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Chapter 03 Contents

JEE (MAIN) TOPICWISE TEST PAPERS JANUARY & APRIL 2019

MATHEMATICS

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MATHEMATICS

JANUARY & APRIL 2019 ATTEMPT (MATHEMATICS)

COMPOUND ANGLE

1. For any $\theta \in \left(\frac{\pi}{4},\frac{\pi}{2}\right)$, the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals :

(1) $13 - 4\cos^6\theta$

(2) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

(3) $13 - 4\cos^2\theta + 6\cos^4\theta$

(4) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

2. The value of $\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\cdot.....\cdot\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$ is :

(1) $\frac{1}{256}$ (2) $\frac{1}{2}$

(3) $\frac{1}{512}$ (4) $\frac{1}{1024}$

3. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for k = 1, 2, 3, ... Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to :-

(1) $\frac{5}{12}$ (2) $\frac{-1}{12}$ (3) $\frac{1}{4}$ (4) $\frac{1}{12}$

4. The maximum value of $3\cos\theta+5\sin\left(\theta-\frac{\pi}{6}\right)$ for any real value of θ is :

(1) $\sqrt{19}$ (2) $\frac{\sqrt{79}}{2}$ (3) $\sqrt{31}$ (4) $\sqrt{34}$

5. If $\cos(\alpha+\beta)=\frac{3}{5}, \sin(\alpha-\beta)=\frac{5}{13}$ and $0<\alpha, \beta<\frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to :

(1) $\frac{21}{16}$ (2) $\frac{63}{52}$

(3) $\frac{33}{52}$ (4) $\frac{63}{16}$

6. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is

(1) $\frac{3}{2}(1+\cos 20°)$

(2) $\frac{3}{4}$

(3) $\frac{3}{4}+\cos 20°$

(4) $\frac{3}{2}$

7. The value of sin 10º sin30º sin50º sin70º is :-

(1) $\frac{1}{36}$ (2) $\frac{1}{32}$

(3) $\frac{1}{18}$ (4) $\frac{1}{16}$

QUADRATIC EQUATION

1. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to :

(1) 512 (2) –512 (3) –256 (4) 256

2. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1,5], then m lies in the interval:

(1) (4,5) (2) (3,4)

(3) (5,6) (4) (–5,–4)

3. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2-11x+\alpha = 0$ are rational numbers is :

(1) 2 (2) 5 (3) 3 (4) 4

4. Consider the quadratic equation $(c-5)x^2-2cx + (c-4) = 0, c\neq5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0,2) and its other root lies in the interval (2,3). Then the number of elements in S is :

(1) 11 (2) 18 (3) 10 (4) 12

5. The values of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda) x + 2 = \lambda$ has the least value is :

(1) 2 (2) $\frac{4}{9}$

(3) $\frac{15}{8}$ (4) 1

6. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is

(1) –81 (2) 100 (3) –300 (4) 144

7. Let α and β be the roots of the quadratic equation

$x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$

$(0 < \theta < 45^o)$, and $\alpha < \beta$. Then $\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to :-

(1) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

(2) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$

(3) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$

(4) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

8. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2+m(m-4)x+2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is :

(1) $2-\sqrt{3}$ (2) $4-3\sqrt{2}$

(3) $-2+\sqrt{2}$ (4) $4-2\sqrt{3}$

9. The number of integral values of m for which the quadratic expression. $(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$, $x \in R$, is always positive, is :

(1) 8 (2) 7 (3) 6 (4) 3

10. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is :

(1) 2 (2) 3

(3) 4 (4) 5

11. The sum of the solutions of the equation $\left|\sqrt{x}-2\right| + \sqrt{x}\left(\sqrt{x}-4\right) + 2 = 0,\ (x > 0)$ is equal to :

(1) 4 (2) 9

(3) 10 (4) 12

12. The number of integral values of m for which the equation

$(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$

has no real root is :

(1) infinitely many (2) 2

(3) 3 (4) 1

13. Let $p, q \in R$. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then :

(1) $q^2 + 4p + 14 = 0$ (2) $p^2 - 4q - 12 = 0$

(3) $q^2 - 4p - 16 = 0$ (4) $p^2 - 4q + 12 = 0$

14. If m is chosen in the quadratic equation $(m^2 + 1) x^2 - 3x + (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is :-

(1) $8\sqrt{3}$ (2) $4\sqrt{3}$

(3) $10\sqrt{5}$ (4) $8\sqrt{5}$

15. If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0$, $\theta \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\alpha^{12}+\beta^{12}}{\left(\alpha^{-12}+\beta^{-12}\right)(\alpha-\beta)^{24}}$ is equal to :

(1) $\frac{2^6}{(\sin\theta+8)^{12}}$ (2) $\frac{2^{12}}{(\sin\theta-8)^{6}}$

(3) $\frac{2^{12}}{(\sin\theta-4)^{12}}$ (4) $\frac{2^{12}}{(\sin\theta+8)^{12}}$

SEQUENCE & PROGRESSION

1. If a, b and c be three distinct real numbers in G. P. and a + b + c = xb, then x cannot be :
(1) 4 (2) –3 (3) –2 (4) 2

2. Let $a_1, a_2, ..., a_{30}$ be an A. P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and S – 2T = 75, then a_{10} is equal to :
(1) 57 (2) 47 (3) 42 (4) 52

3. The sum of the follwing series
$$1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2+....+5^2)}{11}+.... \text{ up to}$$
15 terms, is:
(1) 7820 (2) 7830 (3) 7520 (4) 7510

4. Let a, b and c be the 7^{th}, 11^{th} and 13^{th} terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to:
(1) $\frac{1}{2}$ (2) 4 (3) 2 (4) $\frac{7}{13}$

5. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is :
(1) $\frac{4}{9}$ (2) $\frac{2}{9}$ (3) $\frac{2}{3}$ (4) $\frac{1}{3}$

6. Let $a_1, a_2, ..., a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals :
(1) $2(5^2)$ (2) $4(5^2)$ (3) 5^4 (4) 5^3

7. If 19th term of a non-zero A.P. is zero, then its (49th term) : (29th term) is :-
(1) 3 : 1 (2) 4 : 1
(3) 2 : 1 (4) 1 : 3

8. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expression $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$ is :-
(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{m+n}{6mn}$ (4) 1

9. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now from an A.P. Then the sum of the original three terms of the given G.P. is
(1) 36 (2) 24 (3) 32 (4) 28

10. Let $S_k = \frac{1+2+3+....+k}{k}$.
If $S_1^2 + S_2^2 + ... + S_{10}^2 = \frac{5}{12}A$, then A is equal to :
(1) 303 (2) 283 (3) 156 (4) 301

11. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$;
$\alpha, \beta \in [0, \pi]$, then cos(α + β) – cos(α – β) is equal to :
(1) 0 (2) $-\sqrt{2}$ (3) –1 (4) $\sqrt{2}$

12. If the sum of the first 15 tems of the series $\left(\frac{3}{4}\right)^3+\left(1\frac{1}{2}\right)^3+\left(2\frac{1}{4}\right)^3+3^3+\left(3\frac{3}{4}\right)^3+....$ is equal to 225 k, then k is equal to :
(1) 9 (2) 27 (3) 108 (4) 54

13. The sum of all natural numbers 'n' such that 100 < n < 200 and H.C.F. (91, n) > 1 is :
(1) 3221 (2) 3121
(3) 3203 (4) 3303

14. The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to-
(1) $2-\frac{3}{2^{17}}$ (2) $2-\frac{11}{2^{19}}$
(3) $1-\frac{11}{2^{20}}$ (4) $2-\frac{21}{2^{20}}$

15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct?

(1) d,e,f are in A.P.

(2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

(3) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

(4) d,e,f are in G.P.

16. Let the sum of the first n terms of a non-constant A.P., $a_1, a_2, a_3, \ldots$ be $50n + \frac{n(n-7)}{2}A$, wherre A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to

(1) (A, 50+46A) (2) (A, 50+45A)

(3) (50, 50+46A) (4) (50, 50+45A)

17. If the sum and product of the first three term in an A.P. are 33 and 1155, respectively, then a value of its 11th term is :-

(1) –25 (2) 25

(3) –36 (4) –35

18. The sum of the series $1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + \ldots$ upto 11th term is :-

(1) 915 (2) 946

(3) 945 (4) 916

19. The sum

$$\frac{3\times1^3}{1^2} + \frac{5\times(1^3+2^3)}{1^2+2^2} + \frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2} + \ldots\ldots.$$

upto 10th term, is :

(1) 660 (2) 620

(3) 680 (4) 600

20. If $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ are in A.P. and $a_1 + a_4 + a_7 + \ldots\ldots\ldots + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to :

(1) 38 (2) 98

(3) 76 (4) 64

21. The sum $1 + \frac{1^3+2^3}{1+2} + \frac{1^3+2^3+3^3}{1+2+3} + \ldots$

$$+\frac{1^3+2^3+3^3+\ldots+15^3}{1+2+3+\ldots+15} - \frac{1}{2}(1+2+3+\ldots+15)]$$

(1) 1240 (2)1860

(3) 660 (4) 620

22. Let a, b and c be in G. P. with common ratio r, where $a \neq 0$ and $0 < r \leq \frac{1}{2}$. If 3a, 7b and 15c are the first three terms of an A. P., then the 4th term of this A. P. is :

(1) $\frac{7}{3}a$ (2) a

(3) $\frac{2}{3}a$ (4) 5a

23. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then

$\lim_{n\to\infty}\sum_{r=1}^{n}\alpha^r + \lim_{n\to\infty}\sum_{r=1}^{n}\beta^r$ is equal to :

(1) $\frac{21}{346}$ (2) $\frac{29}{358}$ (3) $\frac{1}{12}$ (4) $\frac{7}{116}$

24. Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to :

(1) –320 (2) –260 (3) –380 (4) –410

25. If $a_1, a_2, a_3, \ldots$ are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is :

(1) 200 (2) 280

(3) 120 (4) 150

26. If α,β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to :

(1) βγ (2) 0 (3) αγ (4) αβ

TRIGONOMETRY EQUATION

1. If $0 \le x < \frac{\pi}{2}$, then the number of values of x for which sin x-sin2x+sin3x = 0, is

(1) 2 (2) 1 (3) 3 (4) 4

2. The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is :

(1) $\frac{\pi}{2}$ (2) π

(3) $\frac{3\pi}{8}$ (4) $\frac{5\pi}{4}$

3. Let S = $\{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is

(1) $\frac{13\pi}{6}$ (2) π (3) 2π (4) $\frac{5\pi}{3}$

4. All the pairs (x, y) that satisfy the inequality $2\sqrt{\sin^2 x - 2\sin x + 5} \cdot \frac{1}{4^{\sin^2 y}} \le 1$ also satisfy the eauation.

(1) sin x = |siny| (2) sin x = 2 sin y

(3) 2|sinx| = 3siny (4) 2sin x = siny

5. The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x,\ x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is :

(1) 5 (2) 4 (3) 7 (4) 3

6. Let S be the set of all $\alpha \in R$ such that the equation, cos2x + αsinx = $2\alpha - 7$ has a solution. Then S is equal to :

(1) [2, 6] (2) [3,7] (3) R (4) [1,4]

SOLUTION OF TRIANGLE

1. If 5, 5r, $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to :

(1) $\frac{3}{2}$ (2) $\frac{3}{4}$

(3) $\frac{5}{4}$ (4) $\frac{7}{4}$

2. With the usual notation, in ΔABC, if $\angle A + \angle B = 120°$, $a = \sqrt{3}+1$ and $b = \sqrt{3}-1$, then the ratio $\angle A : \angle B$, is :

(1) 7 : 1 (2) 5 : 3 (3) 9 : 7 (4) 3 : 1

3. In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2 - c^2 = y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is :

(1) $\frac{y}{\sqrt{3}}$ (2) $\frac{c}{\sqrt{3}}$

(3) $\frac{c}{3}$ (4) $\frac{3}{2}y$

4. Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for a ΔABC with usual notation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triad (α, β, γ) has a value :-

(1) (3, 4, 5) (2) (19, 7, 25)

(3) (7, 19, 25) (4) (5, 12, 13)

5. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is :

(1) 5 : 9 : 13 (2) 5 : 6 : 7

(3) 4 : 5 : 6 (4) 3 : 4 : 5

6. The angles A, B and C of a triangle ABC are in A.P. and a : b = 1 : $\sqrt{3}$. If c = 4 cm, then the area (in sq. cm) of this triangle is :

(1) $4\sqrt{3}$ (2) $\frac{2}{\sqrt{3}}$

(3) $2\sqrt{3}$ (4) $\frac{4}{\sqrt{3}}$

HEIGHT & DISTANCE

1. Consider a triangular plot ABC with sides AB=7m, BC=5m and CA=6m. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is:

(1) $7\sqrt{3}$ (2) $\frac{2}{3}\sqrt{21}$

(3) $\frac{3}{2}\sqrt{21}$ (4) $2\sqrt{21}$

2. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30º and the angle of depression of reflection of the cloud in the lake from P be 60º, then the height of the cloud (in meters) from the surface of the lake is :

(1) 42 (2) 50 (3) 45 (4) 60

3. Two vertical poles of heights, 20m and 80m stand a part on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is :

(1) 12 (2) 15

(3) 16 (4) 18

4. Two poles standing on a horizontal ground are of heights 5m and 10 m respectively. The line joining their tops makes an angle of 15º with ground. Then the distance (in m) between the poles, is :-

(1) $\frac{5}{2}(2+\sqrt{3})$ (2) $5(\sqrt{3}+1)$

(3) $5(2+\sqrt{3})$ (4) $10(\sqrt{3}-1)$

5. ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}\left(3\sqrt{2}\right)$ and $\text{cosec}^{-1}\left(2\sqrt{2}\right)$ respectively, then the height of the tower (in metres) is :

(1) $10\sqrt{5}$ (2) $\frac{100}{3\sqrt{3}}$ (3) 20 (4) 25

6. A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/ sec., then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is :

(1) $25\sqrt{3}$ (2) 25 (3) $\frac{25}{\sqrt{3}}$ (4) $\frac{25}{3}$

7. The angle of elevation of the top of vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is:

(1) $15\left(3-\sqrt{3}\right)$ (2) $15\left(3+\sqrt{3}\right)$

(3) $15\left(1+\sqrt{3}\right)$ (4) $15\left(5-\sqrt{3}\right)$

DETERMINANT

1. The system of linear equations.

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2 - 1)z = a + 1$

(1) has infinitely many solutions for a = 4

(2) is inconsistent when $|a| = \sqrt{3}$

(3) is inconsistent when a = 4

(4) has a unique solution for $|a| = \sqrt{3}$

2. If the system of linear equations

$x-4y+7z = g$

$3y - 5z = h$

$-2x + 5y - 9z = k$

is consistent, then :

(1) g + h + k = 0

(2) 2g + h + k = 0

(3) g + h + 2k = 0

(4) g + 2h + k = 0

3. If the system of equations

$x+y+z = 5$

$x+2y+3z = 9$

$x+3y+\alpha z = \beta$

has infinitely many solutions, then $\beta-\alpha$ equals:

(1) 5 (2) 18 (3) 21 (4) 8

4. Let $d\in R$, and

$$A = \begin{bmatrix} -2 & 4+d & (\sin\theta)-2 \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix},$$

$\theta\in[0,2\pi]$. If the minimum value of det(A) is 8, then a value of d is :

(1) –7 (2) $2\left(\sqrt{2}+2\right)$

(3) –5 (4) $2\left(\sqrt{2}+1\right)$

5. Let $a_1,a_2,a_3, ..., a_{10}$ be in G.P. with $a_i > 0$ for $i = 1,2,...., 10$ and S be the set of pairs (r,k), r, $k\in N$ (the set of natural numbers) for which $\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$

Then the number of elements in S, is :

(1) Infinitely many (2) 4

(3) 10 (4) 2

6. The number of values of $\theta\in(0,\pi)$ for which the system of linear equations

$x + 3y + 7z = 0$

$-x + 4y + 7z = 0$

$(\sin 3\theta)x + (\cos 2\theta) y + 2z = 0$

has a non-trivial solution, is :

(1) One (2) Three (3) Four (4) Two

7. If the system of linear equations

$2x + 2y + 3z = a$

$3x - y + 5z = b$

$x - 3y + 2z = c$

where a, b, c are non-zero real numbers, has more then one solution, then :

(1) $b - c - a = 0$ (2) $a + b + c = 0$

(3) $b + c - a = 0$ (4) $b - c + a = 0$

8. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)$ $(x + a + b + c)^2$, $x \neq 0$ and $a + b + c \neq 0$, then x is equal to :-

(1) $-(a + b + c)$ (2) $2(a + b + c)$

(3) abc (4) $-2(a + b + c)$

9. An ordered pair(α,β) for which the system of linear equations

$(1+\alpha)x + \beta y+z = 2$

$\alpha x+(1+\beta)y+z = 3$

$\alpha x+\beta y+2z = 2$ has a unique solution is

(1) (1,–3) (2) (–3,1)

(3) (2, 4) (4) (–4, 2)

10. The set of all values of λ for which the system of linear equations.

$x - 2y - 2z = \lambda x$

$x + 2y + z = \lambda y$

$-x - y = \lambda z$

has a non-trivial solution.

