frac{\sin 5\pi}{6} + \cos\left(\frac{-5\pi}{6}\right) + i\sin\left(\frac{-5\pi}{6}\right)$$

$$= 2\cos\frac{5\pi}{6} < 0$$

I(z) = 0 and Re(z) < 0

Option (4)

5. Ans. (4)

$$\left(-2-\frac{i}{3}\right)^3 = -\frac{(6+i)^3}{27}$$

$$= \frac{-198-107i}{27} = \frac{x+iy}{27}$$

Hence, y – x = 198 – 107 = 91

6. Ans. (4)

$|z| + z = 3 + i$

$z = 3 - |z| + i$

Let $3 - |z| = a \Rightarrow |z| = (3 - a)$

$\Rightarrow z = a + i \Rightarrow |z| = \sqrt{a^2+1}$

$\Rightarrow 9 + a^2 - 6a = a^2 + 1 \Rightarrow a = \frac{8}{6} = \frac{4}{3}$

$\Rightarrow |z| = 3 - \frac{4}{3} = \frac{5}{3}$

7. Ans. (2)

$$\frac{z-\alpha}{z+\alpha} + \frac{\bar{z}-\alpha}{\bar{z}+\alpha} = 0$$

$$z\bar{z} + z\alpha - \alpha\bar{z} - \alpha^2 + z\bar{z} - z\alpha + \bar{z}\alpha - \alpha^2 = 0$$

$|z|^2 = \alpha^2$, a = ±2

8. Ans. (1)

$|z_1| = 9$, $|z_2 - (3+4i)| = 4$

C_1 (0, 0) radius $r_1 = 9$

C_2(3, 4), radius $r_2 = 4$

$C_1C_2 = |r_1 - r_2| = 5$

∴ Circle touches internally

∴ $|z_1 - z_2|_{min} = 0$

9. Official Ans. by NTA (1)

Sol. $z = \frac{\sqrt{3}}{2} + \frac{i}{2} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

$\Rightarrow z^5 = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} = \frac{-\sqrt{3}+i}{2}$

and $z^8 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = -\left(\frac{1+i\sqrt{3}}{2}\right)$

$\Rightarrow \left(1+iz+z^5+iz^8\right)^9 = \left(1+\frac{i\sqrt{3}}{2}-\frac{1}{2}-\frac{\sqrt{3}}{2}+\frac{i}{2}-\frac{i}{2}+\frac{\sqrt{3}}{2}\right)^9$

$= \left(\frac{1+i\sqrt{3}}{2}\right)^9 = \cos 3\pi + i\sin 3\pi = -1$

10. Official Ans. by NTA (1)

Sol. Let $\frac{\alpha+i}{\alpha-i} = z$

$\Rightarrow \frac{|\alpha+i|}{|\alpha-i|} = |z|$

$\Rightarrow 1 = |z|$

⇒ circle of radius 1

11. Official Ans. by NTA (1)

Sol. Roots of the equation $x^2 + x + 1 = 0$ are

$\alpha = \omega$ and $\beta = \omega^2$

where ω, ω^2 are complex cube roots of unity

$$\therefore \Delta = \begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \Delta = y\begin{vmatrix} 1 & 1 & 1 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

Expanding along R_1, we get

$\Delta = y.y^2 \Rightarrow D = y^3$

12. Official Ans. by NTA (3)

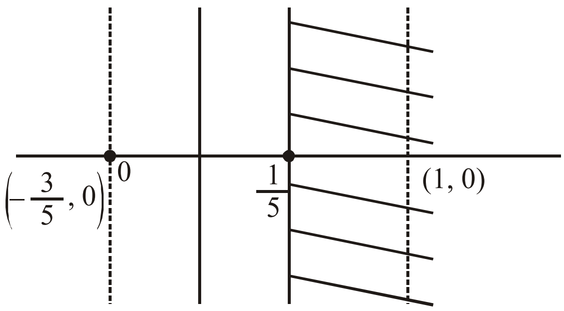
Sol. $|z| < 1$

$5\omega(1 - z) = 5 + 3z$

$5\omega - 5\omega z = 5 + 3z$

$$z = \frac{5\omega-5}{3+5\omega}$$

$$|z| = 5\left|\frac{\omega-1}{3+5\omega}\right| < 1$$



$5|\omega - 1| < |3 + 5\omega|$

$5|\omega - 1| < 5\left|\omega + \frac{3}{5}\right|$

$|\omega - 1| < 5\left|\omega - \left(-\frac{3}{5}\right)\right|$

13. Official Ans. by NTA (3)

Sol. Given a > 0

$z = \frac{(1+i)^2}{a-i} = \frac{2i(a+i)}{a^2+1}$

Also $|z| = \sqrt{\frac{2}{5}} \Rightarrow \frac{2}{\sqrt{a^2+1}} = \sqrt{\frac{2}{5}} \Rightarrow a = 3$

So $\bar{z} = \frac{-2i(3-i)}{10} = \frac{-1-3i}{5}$

14. Official Ans. by NTA (2)

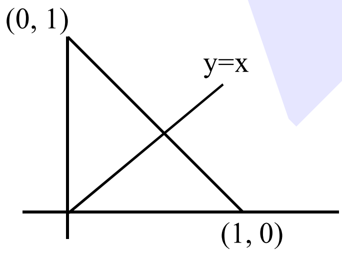
Sol. $|z|.\ |w| = 1 \quad z = re^{i(\theta + \pi/2)}$ and $w = \frac{1}{r}e^{i\theta}$

$\bar{z}.w = e^{-i(\theta+\pi/2)}.e^{i\theta} = e^{-i(\pi/2)} = -i$

$z.\bar{w} = e^{i(\theta+\pi/2)}.e^{-i\theta} = e^{i(\pi/2)} = i$

15. Official Ans. by NTA (4)

Sol.



$|z - i| = |z - 1|$

$y = x$

16. Official Ans. by NTA (3)

Sol. Put $z = x + 10i$

$\therefore 2(x + 10i) - n = (2i - 1) . [2(x+10i) + n]$

compare real and imginary coefficients

$x = -10, n = 40$

PROBABILITY

1. Ans. (2)

Two cards are drawn successively with replacement

4 Aces 48 Non Aces

$P(x=1) = \frac{{}^4C_1}{{}^{52}C_1} \times \frac{48C_1}{52C_1} + \frac{48C_1}{52C_1} \times \frac{4C_1}{52C_1} = \frac{24}{169}$

$P(x=2) = \frac{{}^4C_1}{{}^{52}C_1} \times \frac{{}^4C_1}{{}^{52}C_1} = \frac{1}{169}$

$P(x = 1) + P(x = 2) = \frac{25}{169}$

2. Ans. (2)

E_1 : Event of drawing a Red ball and placing a green ball in the bag

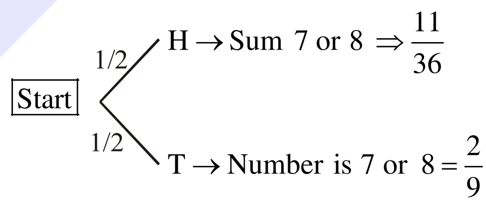
E_2 : Event of drawing a green ball and placing a red ball in the bag

E : Event of drawing a red ball in second draw

$P(E) = P(E_1) \times P\left(\frac{E}{E_1}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right)$

$= \frac{5}{7} \times \frac{4}{7} + \frac{2}{7} \times \frac{6}{7} = \frac{32}{49}$

3. Ans. (3)



$P(A) = \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{9} = \frac{19}{72}$

4. Ans. (2)

$1 - {}^nC_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^n > \frac{5}{6}$

$\frac{1}{6} > \left(\frac{2}{3}\right)^n \Rightarrow 0.1666 > \left(\frac{2}{3}\right)^n$

$n_{min} = 5 \Rightarrow$ Option (2)

5. Ans. (1)

Since sum of two numbers is even so either both are odd or both are even. Hence number of elements in reduced samples space

$= {}^5C_2 + {}^6C_2$

so required probability $= \frac{{}^5C_2}{{}^5C_2 + {}^6C_2}$

6. Ans. (2)

7,

1,6

$$P = \frac{5}{2^{20}}$$

2,5

3,4

1,2,4

7. Ans. (3)

p (probability of getting white ball) = $\frac{30}{40}$

$q = \frac{1}{4}$ and n = 16

mean = np = $16.\frac{3}{4} = 12$

and standard diviation

$$= \sqrt{npq} = \sqrt{16.\frac{3}{4}.\frac{1}{4}} = \sqrt{3}$$

8. Ans. (2)

_ _ _ $\underline{4}$ $\underline{4}$

$$\frac{1}{6^2}\left(\frac{5^3}{6^3} + \frac{2C_1.5^2}{6^3}\right) = \frac{175}{6^5}$$