(1) contains more than two elements

(2) is a singleton

(3) is an empty set

(4) contains exactly two elements

11. The greatest value of $c \in R$ for which the system of linear equations

$x - cy - cz = 0$

$cx - y + cz = 0$

$cx + cy - z = 0$

has a non-trivial solution, is :

(1) $\frac{1}{2}$ (2) –1

(3) 0 (4) 2

12. If the system of linear equations

$x - 2y + kz = 1$

$2x + y + z = 2$

$3x - y - kz = 3$

has a solution (x,y,z), $z \neq 0$, then (x,y) lies on the straight line whose equation is :

(1) $3x - 4y - 1 = 0$ (2) $3x - 4y - 4 = 0$

(3) $4x - 3y - 4 = 0$ (4) $4x - 3y - 1 = 0$

13. If the system of equations 2x + 3y – z = 0, x + ky – 2z = 0 and 2x – y + z = 0 has a non-trival solution (x, y, z), then $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+k$ is equal to:-

(1) $\frac{3}{4}$ (2) –4 (3) $\frac{1}{2}$ (4) $-\frac{1}{4}$

14. If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and

$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$, $x \neq 0$; then for all $\theta \in \left(0, \frac{\pi}{2}\right)$:

(1) $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$
(2) $\Delta_1 + \Delta_2 = -2x^3$
(3) $\Delta_1 - \Delta_2 = -2x^3$
(4) $\Delta_1 + \Delta_2 = -2(x^3 + x - 1)$

15. Let λ be a real number for which the system of linear equations
x + y + z = 6
4x + λy – λz = λ – 2
3x + 2y – 4z = –5
has infinitely many solutions. Then λ is a root of the quadratic equation.

(1) $\lambda^2 - 3\lambda - 4 = 0$ (2) $\lambda^2 - \lambda - 6 = 0$
(3) $\lambda^2 + 3\lambda - 4 = 0$ (4) $\lambda^2 + \lambda - 6 = 0$

16. The sum of the real roots of the equatuion

$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$, is equal to :

(1) 6 (2) 1 (3) 0 (4) – 4

17. A value of θ ∈ (0, π/3), for which

$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$, is :

(1) $\frac{7\pi}{24}$ (2) $\frac{\pi}{18}$ (3) $\frac{\pi}{9}$ (4) $\frac{7\pi}{36}$

18. If [x] denotes the greatest integer ≤ x, then the system of linear equations $[\sin\theta]x + [-\cos\theta]y = 0$
$[\cot\theta]x + y = 0$

(1) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(2) have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

(3) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

(4) has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

STRAIGHT LINE

1. Consider the set of all lines px + qy + r = 0 such that 3p + 2q + 4r = 0. Which one of the following statements is true ?

(1) The lines are all parallel.
(2) Each line passes through the origin.
(3) The lines are not concurrent
(4) The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$

2. Let the equations of two sides of a triangle be 3x – 2y + 6 = 0 and 4x + 5y – 20 = 0. If the orthocentre of this triangle is at (1,1), then the equation of its third side is :

(1) 122y – 26x – 1675 = 0
(2) 26x + 61y + 1675 = 0
(3) 122y + 26x + 1675 = 0
(4) 26x – 122y – 1675 = 0

3. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50sq. units, then the number of elements in the set S is:

(1) 9 (2) 18 (3) 32 (4) 36

4. If the line $3x + 4y - 24 = 0$ intersects the x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is

(1) (3, 4) (2) (2, 2) (3) (4, 4) (4) (4, 3)

5. A point P moves on the line $2x - 3y + 4 = 0$. IfQ(1,4) and R(3,–2) are fixed points, then the locus of the centroid of ΔPQR is a line :

(1) parallel to x-axis (2) with slope $\frac{2}{3}$

(3) with slope $\frac{3}{2}$ (4) parallel to y-axis

6. Two vertices of a triangle are (0,2) and (4,3). If its orthocentre is at the origin, then its third vertex lies in which quadrant ?

(1) Fourth (2) Second (3) Third (4) First

7. Two sides of a parallelogram are along the lines, $x + y = 3$ and $x - y + 3 = 0$. If its diagonals intersect at (2,4), then one of its vertex is :

(1) (2,6) (2) (2,1) (3) (3,5) (4) (3,6)

8. If in a parallelogram ABDC, the coordinates of A, B and C are respectively (1, 2), (3, 4) and (2, 5), then the equation of the diagonal AD is:-

(1) $5x + 3y - 11 = 0$ (2) $3x - 5y + 7 = 0$
(3) $3x + 5y - 13 = 0$ (4) $5x - 3y + 1 = 0$

9. If the straight line, $2x-3y+17 = 0$ is perpendicular to the line passing through the points (7, 17) and (15, β), then β equals :-

(1) –5 (2) $-\frac{35}{3}$ (3) $\frac{35}{3}$ (4) 5

10. If a straight line passing thourgh the point P(–3, 4) is such that its intercepted portion between the coordinate axes is bisected at P, then its equation is :

(1) $x - y + 7 = 0$ (2) $3x - 4y + 25 = 0$
(3) $4x + 3y = 0$ (4) $4x - 3y + 24 = 0$

11. If a circle of radius R passes through the origin O and intersects the coordinate axes at A and B, then the locus of the foot of perpendicular from O on AB is :

(1) $(x^2 + y^2)^2 = 4Rx^2y^2$
(2) $(x^2 + y^2)(x + y) = R^2xy$
(3) $(x^2 + y^2)^3 = 4R^2x^2y^2$
(4) $(x^2 + y^2)^2 = 4R^2x^2y^2$

12. A point on the straight line, $3x + 5y = 15$ which is equidistant from the coordinate axes will lie only in :

(1) 1st and 2nd quadrants
(2) 4th quadrant
(3) 1st, 2nd and 4th quadrant
(4) 1st quadrant

13. Suppose that the points (h,k), (1,2) and (–3,4) lie on the line L_1. If a line L_2 passing through the points (h,k) and (4,3) is perpendicular to L_1, then $\frac{k}{h}$ equals :

(1) 3 (2) $-\frac{1}{7}$ (3) $\frac{1}{3}$ (4) 0

14. Slope of a line passing through P(2, 3) and intersecting the line, $x + y = 7$ at a distance of 4 units from P, is

(1) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$ (2) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$

(3) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$ (4) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$

15. If the two lines $x + (a - 1) y = 1$ and $2x + a^2y = 1 (a \in R - \{0, 1\})$ are perpendicular, then the distance of their point of intersection from the origin is :-

(1) $\frac{2}{5}$ (2) $\frac{2}{\sqrt{5}}$ (3) $\frac{\sqrt{2}}{5}$ (4) $\sqrt{\frac{2}{5}}$

16. A rectangle is inscribed in a circle with a diameter lying along the line $3y = x + 7$. If the two adjacent vertices of the rectangle are (–8, 5) and (6, 5), then the area of the rectangle (in sq. units) is :-

(1) 72 (2) 84 (3) 98 (4) 56

17. The region represented by $|x-y| \leq 2$ and $|x+y| \leq 2$ is bounded by a :

(1) square of side length $2\sqrt{2}$ units

(2) rhombus of side length 2 units

(3) square of area 16 sq, units

(4) rhombus of area $8\sqrt{2}$ sq. units

18. Lines are drawn parallel to the line $4x-3y+2=0$, at a distance $\frac{3}{5}$ from the origin.

Then which one of the following points lies on any of these lines ?

(1) $\left(-\frac{1}{4},\frac{2}{3}\right)$ (2) $\left(\frac{1}{4},\frac{1}{3}\right)$

(3) $\left(-\frac{1}{4},-\frac{2}{3}\right)$ (4) $\left(\frac{1}{4},-\frac{1}{3}\right)$

19. The equation $y = \sin x \sin(x + 2) - \sin^2(x+1)$ represents a straight line lying in :

(1) second and third quadrants only

(2) third and fourth quadrants only

(3) first, third and fourth quadrants

(4) first, second and fourth quadrants

20. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60° with the line $x + y = 0$. Then an equation of the line L is :

(1) $\left(\sqrt{3}+1\right)x+\left(\sqrt{3}-1\right)y=8\sqrt{2}$

(2) $\left(\sqrt{3}-1\right)x+\left(\sqrt{3}+1\right)y=8\sqrt{2}$

(3) $\sqrt{3}x+y=8$

(4) $x+\sqrt{3}y=8$

21. A triangle has a vertex at (1, 2) and the mid points of the two sides through it are (–1, 1) and (2,3). Then the centroid of this triangle is :

(1) $\left(\frac{1}{3},1\right)$ (2) $\left(\frac{1}{3},2\right)$

(3) $\left(1,\frac{7}{3}\right)$ (4) $\left(\frac{1}{3},\frac{5}{3}\right)$

CIRCLE

1. Three circles of radii a, b, c(a < b < c) touch each other externally. If they have x-axis as a common tangent, then :

(1) $\frac{1}{\sqrt{a}}=\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{c}}$

(2) a, b, c are in A. P.

(3) $\sqrt{a},\sqrt{b},\sqrt{c}$ are in A. P.

(4) $\frac{1}{\sqrt{b}}=\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{c}}$

2. If the circles $x^2 + y^2 - 16x-20y + 164 = r^2$ and $(x-4)^2 + (y - 7)^2 = 36$ intersect at two distinct points, then:

(1) $0 < r < 1$ (2) $1 < r < 11$

(3) $r > 11$ (4) $r = 11$

3. If a circle C passing through the point (4,0) touches the circle $x^2 + y^2 + 4x - 6y = 12$ externally at the point (1, –1), then the radius of C is :

(1) $\sqrt{57}$ (2) 4 (3) $2\sqrt{5}$ (4) 5

4. If the area of an equilateral triangle inscribed in the circle, $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq. units then c is equal to :

(1) 20 (2) 25 (3) 13 (4) –25

5. A square is inscribed in the circle $x^2 + y^2 - 6x + 8y - 103 = 0$ with its sides parallel to the corrdinate axes. Then the distance of the vertex of this square which is nearest to the origin is :-

(1) 13 (2) $\sqrt{137}$ (3) 6 (4) $\sqrt{41}$

6. The straight line $x + 2y = 1$ meets the coordinate axes at A and B. A circle is drawn through A, B and the origin. Then the sum of perpendicular distances from A and B on the tangent to the circle at the origin is :

(1) $\frac{\sqrt{5}}{4}$ (2) $\frac{\sqrt{5}}{2}$ (3) $2\sqrt{5}$ (4) $4\sqrt{5}$

7. Two circles with equal radii are intersecting at the points (0, 1) and (0, –1). The tangent at the point (0, 1) to one of the circles passes through the centre of the other circle. Then the distance between the centres of these circles is :

(1) 1 (2) $\sqrt{2}$ (3) $2\sqrt{2}$ (4) 2

8. A circle cuts a chord of length 4a on the x-axis and passes through a point on the y-axis, distant 2b from the origin. Then the locus of the centre of this circle, is :-

(1) A hyperbola (2) A parabola
(3) A straight line (4) An ellipse

9. If a variable line, $3x+4y-\lambda=0$ is such that the two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2+y^2-18x-2y+78 = 0$ are on its opposite sides, then the set of all values of λ is the interval :-

(1) [12, 21] (2) (2, 17)
(3) (23, 31) (4) [13, 23]

10. Let C_1 and C_2 be the centres of the circles $x^2+y^2-2x-2y-2 = 0$ and $x^2+y^2-6x-6y+14 = 0$ respectively. If P and Q are the points of intersection of these circles, then the area (in sq. units) of the quadrilateral PC_1QC_2 is :

(1) 8 (2) 6 (3) 9 (4) 4

11. Let O(0, 0) and A(0, 1) be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is :

(1) $8x^2 - 9y^2 + 9y = 18$
(2) $9x^2 + 8y^2 - 8y = 16$
(3) $8x^2 + 9y^2 - 9y = 18$
(4) $9x^2 - 8y^2 + 8y = 16$

12. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2 + y^2 = 16$, by the lines, $x + y = n$, $n \in N$, where N is the set of all natural numbers, is :

(1) 320 (2) 160
(3) 105 (4) 210

13. The tangent and the normal lines at the point $(\sqrt{3},1)$ to the circle $x^2 + y^2 = 4$ and the x-axis form a triangle. The area of this triangle (in square units) is :

(1) $\frac{1}{3}$ (2) $\frac{4}{\sqrt{3}}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{2}{\sqrt{3}}$

14. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is

(1) $x^2 + y^2 - 2xy = 0$
(2) $x^2 + y^2 - 16x^2y^2 = 0$
(3) $x^2 + y^2 - 4x^2y^2 = 0$
(4) $x^2 + y^2 - 2x^2y^2 = 0$

15. The common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point :-

(1) (–4, 6) (2) (6, –2)
(3) (–6, 4) (4) (4, –2)

16. If the circles $x^2 + y^2 + 5Kx + 2y + K = 0$ and $2(x^2+y^2)+2Kx+3y-1=0$, $(K \in R)$, intersect at the points P and Q, then the line $4x + 5y - K = 0$ passes through P and Q for :

(1) exactly two values of K
(2) exactly one value of K
(3) no value of K.
(4) infinitely many values of K

17. The line x = y touches a circle at the point (1, 1). If the circle also passes through the point (1, –3), then its radius is :

(1) $3\sqrt{2}$ (2) 3 (3) $2\sqrt{2}$ (4) 2

18. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the y-axis and lie in the first quadrant, is :

(1) $y = \sqrt{1+4x}$, $x \ge 0$
(2) $x = \sqrt{1+4y}$, $y \ge 0$
(3) $x = \sqrt{1+2y}$, $y \ge 0$
(4) $y = \sqrt{1+2x}$, $x \ge 0$

19. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is :

(1) $\frac{60}{13}$ (2) $\frac{120}{13}$ (3) $\frac{13}{2}$ (4) $\frac{13}{5}$

20. A circle touching the x-axis at (3, 0) and making an intercept of length 8 on the y-axis passes through the point :

(1) (3, 10) (2) (2,3) (3) (1,5) (4) (3,5)

PERMUTATION & COMBINATION

1. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is:

(1) 200 (2) 300 (3) 500 (4) 350

2. The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repitition of digits allowed) is equal to :

(1) 250 (2) 374 (3) 372 (4) 375

3. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is :

(1) 1365 (2) 1256 (3) 1465 (4) 1356

4. Let S = {1,2,3, ..., 100}. The number of non-empty subsets A of S such that the product of elements in A is even is :-

(1) $2^{50}(2^{50}-1)$ (2) $2^{100}-1$

(3) $2^{50}-1$ (4) $2^{50}+1$

5. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is :

(1) 9 (2) 11 (3) 12 (4) 7

6. If ${}^{n}C_4$, ${}^{n}C_5$ and ${}^{n}C_6$ are in A.P., then n can be:

(1) 14 (2) 11 (3) 9 (4) 12

7. All possible numbers are formed using the digits 1, 1, 2, 2, 2, 2, 3, 4, 4 taken all at a time. The number of such numbers in which the odd digits occupy even places is :

(1) 175 (2) 162

(3) 160 (4) 180

8. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0,1,2,3,4,5 (repetition of digits is allowed) is :

(1) 288 (2) 306 (3) 360 (4) 310

9. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then :

(1) m = n = 78 (2) n = m – 8

(3) m + n = 68 (4) m = n = 68

10. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are addded to the total number of balls used in forming the equilaterial triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is :-

(1) 190 (2) 262 (3) 225 (4) 157

11. The number of 6 digit numbers that can be formed using the digits 0, 1, 2, 5, 7 and 9 which are divisible by 11 and no digit is repeated, is :

(1) 36 (2) 60 (3) 48 (4) 72

12. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all its non-adjacent pillars, then the total number beams is :

(1) 210 (2) 190

(3) 170 (4) 180

13. A group of students comprises of 5 boys and n girls. If the number of ways, in which a team of 3 students can randomly be selected from this group such that there is at least one boy and at least one girl in each team, is 1750, then n is equal to :