9. Ans. (4)

No. of ways = $10C_3 = 120$

10. Ans. (3)

Expected Gain/ Loss

= w × 100 + Lw (–50 + 100) + L^2w (–50 –50 + 100) + L^3 (–150)

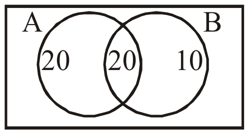
$$= \frac{1}{3} \times 100 + \frac{2}{3}.\frac{1}{3}(50) + \left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)(0)$$

$$+ \left(\frac{2}{3}\right)^3(-150) = 0$$

here w denotes probability that outcome 5 or 6 ($w = \frac{2}{6} = \frac{1}{3}$)

here L denotes probability that outcome 1,2,3,4 ($L = \frac{4}{6} = \frac{2}{3}$)

11. Ans. (2)



A → opted NCC

B → opted NSS

$\therefore$ P (neither A nor B) = $\frac{10}{60} = \frac{1}{6}$

12. Official Ans. by NTA (4)

Sol. $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$

$(\text{as } A \subset B \Rightarrow P(A \cap B) = P(A))$

$\Rightarrow \quad P(A|B) \geq P(A)$

13. Official Ans. by NTA (4)

Sol. Probability of observing at least one head out of n tosses

$$= 1 - \left(\frac{1}{2}\right)^n \geq 0.9$$

$$\Rightarrow \left(\frac{1}{2}\right)^n \leq 0.1$$

$\Rightarrow n \geq 4$

$\Rightarrow$ minimum number of tosses = 4

14. Official Ans. by NTA (3)

Sol. Let persons be A,B,C,D

P(Hit) = 1 – P(none of them hits)

$$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D})$$

$$= 1 - P(\bar{A}).P(\bar{B}).P(\bar{C}).P(\bar{D})$$

$$= 1 - \frac{1}{2}.\frac{2}{3}.\frac{3}{4}.\frac{7}{8}$$

$$= \frac{25}{32}$$

15. Official Ans. by NTA (1)

Sol. P(B) = P(G) = 1/2

Required Proballity =

$$\frac{\text{all 4 girls}}{(\text{all 4 girls}) + (\text{exactly 3 girls} + 1\text{boy}) + (\text{exactly 2 girls} + 2\text{boys})}$$

$$= \frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^4 + {}^4C_3\left(\frac{1}{2}\right)^4 + {}^4C_2\left(\frac{1}{2}\right)^4} = \frac{1}{11}$$

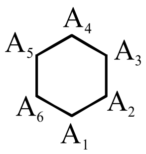
16. Official Ans. by NTA (3)

Sol. $1-\left(\frac{1}{2}\right)^n > \frac{99}{100}$

$\Rightarrow \left(\frac{1}{2}\right)^n < \frac{1}{100} \Rightarrow n = 7.$

17. Official Ans. by NTA (2)

Sol.



Only two equilateral tringles are possible $A_1 A_3 A_5$ and $A_2A_5A_6$

$\frac{2}{6_{C_3}} = \frac{2}{20} = \frac{1}{10}$

18. Official Ans. by NTA (4)

Sol. np = 8

npq = 4

$q = \frac{1}{2} \Rightarrow p = \frac{1}{2}$

n = 16

$p\,(x = r) = {}^{16}C_r\left(\frac{1}{2}\right)^{16}$

$p\,(x \le 2) = \frac{{}^{16}C_0 + {}^{16}C_1 + {}^{16}C_2}{2^{16}}$

$= \frac{137}{2^{16}}$

19. Official Ans. by NTA (2)

Sol. Let X be random varibale which denotes number of problems that candidate is unbale to solve

$\because\ p = \frac{1}{5}$ and $X < 2$

$\Rightarrow P(X<2) = P(X=0) + P(X=1)$

$= \left(\frac{4}{5}\right)^{50} + {}^{50}C_1 . \left(\frac{1}{5}\right) . \left(\frac{4}{5}\right)^{49}$

20. Official Ans. by NTA (2)

Sol. win Rs.15 → number of cases = 6

win Rs.12 → number of cases = 4

loss Rs.6 → number of cases = 26

p(expected gain/loss) $= 15 \times \frac{6}{36} + 12 \times \frac{4}{36} - 6 \times \frac{26}{36} = -\frac{1}{2}$

STATISTICS

1. Ans. (2)

Given $\bar{x} = \frac{\Sigma x_i}{5} = 150$

$\Rightarrow \sum_{i=1}^{5} x_i = 750$ .....(i)

$\frac{\Sigma x_i^2}{5} - (\bar{x})^2 = 18$

$\frac{\Sigma x_i^2}{5} - (150)^2 = 18$

$\Sigma x_i^2 = 112590$ ....(ii)

Given height of new student

$x_6 = 156$

Now, $\bar{x}_{new} = \frac{\sum_{i=1}^{6} x_i}{6} = \frac{750+156}{6} = 151$

Also, New variance $= \frac{\sum_{i=1}^{6} x_i^2}{6} - (\bar{x}_{new})^2$

$= \frac{112590 + (156)^2}{6} - (151)^2$

$= 22821 - 22801 = 20$

2. Ans. (2)

$\sum (x_i + 1)^2 = 9n$ ...(1)

$\sum (x_i - 1)^2 = 5n$ ...(2)

(1) + (2) $\Rightarrow \sum (x_1^2 + 1) = 7n$

$\Rightarrow \frac{\sum x_i^2}{n} = 6$

(1) - (2) $\Rightarrow 4\Sigma x_i = 4n$

$\Rightarrow \Sigma x_i = n$

$\Rightarrow \frac{\Sigma x_i}{n} = 1$

$\Rightarrow$ variance = 6 – 1 = 5

$\Rightarrow$ Standard diviation $= \sqrt{5}$

3. Ans. (1)

Let two observations are x_1 & x_2

mean $= \frac{\sum x_i}{5} = 5 \Rightarrow 1 + 3 + 8 + x_1 + x_2 = 25$

$\Rightarrow x_1 + x_2 = 13$ ....(1)

variance $(\sigma^2) = \frac{\sum x_i^2}{5} - 25 = 9.20$

$\Rightarrow \sum x_i^2 = 171$

$\Rightarrow x_1^2 + x_2^2 = 97$ .....(2)

by (1) & (2)

$(x_1 + x_2)^2 - 2x_1x_2 = 97$

or $x_1x_2 = 36$

$\therefore x_1 : x_2 = 4 : 9$

4. Ans. (2)

$\bar{x} = 10 \Rightarrow \sum_{i=1}^{5} x_i = 50$

$$\text{S.D.} = \sqrt{\frac{\sum_{i=1}^{5} x_i^2}{5} - (\bar{x})^2} = 8$$

$\Rightarrow \sum_{i=1}^{5} (x_i)^2 = 109$

$$\text{variance} = \frac{\sum_{i=1}^{5} (x_i)^2 + (-50)^2}{6} - \left(\sum_{i=1}^{5} \frac{x_i - 50}{6}\right)$$

$= 507.5$

Option (2)

5. Ans. (4)

Variance is independent of origin. So we shift the given data by $\frac{1}{2}$.

so, $\frac{10d^2 + 10 \times 0^2 + 10d^2}{30} - (0)^2 = \frac{4}{3}$

$\Rightarrow d^2 = 2 \Rightarrow |d| = \sqrt{2}$

6. Ans. (4)

$\sum_{i=1}^{50} (x_i - 30) = 50$

$\Sigma x_i = 50 \times 30 = 50$

$\Sigma x_i = 50 + 50 + 30$

Mean $= \bar{x} = \frac{\Sigma x_i}{n} = \frac{50 \times 30 + 50}{50} = 30 + 1 = 31$

7. Ans. (3)

mean $\bar{x} = 4, \sigma^2 = 5.2, n = 5,. x_1 = 3\ x_2 = 4 = x_3$

$\sum x_i = 20$

$x_4 + x_5 = 9$ ....(i)

$\frac{\sum x_i^2}{x} - (\bar{x})^2 = \sigma^2 \Rightarrow \sum x_i^2 = 106$

$x_4^2 + x_5^2 = 65$ ....(ii)