(1) 25 (2) 28 (3) 27 (4) 24

14. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is :
(1) 2^{20} (2) $2^{20} - 1$
(3) $2^{20} + 1$ (4) 2^{21}

BINOMIAL THEOREM

1. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to :
(1) 14 (2) 6 (3) 4 (4) 8

2. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is
(1) 12 (2) 15 (3) 10 (4) 14

3. $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$, then k equals :
(1) 200 (2) 50 (3) 100 (4) 400

4. If the third term in the binomial expansion of $\left(1+x^{\log_2 x}\right)^5$ equals 2560, then a possible value of x is:
(1) $2\sqrt{2}$ (2) $\frac{1}{8}$ (3) $4\sqrt{2}$ (4) $\frac{1}{4}$

5. The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is :
(1) $\sqrt{5}$ (2) 4 (3) $2\sqrt{2}$ (4) 3

6. If $\sum_{r=0}^{25}\left\{{}^{50}C_r \cdot {}^{50-r}C_{25-r}\right\} = K\left({}^{50}C_{25}\right)$, then K is equal to :
(1) $2^{25} - 1$ (2) $(25)^2$ (3) 2^{25} (4) 2^{24}

7. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equals 5670 is :
(1) 6 (2) 8 (3) 0 (4) 4

8. The value of r for which
${}^{20}C_r\,{}^{20}C_0 + {}^{20}C_{r-1}\,{}^{20}C_1 + {}^{20}C_{r-2}\,{}^{20}C_2 + \ldots\ {}^{20}C_0\,{}^{20}C_r$
is maximum, is
(1) 20 (2) 15 (3) 11 (4) 10

9. Let $(x + 10)^{50} + (x - 10)^{50} = a_0 + a_1x + a_2x^2 + \ldots + a_{50}x^{50}$, for all $x \in R$, then $\frac{a_2}{a_0}$ is equal to:-
(1) 12.50 (2) 12.00 (3) 12.75 (4) 12.25

10. Let $S_n = 1 + q + q^2 + \ldots\ldots + q^n$ and $T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + \ldots\ldots + \left(\frac{q+1}{2}\right)^n$,
where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2.S_1 + \ldots\ldots + {}^{101}C_{101}.S_{100} = \alpha T_{100}$, then α is equal to :-
(1) 2^{100} (2) 200 (3) 2^{99} (4) 202

11. A ratio of the 5^{th} term from the beginning to the 5^{th} term from the end in the binomial expansion of $\left(2^{1/3}+\frac{1}{2(3)^{1/3}}\right)^{10}$ is :
(1) $1 : 4(16)^{\frac{1}{3}}$ (2) $1 : 2(6)^{\frac{1}{3}}$
(3) $2(36)^{\frac{1}{3}} : 1$ (4) $4(36)^{\frac{1}{3}} : 1$

12. The total number of irrational terms in the binomial expansion of $\left(7^{1/5} - 3^{1/10}\right)^{60}$ is :
(1) 55 (2) 49 (3) 48 (4) 54

13. The sum of the series $2.{}^{20}C_0 + 5.{}^{20}C_1 + 8.{}^{20}C_2 + 11.{}^{20}C_3 + \ldots + 62.{}^{20}C_{20}$ is equal to :
(1) 2^{24} (2) 2^{25}
(3) 2^{26} (4) 2^{23}

14. The sum of the co-efficients of all even degree terms in x in the expansion of $\left(x+\sqrt{x^3-1}\right)^6 + \left(x-\sqrt{x^3-1}\right)^6, (x > 1)$ is equal to :
(1) 32 (2) 26 (3) 29 (4) 24

15. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10}x}}}+x^{\frac{1}{12}}\right)^6$ is equal to 200, and x > 1, then the value of x is :

(1) 10^3 (2) 100 (3) 10^4 (4) 10

16. If the fourth term in the binomial expansion of $\left(\frac{2}{x}+x^{\log_8 x}\right)^6$ (x > 0) is 20×8^7, then a value of x is :

(1) 8 (2) 8^2 (3) 8^{-2} (4) 8^3

17. If some three consecutive in the binomial expansion of $(x + 1)^n$ is powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficient is :-

(1) 964 (2) 625 (3) 227 (4) 232

18. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 - 3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to :

(1) (28, 315) (2) (–54, 315)
(3) (–21, 714) (4) (24, 861)

19. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2+\frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is :

(1) 35 (2) 38 (3) 23 (4) 58

20. The coefficient of x^{18} in the product $(1+x)(1-x)^{10}(1 + x + x^2)^9$ is :

(1) –84 (2) 84 (3) 126 (4) –126

21. If ${}^{20}C_1 + (2^2)\,{}^{20}C_2 + (3^2)\,{}^{20}C_3 + ... + (20^2)\,{}^{20}C_{20} = A(2^\beta)$, then the ordered pair (A, β) is equal to:

(1) (420, 18) (2) (380, 19)
(3) (380, 18) (4) (420, 19)

22. The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to :

(1) 36 (2) – 108 (3) – 72 (4) – 36

SET

1. In a class of 140 students numbered 1 to 140, all even numbered students opted mathematics course, those whose number is divisible by 3 opted Physics course and theose whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is :

(1) 102 (2) 42 (3) 1 (4) 38

2. Two newspapers A and B are published in a city. It is known that 25% of the city populations reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisement is :-

(1) 12.8 (2) 13.5 (3) 13.9 (4) 13

3. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true?

(1) If $(A - C) \subseteq B$, then $A \subseteq B$

(2) $(C \cup A) \cap (C \cup B) = C$

(3) If $(A - B) \subseteq C$, then $A \subseteq C$

(4) $B \cap C \neq \phi$

RELATION

1. Let Z be the set of integers. If $A = \left\{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\right\}$ and $B = \left\{x \in Z : -3 < 2x - 1 < 9\right\}$, then the number of subsets of the set A × B, is:

(1) 2^{18} (2) 2^{10} (3) 2^{15} (4) 2^{12}

FUNCTION

1. For $x \in R - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 o J o f_1)(x) = f_3(x)$ then J(x) is equal to :-

(1) $f_3(x)$

(2) $f_1(x)$

(3) $f_2(x)$

(4) $\frac{1}{x} f_3(x)$

2. Let A={xεR:x is not a positive integer}. Define a function f :A→R as $f(x)=\frac{2x}{x-1}$ then f is

(1) injective but not surjective

(2) not injective

(3) surjective but not injective

(4) neither injective nor surjective

3. Let N be the set of natural numbers and two functions f and g be defined as f,g : N→N such that : $f(n)=\begin{cases} \frac{n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$ and $g(n) = n-(-1)^n$. The fog is :

(1) Both one-one and onto

(2) One-one but not onto

(3) Neither one-one nor onto

(4) onto but not one-one

4. Let f : R → R be defined by $f(x) = \frac{x}{1+x^2}$, $x \in R$. Then the range of f is :

(1) $(-1, 1) - \{0\}$ (2) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(3) $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$ (4) $R - [-1, 1]$

5. Let a function f : (0, ∞) → (0, ∞) be defined by $f(x) = \left|1 - \frac{1}{x}\right|$. Then f is :-

(1) Injective only

(2) Not injective but it is surjective

(3) Both injective as well as surjective

(4) Neither injective nor surjective

6. The number of functions f from {1, 2, 3, ..., 20} onto {1, 2, 3, ..., 20} such that f(k) is a multiple of 3, whenever k is a multiple of 4, is :-

(1) $(15)! \times 6!$ (2) $5^6 \times 15$

(3) $5! \times 6!$ (4) $6^5 \times (15)!$

7. If $f(x)=\log_e\left(\frac{1-x}{1+x}\right), |x|<1$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to :

(1) $2f(x)$ (2) $2f(x^2)$

(3) $(f(x))^2$ (4) $-2f(x)$

8. Let $f(x) = a^x$ $(a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function of $f_2(x)$ is an odd function. Then $f_1(x + y) + f_1(x - y)$ equals

(1) $2f_1(x)f_1(y)$

(2) $2f_1(x)f_2(y)$

(3) $2f_1(x + y)f_2(x - y)$

(4) $2f_1(x + y)f_1(x - y)$

9. Let $\sum_{k=1}^{10} f(a+k)=16(2^{10}-1)$, where the function f satisfies $f(x + y) = f(x)f(y)$ for all natural numbers x, y and $f(1) = 2$. then the natural number 'a' is

(1) 4 (2) 3 (3) 16 (4) 2

10. If the function f : R – {1, –1} → A defined by $f(x)=\frac{x^2}{1-x^2}$, is surjective, then A is equal to

(1) R – [–1, 0)

(2) R – (–1, 0)

(3) R – {–1}

(4) [0, ∞)

11. The domain of the definition of the function $f(x)=\frac{1}{4-x^2}+\log_{10}(x^3-x)$ is :-

(1) $(1, 2) \cup (2, \infty)$

(2) $(-1, 0) \cup (1, 2) \cup (3, \infty)$

(3) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

(4) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

12. Le $f(x) = x^2$, $x \in R$. For any $A \subseteq R$, define $g(A) = \{x \in R, f(x) \in A\}$. If $S = [0, 4]$, then which one of the following statements is not true ?

(1) $f(g(S)) \neq f(S)$ (2) $f(g(S)) = S$

(3) $g(f(S)) = g(S)$ (4) $g(f(S)) \neq S$

13. The number of real roots of the equation $5 + |2^x - 1| = 2^x (2^x - 2)$ is :

(1) 2 (2) 3 (3) 4 (4) 1

14. For $x \in \left(0, \frac{3}{2}\right)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x)=\frac{1-x^2}{1+x^2}$. If $\phi(x) = ((hof)og)(x)$, then $\phi = \left(\frac{\pi}{3}\right)$ is equal to :

(1) $\tan\frac{\pi}{12}$ (2) $\tan\frac{7\pi}{12}$

(3) $\tan\frac{11\pi}{12}$ (4) $\tan\frac{5\pi}{12}$

15. For $x \in R$, let $[x]$ denote the greatest integer $\leq$ x, then the sum of the series

$$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+.....+\left[-\frac{1}{3}-\frac{99}{100}\right]$$

is

(1) –153 (2) –133

(3) –131 (4) –135

INVERSE TRIGONOMETRY FUNCTION

1. If $\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}\left(x>\frac{3}{4}\right)$ then x is equal to :

(1) $\frac{\sqrt{145}}{12}$ (2) $\frac{\sqrt{145}}{10}$

(3) $\frac{\sqrt{146}}{12}$ (4) $\frac{\sqrt{145}}{11}$

2. If $x = \sin^{-1}(\sin 10)$ and $y=\cos^{-1}(\cos 10)$, then y–x is equal to:

(1) π (2) 7π (3) 0 (4) 10

3. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is :

(1) $\frac{22}{23}$ (2) $\frac{23}{22}$

(3) $\frac{21}{19}$ (4) $\frac{19}{21}$

4. All x satisfying the inequality $(\cot^{-1} x)^2 - 7 (\cot^{-1} x) + 10 > 0$, lie in the interval:-

(1) $(-\infty, \cot 5) \cup (\cot 4, \cot 2)$

(2) $(\cot 5, \cot 4)$

(3) $(\cot 2, \infty)$

(4) $(-\infty, \cot 5) \cup (\cot 2, \infty)$

5. Considering only the principal values of inverse functions, the set $A=\left\{x\geq 0:\tan^{-1}(2x)+\tan^{-1}(3x)=\frac{\pi}{4}\right\}$

(1) is an empty set

(2) Contains more than two elements

(3) Contains two elements

(4) is a singleton

6. If $\alpha = \cos^{-1}\left(\frac{3}{5}\right), \beta = \tan^{-1}\left(\frac{1}{3}\right)$,

where $0 < \alpha, \beta < \frac{\pi}{2}$, then $\alpha - \beta$ is equal to :

(1) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (2) $\tan^{-1}\left(\frac{9}{14}\right)$

(3) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (4) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

7. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$,

where $-1 \le x \le 1, -2 \le y \le 2, x \le \frac{y}{2}$, then for all x, y, $4x^2 - 4xy\cos\alpha + y^2$ is equal to

(1) $4\sin^2\alpha - 2x^2y^2$ (2) $4\cos^2\alpha + 2x^2y^2$

(3) $4\sin^2\alpha$ (4) $2\sin^2\alpha$

8. The value of $\sin^{-1}\left(\frac{12}{13}\right) - \sin^{-1}\left(\frac{3}{5}\right)$ is equal to:

(1) $\pi - \sin^{-1}\left(\frac{63}{65}\right)$ (2) $\pi - \cos^{-1}\left(\frac{33}{65}\right)$

(3) $\frac{\pi}{2} - \sin^{-1}\left(\frac{56}{65}\right)$ (4) $\frac{\pi}{2} - \cos^{-1}\left(\frac{9}{65}\right)$

LIMIT

1. $\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4}$

(1) exists and equals $\frac{1}{4\sqrt{2}}$

(2) does not exist

(3) exists and equals $\frac{1}{2\sqrt{2}}$

(4) exists and equals $\frac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

2. For each $x \varepsilon R$, let [x] be the greatest integer less than or equal to x. Then $\lim_{x\to 0^-}\frac{x\left([x]+|x|\right)\sin[x]}{|x|}$ is equal to

(1) – sin1 (2) 0

(3) 1 (4) sin1

3. For each $t \in R$, let [t] be the greatest integer less than or equal to t. Then,

$$\lim_{x\to 1+}\frac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$$

(1) equals –1

(2) equals 1

(3) does not exist

(4) equals 0

4. Let [x] denote the greatest integer less than or equal to x. Then :-

$$\lim_{x\to 0}\frac{\tan(\pi\sin^2 x) + \left(|x| - \sin(x[x])\right)^2}{x^2}$$

(1) equals π (2) equals 0

(3) equals $\pi + 1$ (4) does not exist

5. $\lim_{x\to 0}\frac{x\cot(4x)}{\sin^2 x\cot^2(2x)}$ is equal to :-

(1) 2 (2) 0

(3) 4 (4) 1

6. $\lim_{x\to \pi/4}\frac{\cot^3 x - \tan x}{\cos\left(x+\pi/4\right)}$ is :

(1) 4 (2) $8\sqrt{2}$

(3) 8 (4) $4\sqrt{2}$

7. $\lim_{x\to 1-}\frac{\sqrt{\pi} - \sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ equal to :

(1) $\frac{1}{\sqrt{2\pi}}$ (2) $\sqrt{\frac{\pi}{2}}$

(3) $\sqrt{\frac{2}{\pi}}$ (4) $\sqrt{\pi}$

8. $\lim_{x\to 0}\frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$ equals :

(1) $2\sqrt{2}$ (2) $4\sqrt{2}$

(3) $\sqrt{2}$ (4) 4

9. Let $f : R \to R$ be a differentiable function satisfying $f'(3) + f'(2) = 0$.

Then $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to

(1) e^2 (2) e (3) e^{-1} (4) 1

10. If $f(x)=[x]-\left[\frac{x}{4}\right], x\in R$, where [x] denotes the greatest integer function, then :

(1) Both $\lim_{x\to 4-} f(x)$ and $\lim_{x\to 4+} f(x)$ exist but are not equal

(2) $\lim_{x\to 4-} f(x)$ exists but $\lim_{x\to 4+} f(x)$ does not exist

(3) $\lim_{x\to 4+} f(x)$ exists but $\lim_{x\to 4-} f(x)$ does not exist

(4) f is continuous at x = 4

11. If $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$, then k is :

(1) $\frac{3}{8}$ (2) $\frac{3}{2}$ (3) $\frac{4}{3}$ (4) $\frac{8}{3}$

12. If $\lim_{x\to 1}\frac{x^2-ax+b}{x-1}=5$, then a + b is equal to :-

(1) –7 (2) – 4 (3) 5 (4) 1

13. $\lim_{x\to 0}\frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$ is :

(1) 3 (2) 2 (3) 6 (4) 1

14. Let f(x) = 5– |x–2| and g(x) = |x + 1|, x ∈ R. If f(x) attains maximum value at α and g(x) attains minimum value at β, then

$\lim_{x\to -\alpha\beta}\frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to :

(1) 1/2 (2) –3/2 (3) 3/2 (4) –1/2

CONTINUITY

1. Let f : R → R be a function defined as :

$$f(x)=\begin{cases}5, & \text{if } x\le 1\\ a+bx, & \text{if } 1<x<3\\ b+5x, & \text{if } 3\le x<5\\ 30, & \text{if } x\ge 5\end{cases}$$

Then, f is :