Using (i) and (ii) $(x_4 - x_5)^2 = 49$

$|x_4 - x_5| = 7$

8. Official Ans. by NTA (3)

Sol. Let 7 observations be $x_1, x_2, x_3, x_4, x_5, x_6, x_7$

$\bar{x} = 8 \Rightarrow \sum_{i=1}^{7} x_i = 56$ ......(1)

Also $\sigma^2 = 16$

$\Rightarrow 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - (\bar{x})^2$

$\Rightarrow 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - 64$

$\Rightarrow \left(\sum_{i=1}^{7} x_i^2\right) = 560$ .....(2)

Now, $x_1 = 2, x_2 = 4, x_3 = 10, x_4 = 12, x_5 = 14$

$\Rightarrow x_6 + x_7 = 14$ (from (1))

& $x_6^2 + x_7^2 = 100$ (from (2))

$\therefore x_6^2 + x_7^2 = (x_6 + x_7)^2 - 2x_6.x_7 \Rightarrow x_6.x_7 = 48$

9. Official Ans. by NTA (1)

Sol. Let x be the 6^{th} observation

$\Rightarrow 45 + 54 + 41 + 57 + 43 + x = 48 \times 6 = 288$

$\Rightarrow x = 48$

$\text{variance} = \left(\frac{\Sigma x_i^2}{6} - (\bar{x})^2\right)$

$\Rightarrow$ variance $= \frac{14024}{6} - (48)^2$

$= \frac{100}{3}$

$\Rightarrow$ standard deviation $= \frac{10}{\sqrt{3}}$

10. Official Ans. by NTA (2)

Sol. $S.D = \sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}}$

$\bar{x} = \dfrac{\Sigma x}{4} = \dfrac{-1+0+1+k}{4} = \dfrac{k}{4}$

Now $\sqrt{5} = \sqrt{\dfrac{\left(-1-\frac{k}{4}\right)^2 + \left(0-\frac{k}{4}\right)^2 + \left(1-\frac{k}{4}\right)^2 + \left(k-\frac{k}{4}\right)^2}{4}}$

$\Rightarrow 5 \times 4 = 2\left(1+\dfrac{k}{16}\right)^2 + \dfrac{5k^2}{8}$

$\Rightarrow 18 = \dfrac{3k^2}{4}$

$\Rightarrow k^2 = 24$

$\Rightarrow k = 2\sqrt{6}$

11. Official Ans. by NTA (1)

Sol. $\dfrac{34+x}{2} = 35$

$x = 36$

$42 = \dfrac{10+22+26+29+34+36+42+67+70+y}{10}$

$420 - 336 = y \Rightarrow y = 84$

$\dfrac{y}{x} = \dfrac{84}{36} = \dfrac{7}{3}$

12. Official Ans. by NTA (1)

Sol. $\sum f_i = 20 = 2x^2 + 2x - 4$

$\Rightarrow x^2 + 2x - 24 = 0$

$x = 3, -4$ (rejected)

$\bar{x} = \dfrac{\sum x_i f_i}{\sum f_i} = 2.8$

13. Official Ans. by NTA (4)

Sol. Mean (μ) = $\dfrac{\sum x_i}{50} = 16$

standard deviation (σ) = $\sqrt{\dfrac{\sum x_i^2}{50} - (\mu)^2} = 16$

$\Rightarrow (256) \times 2 = \dfrac{\sum x_i^2}{50}$

$\Rightarrow$ New mean

$= \dfrac{\sum (x_i - 4)^2}{50} = \dfrac{\sum x_i^2 + 16 \times 50 - 8\sum x_i}{50}$

$= (256) \times 2 + 16 - 8 \times 16 = 400$

14. Official Ans. by NTA (2)

Sol. $x_1 + \ldots + x_4 = 44$

$x_5 + \ldots + x_{10} = 96$

$\bar{x} = 14, \Sigma x_i = 140$

Variance $= \dfrac{\Sigma x_i^2}{n} - \bar{x}^2 = 4$

Standard deviation = 2

REASONING

1. Ans. (1)

$(p \oplus q) \wedge (\sim p \odot q) \equiv p \wedge q$ (given)

p	q	$\sim p$	$p\wedge q$	$p\vee q$	$\sim p\vee q$	$\sim p\wedge q$	$(p\wedge q)\wedge(\sim p\vee q)$
T	T	F	T	T	T	F	T
T	F	F	F	T	F	F	F
F	T	T	F	T	T	T	F
F	F	T	F	F	T	F	F

from truth table $(\oplus, \odot) = (\wedge, \vee)$

2. Ans. (1)

$s[\sim(\sim p \vee q) \wedge (p \wedge r)] \cap (\sim q \wedge r)$

$\equiv [(p \wedge \sim q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$

$\equiv [p \wedge (\sim q \vee r)] \wedge (\sim q \wedge r)$

$\equiv p \wedge (\sim q \wedge r)$

$\equiv (p \wedge r) \sim q$

3. Ans. (4)

It is obvious

$\therefore$ Option (4)

4. Ans. (4)

Given q is F and $(p \wedge q) \leftrightarrow r$ is T

$\Rightarrow p \wedge q$ is F which implies that r is F

$\Rightarrow$ q is F and r is F

$\Rightarrow (p \wedge r)$ is always F

$\Rightarrow (p \wedge r) \rightarrow (p \vee r)$ is tautology.

5. Ans. (1)

Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

6. **Ans. (3)**

7. **Ans. (1)**

p	q	~p	~p→q	~(~p→q)	(~p ^ ~q)
T	T	F	T	F	F
F	T	T	T	F	F
T	F	F	T	F	F
F	F	T	F	T	T

8. **Official Ans. by NTA (2)**

Sol. The contrapositive of statement

$p \to q$ is $\sim q \to \sim p$

Here, p : you are born in India.

q : you are citizen of India.

So, contrapositive of above statement is

" If you are not a citizen of India, then you are not born in India".

9. **Official Ans. by NTA (4)**

Sol. Tautology

(1)

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Tautology

(2)

p	q	$p \wedge q$	$\sim p \vee q$	$(p \wedge q) \to (\sim p) \vee q$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

Tautology

(3)

p	q	$p \vee q$	$p \to p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Tautology

(4)

p	q	$p \vee q$	$\sim p$	$p \vee \sim q$	$p \vee q \to p \wedge (\sim q)$
T	T	T	F	F	F
T	F	T	F	T	T
F	T	T	T	F	T
F	F	F	T	F	T

10. **Official Ans. by NTA (4)**

Sol. $\sim(p \vee (\sim p \wedge q))$

$= \sim p \wedge \sim(\sim p \wedge q)$

$= \sim p \wedge (p \vee \sim q)$

$= (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

$= c \vee (\sim p \wedge \sim q)$

$= (\sim p \wedge \sim q)$

11. **Official Ans. by NTA (2)**

Sol. $P \Rightarrow (q \vee r)$: F

P : T $q \vee r$: F

P : T : q : F : r : F

12. **Official Ans. by NTA (4)**

Sol. (1) $(p \vee q) \wedge (\sim p \vee \sim q) \equiv (p \vee q) \wedge \sim (p \wedge q)$

$\to$ Not tautology (Take both p and q as T)

(2) $(p \wedge q) \vee (p \wedge \sim q) \equiv p \wedge (q \vee \sim q) \equiv p \wedge t \equiv p$

(3) $(p \vee q) \wedge (p \vee \sim q) \equiv p \vee (q \wedge \sim q) \equiv p \vee c \equiv p$

(4) $(p \vee q) \vee (p \vee \sim q) \equiv p \vee (q \vee \sim q) \equiv p \vee t \equiv t$

13. **Official Ans. by NTA (2)**

Sol. $\sim(\sim s \vee (\sim r \wedge s))$

$s \wedge (r \vee \sim s)$

$(s \wedge r) \vee (s \wedge \sim s)$

$(s \wedge r) \vee (c)$

$(s \wedge r)$

14. **Official Ans. by NTA (3)**

Sol. $P \to (\sim q \vee r)$

$\sim p \vee (\sim q \vee r)$

$$\left.\begin{array}{l} \sim p \to F \\ \sim q \to F \\ r \to F \end{array}\right\} \Rightarrow \left.\begin{array}{l} p \to T \\ q \to T \\ r \to F \end{array}\right\}$$

15. **Official Ans. by NTA (4)**

Sol. $\sim(p \to (\sim q)) = \sim(\sim p \vee \sim q)$

$= p \wedge q$

MATHEMATICAL INDUCTION

1. **Ans. (4)**

P(n) : $n^2 - n + 41$ is prime

P(5) = 61 which is prime

P(3) = 47 which is also prime

Important Notes