(1) continuous if a = 5 and b = 5

(2) continuous if a = –5 and b = 10

(3) continuous if a = 0 and b = 5

(4) not continuous for any values of a and b

2. Let $f : [-1,3] \to R$ be defined as

$$f(x)=\begin{cases}|x|+[x], & -1\le x<1\\ x+|x|, & 1\le x<2\\ x+[x], & 2\le x\le 3\end{cases},$$

where [t] denotes the greatest integer less than or equal to t. Then, f is discontinuous at:

(1) four or more points

(2) only one point

(3) only two points

(4) only three points

3. If the function f defined on $\left(\frac{\pi}{6},\frac{\pi}{3}\right)$ by

$$f(x)=\begin{cases}\frac{\sqrt{2}\cos x-1}{\cot x-1}, & x\ne\frac{\pi}{4}\\ k, & x=\frac{\pi}{4}\end{cases}$$ is continuous,

then k is equal to

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) 2

4. If the function $f(x)=\begin{cases}a|\pi-x|+1, & x\le 5\\ b|x-\pi|+3, & x>5\end{cases}$ is continuous at x = 5, then the value of a – b is:-

(1) $\frac{2}{5-\pi}$ (2) $\frac{2}{\pi-5}$

(3) $\frac{2}{\pi+5}$ (4) $\frac{-2}{\pi+5}$

5. If $f(x) = \begin{cases} \dfrac{\sin(p+1)+\sin x}{x} & , x<0 \\ q & , x=0 \\ \dfrac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}} & , x>0 \end{cases}$

is continuous at x = 0, then the ordered pair (p,q) is equal to :

(1) $\left(\frac{5}{2},\frac{1}{2}\right)$ (2) $\left(-\frac{3}{2},-\frac{1}{2}\right)$

(3) $\left(-\frac{1}{2},\frac{3}{2}\right)$ (4) $\left(-\frac{3}{2},\frac{1}{2}\right)$

DIFFERENTIABILITY

1. Let f be a differentiable function from R to R such that $|f(x)-f(y)| \le 2|x-y|^{\frac{3}{2}}$, for all x, y ε R. If $f(0) = 1$ then $\int_0^1 f^2(x)dx$ is equal to

(1) 0 (2) $\frac{1}{2}$ (3) 2 (4) 1

2. Let $f(x)=\begin{cases} \max\{|x|,x^2\}, & |x|\le 2 \\ 8-2|x|, & 2<|x|\le 4 \end{cases}$

Let S be the set of points in the interval (–4,4) at which f is not differentiable. Then S:

(1) is an empty set

(2) equals {–2, –1, 1, 2}

(3) equals {–2, –1, 0, 1, 2}

(4) equals {–2, 2}

3. Let f : (–1,1)→R be a function defined by $f(x) = \max\left\{-|x|,-\sqrt{1-x^2}\right\}$. If K be the set of all points at which f is not differentiable, then K has exactly :

(1) Three elements (2) One element

(3) Five elements (4) Two elements

4. Let K be the set of all real values of x where the function $f(x) = \sin|x| - |x| + 2(x-\pi)\cos|x|$ is not differentiable. Then the set K is equal to :-

(1) {π} (2) {0}

(3) ϕ (an empty set) (4) {0, π}

5. Let S be the set of all points in (–π,π) at which the function, f(x) = min {sinx, cosx} is not differentiable. Then S is a subset of which of the following?

(1) $\left\{-\frac{3\pi}{4},-\frac{\pi}{4},\frac{3\pi}{4},\frac{\pi}{4}\right\}$

(2) $\left\{-\frac{3\pi}{4},-\frac{\pi}{2},\frac{\pi}{2},\frac{3\pi}{4}\right\}$

(3) $\left\{-\frac{\pi}{2},-\frac{\pi}{4},\frac{\pi}{4},\frac{\pi}{2}\right\}$

(4) $\left\{-\frac{\pi}{4},0,\frac{\pi}{4}\right\}$

6. Let $f(x) = \begin{cases} -1, & -2 \le x < 0 \\ x^2-1, & 0 \le x \le 2 \end{cases}$ and $g(x) = |f(x)| + f(|x|)$. Then, in the interval 1(–2, 2), g is :-

(1) differentiable at all points

(2) not differentiable at two points

(3) not continuous

(4) not differentiable at one point

7. Let $f(x) = 15 - |x - 10|$; $x \in R$. Then the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is :

(1) {5,10,15,20} (2) {10,15}

(3) {5,10,15} (4) {10}

8. Let $f : R \to R$ be differentiable at c ∈ R and $f(c) = 0$. If $g(x) = |f(x)|$, then at x = c, g is :

(1) differentiable if $f'(c) = 0$

(2) not differentiable

(3) differentiable if $f'(c) \ne 0$

(4) not differentiable if $f'(c) = 0$

METHOD OF DIFFERENTIATION

1. If x = 3 tan t and y = 3 sec t, then the value of $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{4}$, is:

(1) $\frac{3}{2\sqrt{2}}$ (2) $\frac{1}{3\sqrt{2}}$ (3) $\frac{1}{6}$ (4) $\frac{1}{6\sqrt{2}}$

2. Let $f : R \to R$ be a function such that $f(x) = x^3+x^2f'(1) + xf''(2)+f'''(3)$, $x \in R$. Then $f(2)$ equal :

(1) 8 (2) –2 (3) –4 (4) 30

3. If $x\log_e(\log_e x) - x^2 + y^2 = 4 (y > 0)$, then dy/dx at x = e is equal to :

(1) $\frac{e}{\sqrt{4+e^2}}$ (2) $\frac{(1+2e)}{2\sqrt{4+e^2}}$

(3) $\frac{(2e-1)}{2\sqrt{4+e^2}}$ (4) $\frac{(1+2e)}{\sqrt{4+e^2}}$

4. For x >1, if $(2x)^{2y} = 4e^{2x-2y}$, then $(1+\log_e 2x)^2 \frac{dy}{dx}$ is equal to :

(1) $\log_e 2x$ (2) $\frac{x\log_e 2x + \log_e 2}{x}$

(3) $x\log_e 2x$ (4) $\frac{x\log_e 2x - \log_e 2}{x}$

5. Let f be a differentiable function such that $f(1)=2$ and $f'(x)=f(x)$ for all $x\in R$. If $h(x)=f(f(x))$, then $h'(1)$ is equal to :

(1) 4e (2) $4e^2$ (3) 2e (4) $2e^2$

6. If $2y = \left(\cot^{-1}\left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right)^2, x \in \left(0, \frac{\pi}{2}\right)$, then $\frac{dy}{dx}$ is equal to :

(1) $2x - \frac{\pi}{3}$ (2) $\frac{\pi}{3} - x$

(3) $\frac{\pi}{6} - x$ (4) $x - \frac{\pi}{6}$

7. If $f(1) = 1$, $f'(1) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at x = 1 is :

(1) 12 (2) 33 (3) 9 (4) 15

8. Let $f(x) = \log_e(\sin x)$, $(0 < x < \pi)$ and $g(x) = \sin^{-1}(e^{-x})$, $(x \ge 0)$. If α is a positive real number such that $a = (fog)'(\alpha)$ and $b = (fog)(\alpha)$, then :

(1) $a\alpha^2 - b\alpha - a = 0$

(2) $a\alpha^2 + b\alpha - a = -2\alpha^2$

(3) $a\alpha^2 + b\alpha + a = 0$

(4) $a\alpha^2 - b\alpha - a = 1$

9. If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at x = 0 is equal to :

(1) $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$ (2) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$

(3) $\left(\frac{1}{e}, -\frac{1}{e^2}\right)$ (4) $\left(-\frac{1}{e}, -\frac{1}{e^2}\right)$

10. The derivative of $\tan^{-1}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$, with respect to $\frac{x}{2}$, where $\left(x \in \left(0, \frac{\pi}{2}\right)\right)$ is :

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) 1 (4) 2

INDEFINITE INTEGRATION

1. For $x^2 \ne n\pi + 1$, $n \in N$ (the set of natural numbers), the integral

$$\int x\sqrt{\frac{2\sin(x^2-1) - \sin 2(x^2-1)}{2\sin(x^2-1) + \sin 2(x^2-1)}}dx$$

is equal to :
(where c is a constant of integration)

(1) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$

(2) $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right| + c$

(3) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right| + c$

(4) $\frac{1}{2}\log_e\left|\sec(x^2-1)\right| + c$

2. If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2+1+2x^7)^2}dx, (x \ge 0)$ and $f(0) = 0$, then the value of $f(1)$ is :

(1) $-\frac{1}{2}$ (2) $\frac{1}{2}$ (3) $-\frac{1}{4}$ (4) $\frac{1}{4}$

3. Let $n \geq 2$ be a natural number and $0<\theta<\pi/2$.

Then $\int \frac{(\sin^n \theta - \sin\theta)^{\frac{1}{n}} \cos\theta}{\sin^{n+1}\theta} d\theta$ is equal to :

(Where C is a constant of integration)

(1) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}} + C$

(2) $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

(3) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

(4) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}} + C$

4. If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then f(x) is equal to :

(1) $-4x^3 - 1$ (2) $4x^3 + 1$

(3) $-2x^3 - 1$ (4) $-2x^3 + 1$

5. If $\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function A(x), where C is a constant of integration then $(A(x))^m$ equals :

(1) $\frac{-1}{3x^3}$ (2) $\frac{-1}{27x^9}$ (3) $\frac{1}{9x^4}$ (4) $\frac{1}{27x^6}$

6. If $\int \frac{x+1}{\sqrt{2x-1}} dx = f(x)\sqrt{2x-1} + C$, where C is a constant of integration, then f(x) is equal to :-

(1) $\frac{1}{3}(x+4)$ (2) $\frac{1}{3}(x+1)$

(3) $\frac{2}{3}(x+2)$ (4) $\frac{2}{3}(x-4)$

7. The integral $\int \cos(\log_e x) dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{x}{2}[\sin(\log_e x) - \cos(\log_e x)] + C$

(2) $\frac{x}{2}[\cos(\log_e x) + \sin(\log_e x)] + C$

(3) $x[\cos(\log_e x) + \sin(\log_e x)] + C$

(4) $x[\cos(\log_e x) - \sin(\log_e x)] + C$

8. The integral $\int \frac{3x^{13} + 2x^{11}}{\left(2x^4 + 3x^2 + 1\right)^4} dx$ is equal to :

(where C is a constant of integration)

(1) $\frac{x^4}{\left(2x^4+3x^2+1\right)^3} + C$

(2) $\frac{x^{12}}{6\left(2x^4+3x^2+1\right)^3} + C$

(3) $\frac{x^4}{6\left(2x^4+3x^2+1\right)^3} + C$

(4) $\frac{x^{12}}{\left(2x^4+3x^2+1\right)^3} + C$

9. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}} dx$ is equal to :

(where c is a constant of integration)

(1) 2x + sinx + 2sin2x + c

(2) x + 2sinx + 2sin2x + c

(3) x + 2sinx + sin2x + c

(4) 2x + sinx + sin2x + c

10. If $\int \frac{dx}{x^3\left(1+x^6\right)^{2/3}} = x f(x)\left(1+x^6\right)^{\frac{1}{3}} + C$

where C is a constant of integration, then the function $f(x)$ is equal to-

(1) $-\frac{1}{6x^3}$ (2) $\frac{3}{x^2}$

(3) $-\frac{1}{2x^2}$ (4) $-\frac{1}{2x^3}$

11. The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x\, dx$ is equal to

(Hence C is a constant of integration)

(1) $3\tan^{-1/3}x + C$ (2) $-\frac{3}{4}\tan^{-4/3}x + C$

(3) $-3\cot^{-1/3}x + C$ (4) $-3\tan^{-1/3}x + C$

12. If $\int \frac{dx}{\left(x^2-2x+10\right)^2}$

$= A\left(\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{f(x)}{x^2-2x+10}\right) + C$

where C is a constant of integration, then :

(1) $A = \frac{1}{27}$ and $f(x) = 9(x-1)$

(2) $A = \frac{1}{81}$ and $f(x) = 3(x-1)$

(3) $A = \frac{1}{54}$ and $f(x) = 9(x-1)^2$

(4) $A = \frac{1}{54}$ and $f(x) = 3(x-1)$

13. If $\int x^5 e^{-x^2} dx = g(x)e^{-x^2} + c$, where c is a constant of integration, then g(–1) is equal to :

(1) $-\frac{5}{2}$ (2) 1

(3) $-\frac{1}{2}$ (4) –1

14. The integral $\int \frac{2x^3-1}{x^4+x} dx$ is equal to :

(Here C is a constant of integration)

(1) $\log_e\left|\frac{x^3+1}{x}\right| + C$

(2) $\frac{1}{2}\log_e\frac{(x^3+1)^2}{|x^3|} + C$

(3) $\frac{1}{2}\log_e\frac{|x^3+1|}{x^2} + C$

(4) $\log_e\frac{|x^3+1|}{x^2} + C$

15. Let $\alpha \in (0, \pi/2)$ be fixed. If the integral

$\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx =$

$A(x)\cos 2\alpha + B(x)\sin 2\alpha + C$, where C is a constant of integration, then the functions A(x) and B(x) are respectively :

(1) $x - \alpha$ and $\log_e|\cos(x-\alpha)|$

(2) $x + \alpha$ and $\log_e|\sin(x-\alpha)|$

(3) $x - \alpha$ and $\log_e|\sin(x-\alpha)|$

(4) $x + \alpha$ and $\log_e|\sin(x+\alpha)|$

16. If $\int e^{\sec x}(\sec x \tan x f(x) + (\sec x \tan x + \sec^2 x)dx =$ $e^{\sec x}f(x) + C$, then a possible choice of f(x) is :-

(1) $\sec x - \tan x - \frac{1}{2}$

(2) $x\sec x + \tan x + \frac{1}{2}$

(3) $\sec x + x\tan x - \frac{1}{2}$

(4) $\sec x + \tan x + \frac{1}{2}$

DEFINITE INTEGRATION

1. The value of $\int_0^{\pi} |\cos x|^3 dx$

(1) 2/3 (2) 0 (3) –4/3 (4) 4/3

2. If $\int_0^{\frac{\pi}{3}} \frac{\tan\theta}{\sqrt{2k\sec\theta}} d\theta = 1 - \frac{1}{\sqrt{2}}, (k>0)$, then the value of k is :

(1) 2 (2) $\frac{1}{2}$ (3) 4 (4) 1

3. Let $I = \int_a^b (x^4 - 2x^2)dx$. If I is minimum then the ordered pair (a, b) is :

(1) $(-\sqrt{2}, 0)$ (2) $(-\sqrt{2}, \sqrt{2})$

(3) $(0, \sqrt{2})$ (4) $(\sqrt{2}, -\sqrt{2})$

4. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x]+[\sin x]+4}$,

where [t] denotes the greatest integer less than or equal to t, is :

(1) $\frac{1}{12}(7\pi+5)$

(2) $\frac{3}{10}(4\pi-3)$

(3) $\frac{1}{12}(7\pi-5)$

(4) $\frac{3}{20}(4\pi-3)$

5. If $\int_0^x f(t)dt = x^2 + \int_x^1 t^2 f(t)dt$, then f'(1/2) is :

(1) $\frac{6}{25}$ (2) $\frac{24}{25}$ (3) $\frac{18}{25}$ (4) $\frac{4}{5}$

6. The value of the integral $\int_{-2}^{2} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} dx$

(where [x] denotes the greatest integer less than or equal to x) is :

(1) 4 (2) 4 – sin4(3) sin 4 (4) 0

7. The integral $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x(\tan^5 x + \cot^5 x)}$ equals :

(1) $\frac{1}{10}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

(2) $\frac{1}{5}\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

(3) $\frac{\pi}{10}$

(4) $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$

8. Let f and g be continuous functions on [0, a] such that f(x) = f(a–x) and g(x)+g(a–x)=4, then $\int_0^a f(x)g(x)dx$ is equal to :-

(1) $4\int_0^a f(x)dx$ (2) $2\int_0^a f(x)dx$

(3) $-3\int_0^a f(x)dx$ (4) $\int_0^a f(x)dx$

9. The integral $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^x\right\}\log_e x dx$ is equal to :

(1) $\frac{1}{2} - e - \frac{1}{e^2}$ (2) $\frac{3}{2} - \frac{1}{e} - \frac{1}{2e^2}$

(3) $-\frac{1}{2} + \frac{1}{e} - \frac{1}{2e^2}$ (4) $\frac{3}{2} - e - \frac{1}{2e^2}$

10. $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + ... + \frac{1}{5n}\right)$ is equal to :

(1) $\frac{\pi}{4}$ (2) $\tan^{-1}(2)$ (3) $\tan^{-1}(3)$ (4) $\frac{\pi}{2}$

11. If $f(x) = \frac{2 - x\cos x}{2 + x\cos x}$ and g(x) = $\log_e x$, (x > 0)

then the value of integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} g(f(x))dx$ is :

(1) $\log_e 3$ (2) $\log_e 2$

(3) $\log_e e$ (4) $\log_e 1$

12. Let $f(x)=\int_0^x g(t)dt$, where g is a non-zero even function. If $f(x+5)=g(x)$, then $\int_0^x f(t)dt$ equals-

(1) $\int_{x+5}^{5} g(t)dt$ (2) $5\int_{x+5}^{5} g(t)dt$

(3) $\int_{5}^{x+5} g(t)dt$ (4) $2\int_{5}^{x+5} g(t)dt$

13. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x+\cos x}dx$ is

(1) $\frac{\pi-2}{4}$ (2) $\frac{\pi-2}{8}$ (3) $\frac{\pi-1}{4}$ (4) $\frac{\pi-1}{2}$

14. If f : R → R is a differentiable function and f(2) = 6, then $\lim_{x\to 2}\int_6^{f(x)} \frac{2tdt}{(x-2)}$ is :-

(1) 0 (2) 2f'(2)

(3) 12f '(2) (4) 24f '(2)

15. The value of $\int_0^{2\pi}[\sin 2x(1+\cos 3x)]dx$, where [t] denotes the greatest integer function, is :

(1) -2π (2) π (3) $-\pi$ (4) 2π

16. $\lim_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}}+\frac{(n+2)^{1/3}}{n^{4/3}}+......+\frac{(2n)^{1/3}}{n^{4/3}}\right)$ is equal to :

(1) $\frac{4}{3}(2)^{4/3}$ (2) $\frac{3}{4}(2)^{4/3}-\frac{4}{3}$

(3) $\frac{3}{4}(2)^{4/3}-\frac{3}{4}$ (4) $\frac{4}{3}(2)^{3/4}$

17. The integral $\int_{\pi/6}^{\pi/3}\sec^{2/3}x\,\text{cosec}^{4/3}x\,dx$ equal to:

(1) $3^{7/6}-3^{5/6}$ (2) $3^{5/3}-3^{1/3}$

(3) $3^{4/3}-3^{1/3}$ (4) $3^{5/6}-3^{2/3}$

18. If $\int_0^{\frac{\pi}{2}}\frac{\cot x}{\cot x+\text{cosec}x}dx = m(\pi+n)$, then m . n is equal to :

(1) –1 (2) 1 (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

19. The value of the integral $\int_0^1 x\cot^{-1}(1-x^2+x^4)dx$ is :-

(1) $\frac{\pi}{4}-\frac{1}{2}\log_e 2$ (2) $\frac{\pi}{2}-\log_e 2$

(3) $\frac{\pi}{2}-\frac{1}{2}\log_e 2$ (4) $\frac{\pi}{4}-\log_e 2$

20. Let f : R → R be a continuously differentiable function wuch that f(2) = 6 and f(2) = $\frac{1}{48}$.

If $\int_6^{f(x)}4t^3dt=(x-2)g(x)$, then $\lim_{x\to 2}g(x)$ is equal to :

(1) 24 (2) 36 (3) 12 (4) 18

21. A value of α such that

$$\int_{\alpha}^{\alpha+1}\frac{dx}{(x+\alpha)(x+\alpha+1)}=\log_e\left(\frac{9}{8}\right) \text{ is :}$$

(1) $\frac{1}{2}$ (2) 2 (3) $-\frac{1}{2}$ (4) – 2

TANGENT & NORMAL

1. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then |tan θ| is equal to :

(1) 4/9 (2) 7/17 (3) 8/17 (4) 8/15

2. The tangent to the curve $y = x^2 - 5x + 5$, parallel to the line 2y = 4x + 1, also passes through the point.

(1) $\left(\frac{1}{4},\frac{7}{2}\right)$ (2) $\left(\frac{7}{2},\frac{1}{4}\right)$

(3) $\left(-\frac{1}{8},7\right)$ (4) $\left(\frac{1}{8},-7\right)$

3. A helicopter is flying along the curve given by $y - x^{3/2} = 7, (x \geq 0)$. A soldier positioned at the point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is:

(1) $\frac{1}{2}$ (2) $\frac{1}{3}\sqrt{\frac{7}{3}}$ (3) $\frac{1}{6}\sqrt{\frac{7}{3}}$ (4) $\frac{\sqrt{5}}{6}$

4. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points $(1, f(1))$ and $(-1, f(-1))$, then S is equal to :

(1) $\left\{-\frac{1}{3}, -1\right\}$ (2) $\left\{\frac{1}{3}, -1\right\}$

(3) $\left\{-\frac{1}{3}, 1\right\}$ (4) $\left\{\frac{1}{3}, 1\right\}$

5. If the tangent to the curve, $y = x^3 + ax - b$ at the point (1, –5) is perpendicular to the line, $-x + y + 4 = 0$, then which one of the following points lies on the curve ?

(1) (–2, 2) (2) (2, –2)

(3) (2, –1) (4) (–2, 1)

6. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rage of 5 cubic meter per minute. The the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10m; is :-

(1) $2/\pi$ (2) $1/5\pi$ (3) $1/10\pi$ (4) $1/15\pi$

7. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm³/min. When the thickness of the ice is 5cm, then the rate at which the thickness (in cm/min) of the ice decreases, is :

(1) $\frac{1}{9\pi}$ (2) $\frac{5}{6\pi}$ (3) $\frac{1}{18\pi}$ (4) $\frac{1}{36\pi}$

8. If the tangent to the curve $y = \frac{x}{x^2 - 3}$, $x \in R$, $\left(x \neq \pm\sqrt{3}\right)$, at a point $(\alpha, \beta) \neq (0, 0)$ on it is parallel to the line $2x + 6y - 11 = 0$, then :

(1) $|6\alpha + 2\beta| = 19$ (2) $|2\alpha + 6\beta| = 11$

(3) $|6\alpha + 2\beta| = 9$ (4) $|2\alpha + 6\beta| = 19$

MONOTONICITY

1. Let $f(x) = \frac{x}{\sqrt{a^2 + x^2}} - \frac{d - x}{\sqrt{b^2 + (d - x)^2}}$, $x \in R$, where a, b and d are non-zero real constants. Then :-

(1) f is a decreasing function of x

(2) f is neither increasing nor decreasing function of x

(3) f' is not a continuous function of x

(4) f is an increasing function of x

2. If the function f given by $f(x) = x^3 - 3(a - 2)x^2 + 3ax + 7$, for some $a \in R$ is increasing in (0, 1] and decreasing in [1, 5), then a root of the equation, $\frac{f(x) - 14}{(x - 1)^2} = 0 (x \neq 1)$ is :

(1) 6 (2) 5 (3) 7 (4) –7

3. Let $f : [0, 2] \rightarrow R$ be a twice differentiable function such that $f''(x) > 0$, for all $x \in (0, 2)$. If $\phi(x) = f(x) + f(2 - x)$, then ϕ is :

(1) decreasing on (0, 2)

(2) decreasing on (0, 1) and increasing on (1, 2)

(3) increasing on (0, 2)

(4) increasing on (0, 1) and decreasing on (1, 2)

4. Let $f(x) = e^x - x$ and $g(x) = x^2 - x$, $\forall x \in R$. Then the set of all $x \in R$, where the function $h(x) = (f o g)(x)$ is increasing, is :

(1) $\left[-1, \frac{-1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$ (2) $\left[0, \frac{1}{2}\right] \cup [1, \infty)$

(3) $\left[\frac{-1}{2}, 0\right] \cup [1, \infty)$ (4) $[0, \infty)$

5. If m is the minimum value of k for which the function $f(x)=x\sqrt{kx-x^2}$ is increasing in the interval [0, 3] and M is the maximum value of f in [0, 3] when k = m, then the ordered pair (m, M) is equal to :

(1) $(4, 3\sqrt{2})$ (2) $(4, 3\sqrt{3})$

(3) $(3, 3\sqrt{3})$ (4) $(5, 3\sqrt{6})$

MAXIMA & MINIMA

1. The maximum volume (in cu. m) of the right circular cone having slant height 3m is :

(1) $3\sqrt{3}\ \pi$ (2) $6\ \pi$ (3) $2\sqrt{3}\ \pi$ (4) $\frac{4}{3}\ \pi$

2. The shortest distance between the point $\left(\frac{3}{2},0\right)$ and the curve $y=\sqrt{x},(x>0)$ is :

(1) $\frac{\sqrt{5}}{2}$ (2) $\frac{5}{4}$

(3) $\frac{3}{2}$ (4) $\frac{\sqrt{3}}{2}$

3. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S=\{x\in R : x^2+30\le 11x\}$ is :

(1) 122 (2) –222

(3) –122 (4) 222

4. The maximum area (in sq. units) of a rectangle having its base on the x-axis and its other two vertices on the parabola, $y = 12-x^2$ such that the rectangle lies inside the parabola, is :-

(1) $20\sqrt{2}$ (2) $18\sqrt{3}$

(3) 32 (4) 36

5. The shortest distance between the line y = x and the curve $y^2 = x - 2$ is :

(1) $\frac{7}{4\sqrt{2}}$ (2) $\frac{7}{8}$

(3) $\frac{11}{4\sqrt{2}}$ (4) 2

6. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$, $x \in R$, then :

(1) $S_1 = \{-2, 1\}$; $S_2 = \{0\}$

(2) $S_1 = \{-2, 0\}$; $S_2 = \{1\}$

(3) $S_1 = \{-2\}$; $S_2 = \{0, 1\}$

(4) $S_1 = \{-1\}$; $S_2 = \{0, 2\}$

7. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is

(1) $2\sqrt{3}$ (2) $\sqrt{3}$

(3) $\sqrt{6}$ (4) $\frac{2}{3}\sqrt{3}$

8. If $f(x)$ is a non-zero polynomial of degree four, having local extreme points at x = –1, 0, 1; then the set $S = \{x \in R : f(x) = f(0)\}$

Contains exactly :

(1) four irrational numbers.

(2) two irrational and one rational number.

(3) four rational numbers.

(4) two irrational and two rational numbes.

9. Let a_1, a_2, a_3,......be an A. P. with $a_6 = 2$. Then the common difference of this A. P., which maximises the produce $a_1a_4a_5$, is :

(1) $\frac{6}{5}$ (2) $\frac{8}{5}$

(3) $\frac{2}{3}$ (4) $\frac{3}{2}$

DIFFERENTIAL EQUATION

1. If y = y(x) is the solution of the differential equation, $x\frac{dy}{dx}+2y=x^2$ satisfying y(1) = 1, then $y\left(\frac{1}{2}\right)$ is equal to :

(1) $\frac{7}{64}$ (2) $\frac{13}{16}$

(3) $\frac{49}{16}$ (4) $\frac{1}{4}$

2. Let $f:[0,1]\to R$ be such that $f(xy) = f(x).f(y)$ for all $x,y,\varepsilon[0,1]$, and $f(0)\neq 0$. If $y = y(x)$ satisfies the differential equation, $\frac{dy}{dx} = f(x)$ with $y(0) = 1$, then $y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)$ is equal to

(1) 4 (2) 3

(3) 5 (4) 2

3. If $\frac{dy}{dx}+\frac{3}{\cos^2 x}y=\frac{1}{\cos^2 x}, x\in\left(\frac{-\pi}{3},\frac{\pi}{3}\right)$, and $y\left(\frac{\pi}{4}\right)=\frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals :

(1) $\frac{1}{3}+e^6$ (2) $\frac{1}{3}$

(3) $-\frac{4}{3}$ (4) $\frac{1}{3}+e^3$

4. Let f be a differentiable function such that $f'(x)=7-\frac{3}{4}\frac{f(x)}{x},(x>0)$ and $f(1)\neq 4$. Then $\lim_{x\to 0^+} xf\left(\frac{1}{x}\right)$:

(1) Exists and equals 4

(2) Does not exist

(3) Exist and equals 0

(4) Exists and equals $\frac{4}{7}$

5. The curve amongst the family of curves, represented by the differential equation, $(x^2 - y^2)dx + 2xy\,dy = 0$ which passes through (1,1) is :

(1) A circle with centre on the y-axis

(2) A circle with centre on the x-axis

(3) An ellipse with major axis along the y-axis

(4) A hyperbola with transverse axis along the x-axis

6. If y(x) is the solution of the differential equation

$$\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y = e^{-2x},\ x > 0,$$

where $y(1) = \frac{1}{2}e^{-2}$, then :

(1) y(x) is decreasing in (0, 1)

(2) y(x) is decreasing in $\left(\frac{1}{2},1\right)$

(3) $y(\log_e 2) = \frac{\log_e 2}{4}$

(4) $y(\log_e 2) = \log_e 4$

7. The solution of the differential equation, $\frac{dy}{dx}=(x-y)^2$, when y(1) = 1, is :-

(1) $\log_e\left|\frac{2-y}{2-x}\right| = 2(y-1)$

(2) $\log_e\left|\frac{2-x}{2-y}\right| = x-y$

(3) $-\log_e\left|\frac{1+x-y}{1-x+y}\right| = x+y-2$

(4) $-\log_e\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$

8. Lety = y(x) be the solution of the differential equation, $x\frac{dy}{dx}+y = x\log_e x,(x>1)$. If $2y(2) = \log_e 4-1$, then y(e) is equal to :-

(1) $\frac{e^2}{4}$ (2) $\frac{e}{4}$

(3) $-\frac{e}{2}$ (4) $-\frac{e^2}{2}$

9. If a cuver passes through the point (1, –2) and has slope of the tangent at any point (x, y) on it as $\frac{x^2-2y}{x}$, then the curve also passes through the point :

(1) $\left(-\sqrt{2},1\right)$ (2) $\left(\sqrt{3},0\right)$

(3) (–1, 2) (4) (3, 0)

10. Let y = y(x) be the solution of the differential equation, $(x^2+1)^2\frac{dy}{dx}+2x(x^2+1)y=1$ such that y(0) = 0. If $\sqrt{a}\,y(1)=\frac{\pi}{32}$, then the value of 'a' is :

(1) $\frac{1}{2}$ (2) $\frac{1}{16}$

(3) $\frac{1}{4}$ (4) 1

11. Given that the slope of the tangent to a curve y = y(x) at any point (x,y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is :

(1) $x \log_e|y| = 2(x-1)$

(2) $x \log_e|y| = x-1$

(3) $x^2 \log_e|y| = -2(x-1)$

(4) $x \log_e|y| = -2(x-1)$

12. The solution of the differential equation $x\frac{dy}{dx}+2y=x^2$ $(x \neq 0)$ with y(1) = 1, is

(1) $y=\frac{x^3}{5}+\frac{1}{5x^2}$

(2) $y=\frac{4}{5}x^3+\frac{1}{5x^2}$

(3) $y=\frac{3}{4}x^2+\frac{1}{4x^2}$

(4) $y=\frac{x^2}{4}+\frac{3}{4x^2}$

13. If $\cos x\frac{dy}{dx}-y\sin x=6x, (0<x<\frac{\pi}{2})$ and $y\left(\frac{\pi}{3}\right)=0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :-

(1) $-\frac{\pi^2}{4\sqrt{3}}$ (2) $-\frac{\pi^2}{2}$

(3) $-\frac{\pi^2}{2\sqrt{3}}$ (4) $\frac{\pi^2}{2\sqrt{3}}$

14. If y = y (x) is the solution of the differential equation $\frac{dy}{dx}$ = (tanx –y) sec² x, $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, such that y(0) = 0, then $y\left(-\frac{\pi}{4}\right)$ is equal to :

(1) $2+\frac{1}{e}$ (2) $\frac{1}{2}-e$ (3) e – 2 (4) $\frac{1}{2}-e$

15. Let y = y(x) be the solution of the differential equation, $\frac{dy}{dx}+y\tan x=2x+x^2\tan x$, $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, such that y(0) = 1. Then :

(1) $y'\left(\frac{\pi}{4}\right)+y'\left(\frac{-\pi}{4}\right)=-\sqrt{2}$

(2) $y'\left(\frac{\pi}{4}\right)-y'\left(\frac{-\pi}{4}\right)=\pi-\sqrt{2}$

(3) $y\left(\frac{\pi}{4}\right)-y\left(-\frac{\pi}{4}\right)=\sqrt{2}$

(4) $y\left(\frac{\pi}{4}\right)+y\left(-\frac{\pi}{4}\right)=\frac{\pi^2}{2}+2$

16. Consider the differential equation, $y^2dx+\left(x-\frac{1}{y}\right)dy=0$. If value of y is 1 when x = 1, the the value of x for which y = 2, is :

(1) $\frac{1}{2}+\frac{1}{\sqrt{e}}$ (2) $\frac{3}{2}-\sqrt{e}$

(3) $\frac{5}{2}+\frac{1}{\sqrt{e}}$ (4) $\frac{3}{2}-\frac{1}{\sqrt{e}}$

17. The general solution of the differential equation $(y^2 - x^3)\,dx - xy\,dy = 0$ $(x \neq 0)$ is :
(where c is a constant of integration)

(1) $y^2 + 2x^3 + cx^2 = 0$

(2) $y^2 + 2x^2 + cx^3 = 0$

(3) $y^2 - 2x^3 + cx^2 = 0$

(4) $y^2 - 2x^2 + cx^3 = 0$

AREA UNDER THE CURVE

1. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is :

(1) $\frac{14}{3}$ (2) $\frac{56}{3}$ (3) $\frac{8}{3}$ (4) $\frac{32}{3}$

2. The area of the region $A = [(x,y) : 0 \le y \le x|x| + 1 \text{ and } -1 \le x \le 1]$ in sq. units, is :

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) 2 (4) $\frac{4}{3}$

3. If the area enclosed between the curves $y=kx^2$ and $x=ky^2$, (k>0), is 1 square unit. Then k is:

(1) $\frac{1}{\sqrt{3}}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\sqrt{3}$

4. The tangent to the curve, $y = xe^{x^2}$ passing through the point (1,e) also passes through the point :

(1) $\left(\frac{4}{3}, 2e\right)$ (2) (2,3e) (3) $\left(\frac{5}{3}, 2e\right)$ (4) (3,6e)

5. The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$:-

(1) $\frac{5}{4}$ (2) $\frac{9}{8}$ (3) $\frac{3}{4}$ (4) $\frac{7}{8}$

6. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is :

(1) $\frac{15}{4}$ (2) $\frac{15}{2}$ (3) $\frac{21}{2}$ (4) $\frac{17}{4}$

7. The area (in sq. units) of the region $A = \{(x, y) \in R \times R | 0 \le x \le 3, 0 \le y \le 4, y \le x^2 + 3x\}$ is :

(1) $\frac{53}{6}$ (2) $\frac{59}{6}$

(3) 8 (4) $\frac{26}{3}$

8. Let $S(\alpha) = \{(x,y) : y^2 \le x, 0 \le x \le \alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a λ, $0 < \lambda < 4$, $A(\lambda) : A(4) = 2 : 5$, then λ equals

(1) $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$ (2) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

(3) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (4) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$

9. The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is

(1) $\frac{10}{3}$ (2) $\frac{9}{2}$

(3) $\frac{31}{6}$ (4) $\frac{13}{6}$

10. The area (in sq. units) of the region

$A = \{(x,y) : \frac{y^2}{2} \le x \le y + 4\}$ is :-

(1) $\frac{53}{3}$ (2) 18 (3) 30 (4) 16

11. The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is :

(1) $\frac{3}{2} - \frac{1}{\log_e 2}$ (2) $\frac{1}{2}$

(3) $\log_e 2 + \frac{3}{2}$ (4) $\frac{3}{2}$

12. If the area (in sq. units) of the region $\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$ is $a\sqrt{2} + b$, then $a - b$ is equal to :

(1) $\frac{8}{3}$ (2) $\frac{10}{3}$ (3) 6 (4) $-\frac{2}{3}$

13. If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$, then λ is equal to :

(1) 24 (2) 48

(3) $4\sqrt{3}$ (4) $2\sqrt{6}$

MATRIX

1. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to :

(1) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (2) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

(3) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (4) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

2. If $A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$

Then A is-

(1) Invertible only if $t = \frac{\pi}{2}$

(2) not invertible for any tεR

(3) invertible for all tεR

(4) invertible only if t=π

3. Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where $b > 0$. Then the minimum value of $\frac{\det(A)}{b}$ is :

(1) $\sqrt{3}$ (2) $-\sqrt{3}$ (3) $-2\sqrt{3}$ (4) $2\sqrt{3}$

4. Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. It $AA^T = I_3$, then |p| is :

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{\sqrt{5}}$ (3) $\frac{1}{\sqrt{6}}$ (4) $\frac{1}{\sqrt{3}}$

5. Let A and B be two invertible matrices of order 3×3. If $\det(ABA^T) = 8$ and $\det(AB^{-1}) = 8$, then det $(BA^{-1} B^T)$ is equal to :-

(1) 16 (2) $\frac{1}{16}$ (3) $\frac{1}{4}$ (4) 1

6. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q = [q_{ij}]$ be two 3×3 matrices such that $Q - P^5 = I_3$. Then $\frac{q_{21} + q_{31}}{q_{32}}$ is equal to:

(1) 15 (2) 9 (3) 135 (4) 10

7. If $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$; then for all $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$, det(A) lies in the interval :

(1) $\left[\frac{5}{2}, 4\right)$ (2) $\left(\frac{3}{2}, 3\right]$

(3) $\left(0, \frac{3}{2}\right]$ (4) $\left(1, \frac{5}{2}\right]$

8. Let the number 2,b,c be in an A.P. and $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If $\det(A) \in [2,16]$, then c lies in the interval :

(1) [2,3) (2) $(2 + 2^{3/4}, 4)$

(3) $[3, 2 + 2^{3/4}]$ (4) [4,6]

9. Let $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, (\alpha \in R)$ such that $A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Then a value of α is

(1) $\frac{\pi}{16}$ (2) 0 (3) $\frac{\pi}{32}$ (4) $\frac{\pi}{64}$

10. If $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}.\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}.\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}......\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$, then the inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ is

(1) $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$ (2) $\begin{bmatrix}1 & 0\\12 & 1\end{bmatrix}$

(3) $\begin{bmatrix}1 & -12\\0 & 1\end{bmatrix}$ (4) $\begin{bmatrix}1 & 0\\13 & 1\end{bmatrix}$

11. The total number of matrices

$A=\begin{pmatrix}0 & 2y & 1\\2x & y & -1\\2x & -y & 1\end{pmatrix}, (x, y \in R, x \neq y)$ for which

$A^TA = 3I_3$ is :-

(1) 6 (2) 2 (3) 3 (4) 4

12. If $B=\begin{bmatrix}5 & 2\alpha & 1\\0 & 2 & 1\\\alpha & 3 & -1\end{bmatrix}$ is the inverse of a 3 × 3 matrix A, then the sum of all values of α for which det (A) + 1 = 0, is :

(1) 0 (2) 2 (3) 1 (4) –1

13. If a is A symmetric matrix and B is a skew-symmetrix matrix such that $A+B=\begin{bmatrix}2 & 3\\5 & -1\end{bmatrix}$, then AB is equal to :

(1) $\begin{bmatrix}-4 & 2\\1 & 4\end{bmatrix}$ (2) $\begin{bmatrix}-4 & -2\\-1 & 4\end{bmatrix}$

(3) $\begin{bmatrix}4 & -2\\-1 & -4\end{bmatrix}$ (4) $\begin{bmatrix}4 & -2\\1 & -4\end{bmatrix}$

VECTORS

1. Let $\vec{a}=\hat{i}-\hat{j}, \vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c}+\vec{b}=\vec{0}$ and $\vec{a}.\vec{c}=4$, then $|\vec{c}|^2$ is equal to:-

(1) $\frac{19}{2}$ (2) 8

(3) $\frac{17}{2}$ (4) 9

2. Let $\vec{a}=\hat{i}+\hat{j}+\sqrt{2}\hat{k}, \vec{b}=b_1\hat{i}+b_2\hat{j}+\sqrt{2}\hat{k}$ and $\vec{c}=5\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to:

(1) $\sqrt{22}$ (2) 4 (3) $\sqrt{32}$ (4) 6

3. Let $\vec{a}=2\hat{i}+\lambda_1\hat{j}+3\hat{k}$, $\vec{b}=4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}$ and $\vec{c}=3\hat{i}+6\hat{j}+(\lambda_3-1)\hat{k}$ be three vectors such that $\vec{b}=2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1,\lambda_2,\lambda_3)$ is :-

(1) $\left(\frac{1}{2},4,-2\right)$ (2) $\left(-\frac{1}{2},4,0\right)$

(3) (1,3,1) (4) (1,5,1)

4. Let $\vec{\alpha}=(\lambda-2)\vec{a}+\vec{b}$ and $\vec{\beta}=(4\lambda-2)\vec{a}+3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is :

(1) –3 (2) 4 (3) 3 (4) –4

5. Let $\vec{a}=\hat{i}+2\hat{j}+4\hat{k}$, $\vec{b}=\hat{i}+\lambda\hat{j}+4\hat{k}$ and $\vec{c}=2\hat{i}+4\hat{j}+(\lambda^2-1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a}\times\vec{c}$ is :

(1) $-14\hat{i}-5\hat{j}$ (2) $-10\hat{i}-5\hat{j}$

(3) $-10\hat{i}+5\hat{j}$ (4) $-14\hat{i}+5\hat{j}$

6. Let $\sqrt{3}\hat{i}+\hat{j}, \hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is :-

(1) 2 (2) 1

(3) 3 (4) 4

7. The sum of the distinct real values of μ, for which the vectors, $\mu\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+\mu\hat{j}+\hat{k}$, $\hat{i}+\hat{j}+\mu\hat{k}$ are co-planer, is :

(1) 2 (2) 0 (3) –1 (4) 1

8. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{1}{2}\vec{b}$, then $|\alpha - \beta|$ is equal to :

(1) 60° (2) 30° (3) 90° (4) 45°

9. The magnitude of the projection of the vector $2\hat{i}+3\hat{j}+\hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$, is :

(1) $\frac{\sqrt{3}}{2}$ (2) $\sqrt{\frac{3}{2}}$

(3) $\sqrt{6}$ (4) $3\sqrt{6}$

10. Let $\vec{a} = 3\hat{i}+2\hat{j}+x\hat{k}$ and $\vec{b} = \hat{i}-\hat{j}+\hat{k}$, for some real x. Then $|\vec{a}\times\vec{b}| = r$ is possible if :

(1) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$ (2) $0 < r \le \sqrt{\frac{3}{2}}$

(3) $\sqrt{\frac{3}{2}} < r \le 3\sqrt{\frac{3}{2}}$ (4) $r \ge 5\sqrt{\frac{3}{2}}$

11. Let $\vec{\alpha} = 3\hat{i}+\hat{j}$ and $\vec{\beta} = 2\hat{i}-\hat{j}+3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1\times\vec{\beta}_2$ is equal to

(1) $-3\hat{i}+9\hat{j}+5\hat{k}$ (2) $3\hat{i}-9\hat{j}-5\hat{k}$

(3) $\frac{1}{2}(-3\hat{i}+9\hat{j}+5\hat{k})$ (4) $\frac{1}{2}(3\hat{i}-9\hat{j}+5\hat{k})$

12. If a unit vector $\vec{a}$ makes angles $\pi/3$ with $\hat{i}$, $\pi/4$ with $\hat{j}$ and $\theta \in (0, \pi)$ with $\hat{k}$, then a value of θ is :-

(1) $\frac{5\pi}{12}$ (2) $\frac{5\pi}{6}$

(3) $\frac{2\pi}{3}$ (4) $\frac{\pi}{4}$

13. The distance of the point having position vector $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line passing through the point (2, 3, –4) and parallel to the vector, $6\hat{i}+3\hat{j}-4\hat{k}$ is :

(1) 7 (2) $4\sqrt{3}$

(3) $2\sqrt{13}$ (4) 6

14. If the volume of parallelopiped formed by the vectors $\hat{i}+\lambda\hat{j}+\hat{k}$, $\hat{j}+\lambda\hat{k}$ and $\lambda\hat{i}+\hat{k}$ is minimum, then λ is equal to :

(1) $\sqrt{3}$ (2) $-\frac{1}{\sqrt{3}}$

(3) $\frac{1}{\sqrt{3}}$ (4) $-\sqrt{3}$

15. Let $\vec{a} = 3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}-2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ has the magnitude 12 then one such vector is

(1) $4(2\hat{i}+2\hat{j}-\hat{k})$

(2) $4(-2\hat{i}-2\hat{j}+\hat{k})$

(3) $4(2\hat{i}-2\hat{j}-\hat{k})$

(4) $4(2\hat{i}+2\hat{j}+\hat{k})$

16. Let $\alpha \in R$ and the three vectors

$\vec{a} = \alpha\hat{i}+\hat{j}+3\hat{k}$, $\vec{b} = 2\hat{i}+\hat{j}-\alpha\hat{k}$ and $\vec{c} = \alpha\hat{i}-2\hat{j}+3\hat{k}$. Then the set S = {$\alpha$: $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar}

(1) is singleton

(2) Contains exactly two numbers only one of which is positive

(3) Contains exactly two positive numbers

(4) is empty

3D

1. The plane through the intersection of the planes x + y + z = 1 and 2x + 3y – z + 4 = 0 and parallel to y-axis also passes through the point :

(1) (–3, 0, –1) (2) (3, 3, –1)
(3) (3, 2, 1) (4) (–3, 1, 1)

2. The equation of the line passing through (–4, 3, 1), parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is:

(1) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

(2) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(3) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

(4) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

3. The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is:

(1) x + 2y – 2z = 0 (2) x – 2y + z = 0
(3) 5x + 2y – 4z = 0 (4) 3x + 2y – 3z = 0

4. If the lines x = ay+b, z = cy + d and x=a'z + b', y = c'z + d' are perpendicular, then:

(1) cc' + a + a' = 0 (2) aa' + c + c' = 0
(3) ab' + bc' + 1 = 0 (4) bb' + cc' + 1 = 0

5. The plane passing through the point (4, –1, 2) and parallel to the lines $\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2}$ and $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$ also passes through the point :

(1) (–1, –1, –1) (2) (–1, –1, 1)
(3) (1, 1, –1) (4) (1, 1, 1)

6. Let A be a point on the line $\vec{r} = (1-3\mu)\hat{i} + (\mu-1)\hat{j} + (2+5\mu)\hat{k}$ and B(3, 2, 6) be a point in the space. Then the value of μ for which the vector $\overline{AB}$ is parallel to the plane $x - 4y + 3z = 1$ is :

(1) $\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) $\frac{1}{4}$ (4) $\frac{1}{8}$

7. The plane which bisects the line segment joining the points (–3,–3,4) and (3,7,6) at right angles, passes through which one of the following points ?

(1) (4, –1,7) (2) (4,1,–2)
(3) (–2,3,5) (4) (2,1,3)

8. On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane, x + y + z = 2 ?

(1) $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$

(2) $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$

(3) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$

(4) $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$

9. The direction ratios of normal to the plane through the points (0, –1, 0) and (0, 0, 1) and making an anlge $\frac{\pi}{4}$ with the plane y–z+5=0 are:

(1) $2\sqrt{3}$, 1, –1 (2) 2, $\sqrt{2}$, $-\sqrt{2}$
(3) 2, –1, 1 (4) $\sqrt{2}$, 1, –1

10. The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also containing its projection on the plane 2x + 3y – z = 5, contains which one of the following points ?

(1) (2, 0, –2) (2) (–2, 2, 2)
(3) (0, –2, 2) (4) (2, 2, 0)

11. If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to :-

(1) 5 (2) 17 (3) 12 (4) 7

12. Two lines $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates :-

(1) (2, 4, 7) (2) (–2, 4, 7)
(3) (2, –4, –7) (4) (2, –4, 7)

13. The perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is:

(1) $11/\sqrt{6}$ (2) $6\sqrt{11}$ (3) 11 (4) $11\sqrt{6}$

14. A tetrahedron has vertices P(1, 2, 1), Q(2, 1, 3), R(–1,1,2) and O(0, 0, 0). The angle between the faces OPQ and PQR is :

(1) $\cos^{-1}\left(\frac{9}{35}\right)$ (2) $\cos^{-1}\left(\frac{19}{35}\right)$

(3) $\cos^{-1}\left(\frac{17}{31}\right)$ (4) $\cos^{-1}\left(\frac{7}{31}\right)$

15. If an angle between the line, $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z-3}{-2}$ and the plane, $x-2y-kz=3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of k is:

(1) $-\frac{5}{3}$ (2) $\sqrt{\frac{3}{5}}$ (3) $\sqrt{\frac{5}{3}}$ (4) $-\frac{3}{5}$

16. Let S be the set of all real values of λ such that a plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point (–1, –1, 1). Then S is equal to :

(1) $\{\sqrt{3}\}$ (2) $\{\sqrt{3} - \sqrt{3}\}$

(3) {1 , –1} (4) {3, –3}

17. The length of the perpendicular from the point (2, –1, 4) on the straight line, $\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}$ is :

(1) less than 2

(2) greater than 3 but less than 4

(3) greater than 4

(4) greater than 2 but less than 3

18. The equation of a plane containing the line of intersection of the planes $2x - y - 4 = 0$ and $y + 2z - 4 = 0$ and passing through the point (1, 1, 0) is :

(1) $x + 3y + z = 4$ (2) $x - y - z = 0$
(3) $x - 3y - 2z = -2$ (4) $2x - z = 2$

19. The vector equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$ is :

(1) $\vec{r}\times(\hat{i}+\hat{k})+2=0$

(2) $\vec{r}.(\hat{i}-\hat{k})-2=0$

(3) $\vec{r}.(\hat{i}-\hat{k})+2=0$

(4) $\vec{r}\times(\hat{i}-\hat{k})+2=0$

20. If a point R(4,y,z) lies on the line segment joining the points P(2,–3,4) and Q(8,0,10), then the distance of R from the origin is :

(1) $2\sqrt{14}$ (2) 6

(3) $\sqrt{53}$ (4) $2\sqrt{21}$

21. A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0$, also passes through the point

(1) $(-\sqrt{2},1,-4)$ (2) $(\sqrt{2},1,4)$

(3) $(\sqrt{2},-1,4)$ (4) $(-\sqrt{2},-1,-4)$

22. If the line, $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ meets the plane, $x + 2y + 3z = 15$ at a point P, then the distance of P from the origin is

(1) $\frac{9}{2}$ (2) $2\sqrt{5}$ (3) $\frac{\sqrt{5}}{2}$ (4) $\frac{7}{2}$

23. The vertices B and C of a ΔABC lie on the line, $\frac{x+2}{3}=\frac{y-1}{0}=\frac{z}{4}$ such that BC = 5 units. Then the area (in sq. units) of this triangle, given that the point A(1, –1, 2), is :-

(1) $2\sqrt{34}$ (2) $\sqrt{34}$ (3) 6 (4) $5\sqrt{17}$

24. Let P be the plane, which contains the line of intersection of the planes, $x + y + z - 6 = 0$ and $2x + 3y + z + 5 = 0$ and it is perpendicular to the xy-plane. Then the distance of the point (0, 0, 256) from P is equal to :-

(1) $63\sqrt{5}$ (2) $205\sqrt{5}$

(3) $17/\sqrt{5}$ (4) $11/\sqrt{5}$

25. If the system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu$, $(\lambda, \mu \in R)$, has infinitely many solutions, then the value of $\lambda + \mu$ is :

(1) 12 (2) 10 (3) 9 (4) 7

26. Let A(3, 0, –1), B (2, 10, 6) and C(1, 2, 1) be the vertices of a triangle and M be the midpoint of AC. If G divides BM in the ratio, 2 : 1, then $\cos(\angle GOA)$ (O being the origin) is equal to :

(1) $\frac{1}{\sqrt{30}}$ (2) $\frac{1}{6\sqrt{10}}$

(3) $\frac{1}{\sqrt{15}}$ (4) $\frac{1}{2\sqrt{15}}$

27. If the length of the perpendicular from the point $(\beta, 0, \beta)$ $(\beta \neq 0)$ to the line, $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to :

(1) –1 (2) 2 (3) –2 (4) 1

28. If Q(0, –1, –3) is the image of the point P in the plane $3x - y + 4z = 2$ and R is the point (3, –1, –2), then the area (in sq. units) of ΔPQR is :

(1) $\frac{\sqrt{65}}{2}$ (2) $\frac{\sqrt{91}}{4}$ (3) $2\sqrt{13}$ (4) $\frac{\sqrt{91}}{2}$

29. A perpendicular is drawn from a point on the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}$ to the plane $x + y + z = 3$ such that the foot of the perpendicular Q also lies on the plane $x - y + z = 3$. Then the co-ordinates of Q are :

(1) (2, 0, 1) (2) (4, 0, –1)

(3) (–1, 0, 4) (4) (1, 0, 2)

30. If the plane $2x - y + 2z + 3 = 0$ has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes $4x - 2y + 4z + \lambda = 0$ and $2x - y + 2z + \mu = 0$, respectively, then the maximum value of $\lambda + \mu$ is equal to :

(1) 15 (2) 5

(3) 13 (4) 9

31. If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the palne $2x + 3y - z + 13 = 0$ at a point P and the plane $3x + y + 4z = 16$ at a point Q, then PQ is equal to :

(1) $2\sqrt{14}$ (2) $\sqrt{14}$ (3) $2\sqrt{7}$ (4) 14

32. The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines

$\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+2\hat{j}-\hat{k})$ and

$\vec{r}=(\hat{i}+\hat{j})+\mu(-\hat{i}+\hat{j}-2\hat{k})$ is :

(1) $\sqrt{3}$ (2) $\frac{1}{\sqrt{3}}$

(3) $\frac{1}{3}$ (4) 3

33. A plane which bisects the angle between the two given planes $2x - y + 2z - 4 = 0$ and $x + 2y + 2z - 2 = 0$, passes through the point:

(1) (2,4,1) (2) (2, – 4, 1)

(3) (1, 4, – 1) (4) (1, – 4, 1)

PARABOLA

1. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is:

(1) $2\sqrt{3}\,y = 12x + 1$ (2) $2\sqrt{3}\,y = -x - 12$

(3) $\sqrt{3}\,y = x + 3$ (4) $\sqrt{3}\,y = 3x + 1$

2. Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it ?

(1) (4, –4) (2) $(5, 2\sqrt{6})$

(3) (8, 6) (4) $6, 4\sqrt{2}$

3. Let A(4,–4) and B(9,6) be points on the parabola, $y^2 = 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB, is:

(1) $31\frac{3}{4}$ (2) 32 (3) $30\frac{1}{2}$ (4) $31\frac{1}{4}$

4. If the parabolas $y^2=4b(x-c)$ and $y^2=8ax$ have a common normal, then which one of the following is a valid choice for the ordered triad (a,b,c)

(1) (1, 1, 0) (2) $\left(\frac{1}{2},2,3\right)$

(3) $\left(\frac{1}{2},2,0\right)$ (4) (1, 1, 3)

5. The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is :

(1) $2\sqrt{11}$ (2) $3\sqrt{2}$

(3) $6\sqrt{3}$ (4) $8\sqrt{2}$

6. If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2 + 4(x - a^2) = 0$ and the other two vertices are the points of intersection of the parabola and y-axis, is 250 sq. units, then a value of 'a' is :-

(1) $5\sqrt{5}$ (2) $(10)^{2/3}$ (3) $5(2^{1/3})$ (4) 5

7. The area (in sq. units) in the first quadrant bounded by the parabola, $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is :-

(1) $\frac{14}{3}$ (2) $\frac{187}{24}$

(3) $\frac{37}{24}$ (4) $\frac{8}{3}$

8. Let P(4, –4) and Q(9, 6) be two points on the parabola, $y^2= 4x$ and let X be any point on the arc POQ of this parabola, where O is the vertex of this parabola, such that the area of ΔPXQ is maximum. Then this maximum area (in sq. units) is :

(1) $\frac{125}{4}$ (2) $\frac{125}{2}$

(3) $\frac{625}{4}$ (4) $\frac{75}{2}$

9. The equation of a tangent to the parabola, $x^2 = 8y$, which makes an angle θ with the positive direction of x-axis, is :

(1) $x = y\cot\theta + 2\tan\theta$

(2) $x = y\cot\theta - 2\tan\theta$

(3) $y = x\tan\theta - 2\cot\theta$

(4) $y = x\tan\theta + 2\cot\theta$

10. The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant, passes through the point :

(1) $\left(-\frac{1}{3},\frac{4}{3}\right)$ (2) $\left(-\frac{1}{4},\frac{1}{2}\right)$

(3) $\left(\frac{3}{4},\frac{7}{4}\right)$ (4) $\left(\frac{1}{4},\frac{3}{4}\right)$

11. If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1, 4), then the length of this focal chord is

(1) 25 (2) 24 (3) 20 (4) 22

12. If the tangent to the parabola $y^2 = x$ at a point (α, β), $(\beta > 0)$ is also a tangent to the ellipse, $x^2 + 2y^2 = 1$, then α is equal to :

(1) $2\sqrt{2}+1$ (2) $\sqrt{2}-1$

(3) $\sqrt{2}+1$ (4) $2\sqrt{2}-1$

13. The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1, 2) and the x-axis is :-

(1) $4\pi(2-\sqrt{2})$ (2) $8\pi(3-2\sqrt{2})$

(3) $4\pi(3+\sqrt{2})$ (4) $8\pi(2-\sqrt{2})$

14. If the line $ax + y = c$, touches both the curves $x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$, then $|c|$ is equal to:

(1) 1/2 (2) 2

(3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

15. The tangents to the curve $y = (x-2)^2 - 1$ at its points of intersection with the line $x - y = 3$, intersect at the point :

(1) $\left(-\frac{5}{2},-1\right)$ (2) $\left(-\frac{5}{2},1\right)$

(3) $\left(\frac{5}{2},-1\right)$ (4) $\left(\frac{5}{2},1\right)$

16. The equation of a common tangent to the curves, $y^2 = 16x$ and $xy = -4$ is :

(1) $x + y + 4 = 0$ (2) $x - 2y + 16 = 0$

(3) $2x - y + 2 = 0$ (4) $x - y + 4 = 0$

ELLIPSE

1. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted betwen the coordinate axes lie on the curve :

(1) $\frac{x^2}{2}+\frac{y^2}{4}=1$ (2) $\frac{x^2}{4}+\frac{y^2}{2}=1$

(3) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (4) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$

2. Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it ?

(1) $\left(4\sqrt{3}, 2\sqrt{3}\right)$

(2) $\left(4\sqrt{3}, 2\sqrt{2}\right)$

(3) $\left(4\sqrt{2}, 2\sqrt{2}\right)$

(4) $\left(4\sqrt{2}, 2\sqrt{3}\right)$

3. Let S and S' be the foci of the ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right angled triangle with right angle at B and area $(\Delta S'BS) = 8$ sq. units, then the length of a latus rectum of the ellipse is :

(1) $2\sqrt{2}$ (2) 2

(3) 4 (4) $4\sqrt{2}$

4. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points (1, 2) and (a, b) are perpendicular to each other, then a^2 is equal to :

(1) $\frac{64}{17}$ (2) $\frac{2}{17}$

(3) $\frac{128}{17}$ (4) $\frac{4}{17}$

5. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $\left(0, 5\sqrt{3}\right)$, then the length of its latus rectum is:

(1) 10 (2) 8 (3) 5 (4) 6

6. If the line $x - 2y = 12$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $\left(3,\frac{-9}{2}\right)$, then the length of the latus recturm of the ellipse is :

(1) 9 (2) $8\sqrt{3}$ (3) $12\sqrt{2}$ (4) 5

7. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point P(2, 2) meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is :

(1) $\frac{14}{3}$ (2) $\frac{16}{3}$ (3) $\frac{68}{15}$ (4) $\frac{34}{15}$

8. If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through Q(4, 4) then PQ is equal to :

(1) $\frac{\sqrt{221}}{2}$ (2) $\frac{\sqrt{157}}{2}$ (3) $\frac{\sqrt{61}}{2}$ (4) $\frac{5\sqrt{5}}{2}$

9. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points ?

(1) $(1, 2\sqrt{2})$

(2) $(2, \sqrt{2})$

(3) $(2, 2\sqrt{2})$

(4) $(\sqrt{2}, 2)$

HYPERBOLA

1. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta}-\frac{y^2}{\sin^2\theta}=1$ is greater than 2, then the length of its latus rectum lies in the interval :

(1) (2, 3] (2) $(3, \infty)$

(3) (3/2, 2] (4) (1, 3/2]

2. A hyperbola has its centre at the origin, passes through the point (4,2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is :

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{3}{2}$

(3) $\sqrt{3}$ (4) 2

3. The equation of a tangent to the hyperbola $4x^2–5y^2 = 20$ parallel to the line $x–y = 2$ is :

(1) $x–y+9 = 0$

(2) $x–y+7 = 0$

(3) $x–y+1 = 0$

(4) $x–y–3 = 0$

4. Let $S=\left\{(x,y)\in R^2:\frac{y^2}{1+r}-\frac{x^2}{1-r}=1\right\}$, where $r \neq \pm 1$. Then S represents :

(1) A hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, where $0 < r < 1$.

(2) An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, where $r > 1$

(3) A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when $0 < r < 1$.

(4) An ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when $r > 1$

5. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbole $xy = 2$ is :

(1) $x + 2y + 4 = 0$

(2) $x – 2y + 4 = 0$

(3) $x + y + 1 = 0$

(4) $4x + 2y + 1 = 0$

6. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is :-

(1) 2 (2) $\frac{13}{6}$ (3) $\frac{13}{8}$ (4) $\frac{13}{12}$

7. If the vertices of a hyperbola be at (–2, 0) and (2, 0) and one of its foci be at (–3, 0), then which one of the following points does not lie on this hyperbola?

(1) $\left(4,\sqrt{15}\right)$ (2) $\left(-6,2\sqrt{10}\right)$

(3) $\left(6,5\sqrt{2}\right)$ (4) $\left(2\sqrt{6},5\right)$

8. If the eccentricity of the standard hyperbola passing through the point (4,6) is 2, then the equation of the tangent to the hyperbola at (4,6) is-

(1) $2x - y - 2 = 0$

(2) $3x - 2y = 0$

(3) $2x - 3y + 10 = 0$

(4) $x - 2y + 8 = 0$

9. If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24}-\frac{y^2}{18}=1$, then a value of m is

(1) $\frac{\sqrt{5}}{2}$ (2) $\frac{3}{\sqrt{5}}$

(3) $\frac{2}{\sqrt{5}}$ (4) $\frac{\sqrt{15}}{2}$

10. If a directrix of a hyperbola centred at the origin and passing through the point $\left(4,-2\sqrt{3}\right)$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then :

(1) $4e^4 - 24e^2 + 35 = 0$

(2) $4e^4 + 8e^2 - 35 = 0$

(3) $4e^4 - 12e^2 - 27 = 0$

(4) $4e^4 - 24e^2 + 27 = 0$

11. If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is :

(1) $\left(-\frac{5}{3},0\right)$ (2) (5, 0)

(3) (–5, 0) (4) $\left(\frac{5}{3},0\right)$

12. Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $8x^2 - y^2 = 8$. If S and S' denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS' in a ratio:

(1) 5:4 (2) 14:13 (3) 2:1 (4) 13:11

COMPLEX NUMBER

1. Let A $=\left\{\theta\in\left(-\frac{\pi}{2},\pi\right):\frac{3+2i\sin\theta}{1-2i\sin\theta}\text{ is purely imaginary}\right\}$.

Then the sum of the elements in A is :

(1) $\frac{5\pi}{6}$ (2) $\frac{2\pi}{3}$

(3) $\frac{3\pi}{4}$ (4) π

2. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6iz_0^{81} - 3iz_0^{93}$, then arg z is equal to:

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$

(3) 0 (4) $\frac{\pi}{6}$

3. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1| = 4|z_2|$. If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ then:

(1) $|z| = \frac{1}{2}\sqrt{\frac{17}{2}}$ (2) Re(z) = 0

(3) $|z| = \sqrt{\frac{5}{2}}$ (4) Im(z) = 0

4. Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$. If R(z) and I[z] respectively denote the real and imaginary parts of z, then :

(1) $R(z) > 0$ and $I(z) > 0$

(2) $R(z) < 0$ and $I(z) > 0$

(3) $R(z) = -3$

(4) $I(z) = 0$

5. Let $\left(-2 - \frac{1}{3}i\right)^3 = \frac{x + iy}{27} (i = \sqrt{-1})$, where x and y are real numbers, then y – x equals :

(1) –85 (2) 85

(3) –91 (4) 91

6. Let z be a complex number such that $|z| + z = 3 + i$ (where $i = \sqrt{-1}$). Then $|z|$ is equal to :-

(1) $\frac{5}{4}$ (2) $\frac{\sqrt{41}}{4}$ (3) $\frac{\sqrt{34}}{3}$ (4) $\frac{5}{3}$

7. If $\frac{z-\alpha}{z+\alpha} (\alpha \in R)$ is a purely imaginary number and $|z| = 2$, then a value of α is :

(1) 1 (2) 2

(3) $\sqrt{2}$ (4) $\frac{1}{2}$

8. Let Z_1 and Z_2 be two complex numbers satisfying $|Z_1| = 9$ and $|Z_2 - 3 - 4i| = 4$. Then the minimum value of $|Z_1 - Z_2|$ is :

(1) 0 (2) 1

(3) $\sqrt{2}$ (4) 2

9. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2} \left(i = \sqrt{-1}\right)$, then $\left(1 + iz + z^5 + iz^8\right)^9$ is equal to

(1) –1 (2) 1

(3) 0 (4) $(-1 + 2i)^9$

10. All the points in the set $S = \left\{\frac{\alpha + i}{\alpha - i} : \alpha \in R\right\} \left(i = \sqrt{-1}\right)$ lie on a

(1) circle whose radius is 1.

(2) straight line whose slope is 1.

(3) straight line whose slope is –1

(4) circle whose radius is $\sqrt{2}$.

11. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in R,

$$\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$$ is equal to

(1) y^3 (2) $y^3 - 1$

(3) $y(y^2 - 1)$ (4) $y(y^2 - 3)$

12. Let $z \in C$ be such that $|z| < 1$. If $\omega = \frac{5+3z}{5(1-z)}$, then:-

(1) $5\text{Im}(\omega) < 1$ (2) $4\text{Im}(\omega) > 5$

(3) $5\text{Re}(\omega) > 1$ (4) $5\text{Re}(\omega) > 4$

13. If $a > 0$ and $z = \frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to :

(1) $-\frac{3}{5}-\frac{1}{5}i$ (2) $-\frac{1}{5}+\frac{3}{5}i$

(3) $-\frac{1}{5}-\frac{3}{5}i$ (4) $\frac{1}{5}-\frac{3}{5}i$

14. If z and w are two complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$, then :

(1) $\bar{z}w = i$ (2) $\bar{z}w = -i$

(3) $z\bar{w} = \frac{1-i}{\sqrt{2}}$ (4) $z\bar{w} = \frac{-1+i}{\sqrt{2}}$

15. The equation $|z-i| = |z-1|$, $i = \sqrt{-1}$, represents:

(1) the line through the origin with slope –1.

(2) a circle of radius 1.

(3) a circle of radius $\frac{1}{2}$.

(4) the line through the origin with slope 1.

16. Let $z \in C$ with $\text{Im}(z) = 10$ and it satisfies $\frac{2z-n}{2z+n} = 2i-1$ for some natural number n. Then :

(1) n = 20 and Re(z) = – 10

(2) n = 20 and Re(z) = 10

(3) n = 40 and Re(z) = – 10

(4) n = 40 and Re(z) = 10

PROBABILITY

1. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P(X = 1) + P(X = 2) equals :

(1) 52/169 (2) 25/169

(3) 49/169 (4) 24/169

2. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is :

(1) $\frac{26}{49}$ (2) $\frac{32}{49}$ (3) $\frac{27}{49}$ (4) $\frac{21}{49}$

3. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and the sum of the numbers obtained on them is noted. If the toss of the coin results in tail then a card from a well-shuffled pack of nine cards numbered 1,2,3,...,9 is randomly picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is :

(1) $\frac{13}{36}$ (2) $\frac{19}{36}$ (3) $\frac{19}{72}$ (4) $\frac{15}{72}$

4. If the probability of hitting a target by a shooter, in any shot, is 1/3, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is :

(1) 6 (2) 5 (3) 4 (4) 3

5. Two integers are selected at random from the set {1, 2,...., 11}. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is :

(1) $\frac{2}{5}$ (2) $\frac{1}{2}$ (3) $\frac{3}{5}$ (4) $\frac{7}{10}$

6. Let S = {1, 2, ... , 20}. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Then the probability that a randomly chosen subset of S is "nice" is :-

(1) $\frac{6}{2^{20}}$ (2) $\frac{5}{2^{20}}$

(3) $\frac{4}{2^{20}}$ (4) $\frac{7}{2^{20}}$

7. A bag contains 30 white balls and 10 red balls. 16 balls are drawn one by one randomly from the bag with replacement. If X be the number of white balls drawn, the $\left(\frac{\text{mean of X}}{\text{standard deviation of X}}\right)$ is equal to :-

(1) 4 (2) $\frac{4\sqrt{3}}{3}$ (3) $4\sqrt{3}$ (4) $3\sqrt{2}$

8. In a random experiment, a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end in the fifth throw of the die is equal to :

(1) $\frac{150}{6^5}$ (2) $\frac{175}{6^5}$ (3) $\frac{200}{6^5}$ (4) $\frac{225}{6^5}$

9. Consider three boxes, each containing 10 balls labelled 1,2,....,10. Suppose one ball is randomly drawn from each of the boxes. Denote by n_i, the label of the ball drawn from the i^{th} box, (i = 1, 2, 3). Then, the number of ways in which the balls can be chosen such that $n_1 < n_2 < n_3$ is :

(1) 82 (2) 240 (3) 164 (4) 120

10. In a game, a man wins Rs. 100 if he gets 5 of 6 on a throw of a fair die and loses Rs. 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is :

(1) $\frac{400}{3}$ gain (2) $\frac{400}{3}$ loss

(3) 0 (4) $\frac{400}{9}$ loss

11. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the student selected has opted neither for NCC nor for NSS is :

(1) $\frac{2}{3}$ (2) $\frac{1}{6}$

(3) $\frac{1}{3}$ (4) $\frac{5}{6}$

12. Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct ?

(1) $P(A|B) = 1$

(2) $P(A|B) = P(B) - P(A)$

(3) $P(A|B) \leq P(A)$

(4) $P(A|B) \geq P(A)$

13. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is :

(1) 5 (2) 3 (3) 2 (4) 4

14. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. if all hit at the target independently, then the probability that the target would be hit, is

(1) $\frac{25}{192}$ (2) $\frac{1}{192}$

(3) $\frac{25}{32}$ (4) $\frac{7}{32}$

15. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is :

(1) $\frac{1}{11}$ (2) $\frac{1}{17}$

(3) $\frac{1}{10}$ (4) $\frac{1}{12}$

16. Minimum number of times a fair coin must be tossed so that the probability of getting at least one head is more than 99% is :

(1) 5 (2) 6

(3) 7 (4) 8

17. If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is :

(1) $\frac{3}{10}$ (2) $\frac{1}{10}$

(3) $\frac{3}{20}$ (4) $\frac{1}{5}$

18. Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(x \leq 2) = \frac{k}{2^{16}}$, then k is equal to :

(1) 17 (2) 1

(3) 121 (4) 137

19. For and initial screening of an admission test, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability that he is unable to solve less than two problems is :

(1) $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$ (2) $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$

(3) $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$ (4) $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$

20. A person throws two fair dice. He wins Rs. 15 for throwing a doublet (same numbers on the two dice), wins Rs.12 when the throw results in the sum of 9, and loses Rs. 6 for any other outcome on the throw. Then the expected gain/loss (in Rs.) of the person is :

(1) 2 gain (2) $\frac{1}{2}$ loss (3) $\frac{1}{4}$ loss (4) $\frac{1}{2}$ gain

STATISTICS

1. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is:

(1) 22 (2) 20

(3) 16 (4) 18

2. A data consists of n observations:

$x_1, x_2, \ldots, x_n$. If $\sum_{i=1}^{n}(x_i+1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i-1)^2 = 5n$, then the standard deviation of this data is :

(1) 5 (2) $\sqrt{5}$ (3) $\sqrt{7}$ (4) 2

3. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is :

(1) 4 : 9 (2) 6 : 7

(3) 5 : 8 (4) 10 : 3

4. If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations $x_1, x_2, \ldots, x_5$ and –50 is equal to:

(1) 582.5 (2) 507.5 (3) 586.5 (4) 509.5

5. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2} - d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2} + d$ each. If the variance of this outcome data is $\frac{4}{3}$ then $|d|$ equals :-

(1) 2 (2) $\frac{\sqrt{5}}{2}$

(3) $\frac{2}{3}$ (4) $\sqrt{2}$

6. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observation is :

(1) 50 (2) 51 (3) 30 (4) 31

7. The mean and the variance of five observation are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then then absolute value of the difference of the other two observations, is :

(1) 1 (2) 3

(3) 7 (4) 5

8. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is :

(1) 40 (2) 49

(3) 48 (4) 45

9. A student scores the following marks in five tests : 45,54,41,57,43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is

(1) $\frac{10}{\sqrt{3}}$ (2) $\frac{100}{\sqrt{3}}$

(3) $\frac{100}{3}$ (4) $\frac{10}{3}$

10. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where k > 0, then k is equal to

(1) $2\sqrt{\frac{10}{3}}$ (2) $2\sqrt{6}$

(3) $4\sqrt{\frac{5}{3}}$ (4) $\sqrt{6}$

11. The mean and the median of the following ten numbers in increasing order 10, 22, 26, 29, 34, x 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to :-

(1) 7/3 (2) 9/4

(3) 7/2 (4) 8/3

12. If for some $x \in R$, the frequency distribution of the marks obtained by 20 students in a test is :

Marks	2	3	5	7
Frequencey	$(x+1)^2$	$2x-5$	x^2-3x	x

then the mean of the marks is :

(1) 2.8 (2) 3.2

(3) 3.0 (4) 2.5

13. If both the mean and the standard deviation of 50 observations $x_1, x_2, ..., x_{50}$ are equal to 16, then the mean of $(x_1 - 4)^2, (x_2 - 4)^2, ...(x_{50} - 4)^2$ is :

(1) 525 (2) 380

(3) 480 (4) 400

14. If the data $x_1, x_2, ..., x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000; then the standard deviation of this data is :

(1) 4 (2) 2

(3) $\sqrt{2}$ (4) $2\sqrt{2}$

REASONING

1. If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$, then the ordered pair $(\oplus, \odot)$ is:

(1) $(\wedge, \vee)$ (2) $(\vee, \vee)$

(3) $(\wedge, \wedge)$ (4) $(\vee, \wedge)$

2. The logical statement

$[\sim(\sim p \vee q) \vee (p \wedge r) \wedge (\sim q \wedge r)]$ is equivalent to:

(1) $(p \wedge r) \wedge \sim q$

(2) $(\sim p \wedge \sim q) \wedge r$

(3) $\sim p \vee r$

(4) $(p \wedge \sim q) \vee r$

3. Consider the following three statements :

P : 5 is a prime number.

Q : 7 is a factor of 192.

R : L.C.M. of 5 and 7 is 35.

Then the truth value of which one of the following statements is true ?

(1) $(P \wedge Q) \vee (\sim R)$ (2) $(\sim P) \wedge (\sim Q \wedge R)$

(3) $(\sim P) \vee (Q \wedge R)$ (4) $P \vee (\sim Q \wedge R)$

4. If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology?

(1) $(p \vee r) \rightarrow (p \wedge r)$

(2) $p \vee r$

(3) $p \wedge r$

(4) $(p \wedge r) \rightarrow (p \vee r)$

5. Contrapositive of the statement

"If two numbers are not equal, then their squares are not equal." is :-

(1) If the squares of two numbers are equal, then the numbers are equal.

(2) If the squares of two numbers are equal, then the numbers are not equal.

(3) If the squares of two numbers are not equal, then the numbers are equal.

(4) If the squares of two numbers are not equal, then the numbers are not equal.

6. The Boolean expression

$((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ is equivalent to:

(1) $p \wedge (\sim q)$ (2) $p \vee (\sim q)$

(3) $(\sim p) \wedge (\sim q)$ (4) $p \wedge q$

7. The expression $\sim(\sim p \rightarrow q)$ is logically equvalent to :

(1) $\sim p \wedge \sim q$ (2) $p \wedge q$

(3) $\sim p \wedge q$ (4) $p \wedge \sim q$

8. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is :

(1) If you are born in India, then you are not a citizen of India.

(2) If you are not a citizen of India, then you are not born in India.

(3) If you are a citizen of India, then you are born in India.

(4) If you are not born in India, then you are not a citizen of India.

9. Which one of the following statements is not a tautology ?

(1) $(p \wedge q) \rightarrow p$

(2) $(p \wedge q) \rightarrow (\sim p) \vee q$

(3) $p \rightarrow (p \vee q)$

(4) $(p \vee q) \rightarrow (p \vee (\sim q))$

10. For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is

(1) $p \wedge q$ (2) $p \leftrightarrow q$

(3) $\sim p \vee \sim q$ (4) $\sim p \wedge \sim q$

11. If $P \Rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively :-

(1) F, T, T (2) T, F, F

(3) T, T, F (4) F, F, F

12. Which one of the following Boolean expressions is a tautology ?

(1) $(P \vee q) \wedge (\sim p \vee \sim q)$

(2) $(P \wedge q) \vee (p \wedge \sim q)$

(3) $(P \vee q) \wedge (p \vee \sim q)$

(4) $(P \vee q) \vee (p \vee \sim q)$

13. The negation of the boolean expression $\sim s \vee (\sim r \wedge s)$ is equivalent to :

(1) r (2) $s \wedge r$

(3) $s \vee r$ (4) $\sim s \wedge \sim r$

14. If the truth value of the statement $P \rightarrow (\sim p \vee r)$ is false(F), then the truth values of the statements p, q, r are respectively :

(1) F, T, T

(2) T, F, F

(3) T, T, F

(4) T, F, T

15. The Boolean expression $\sim(p \Rightarrow (\sim q))$ is equivalent to :

(1) $(\sim p) \Rightarrow q$ (2) $p \vee q$

(3) $q \Rightarrow \sim p$ (4) $p \wedge q$

MATHEMATICAL INDUCTION

1. Consider the statement : "P(n): $n^2 - n + 41$ is prime." Then which one of the following is true?

(1) P(5) is false but P(3) is true

(2) Both P(3) and P(5) are false

(3) P(3) is false but P(5) is true

(4) Both P(3) and P(5) are true

ANSWER KEY

COMPOUND ANGLE

Que.	1	2	3	4	5	6	7
Ans.	1	3	4	1	4	2	4

QUADRATIC EQUATION

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	1	3	1	1	3	1	2	2	3
Que.	11	12	13	14	15					
Ans.	3	1	2	4	4					

SEQUENCE & PROGRESSION

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	4	1	2	3	3	1	2	4	1
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	2	2	2	2	3	1	1	2	1	3
Que.	21	22	23	24	25	26				
Ans.	4	2	3	1	1	1				

TRIGONOMETRIC EQUATION

Que.	1	2	3	4	5	6
Ans.	1	1	3	1	1	1

SOLUTION OF TRIANGLE

Que.	1	2	3	4	5	6
Ans.	4	1	2	3	3	3

HEIGHT & DISTANCE

Que.	1	2	3	4	5	6	7
Ans.	2	2	3	3	3	3	2

DETERMINANT

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2	2	4	3	1	4	1	4	3	2
Que.	11	12	13	14	15	16	17	18		
Ans.	1	3	3	2	2	3	3	2		

STRAIGHT LINE

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	4	4	2	2	2	4	4	4	4
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	3	1	3	3	4	2	1	1	2	2
Que.	21									
Ans.	2									

CIRCLE										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	1	2	4	2	4	2	4	2	1	4
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	2	4	4	3	2	3	1	4	2	1

PERMUTATION & COMBINATION										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	2	2	4	1	3	1	4	4	1	1
Que.	**11**	**12**	**13**	**14**						
Ans.	2	3	1	1						

BIONOMIAL THEOREM										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	4	2	3	4	2	3	3	1	4	1
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	4	4	2	4	4	2	4	1	2	2
Que.	**21**	**22**								
Ans.	1	4								

SET			
Que.	**1**	**2**	**3**
Ans.	4	3	1

RELATION	
Que.	**1**
Ans.	3

FUNCTION										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	1	1	4	2	Bonus	1	1	1	2	1
Que.	**11**	**12**	**13**	**14**	**15**					
Ans.	3	3	4	3	2					

INVERSE TRIGONOMETRIC FUNCTION								
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Ans.	1	1	3	3	4	1	3	3

LIMIT										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	1	1	4	4	4	3	3	2	4	4
Que.	**11**	**12**	**13**	**14**						
Ans.	4	1	2	1						

CONTINUITY						
Que.	1	2	3	4	5	
Ans.	4	4	1	1	4	

DIFFERENTIABILITY									
Que.	1	2	3	4	5	6	7	8	
Ans.	4	3	1	3	1	4	3	1	

METHOD OF DIFFERENTIATION										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	2	3	4	1	4	2	4	1	4

INDEFINITE INTEGRATION										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1 or 3	4	3	1	2	1	2	2	3	4
Que.	11	12	13	14	15	16				
Ans.	4	4	1	1	3	4				

DEFINITE INTEGRATION										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	1	2	4	2	4	1	2	4	2
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	4	1	3	3	3	3	1	1	1	4
Que.	21									
Ans.	4									

TANGENT & NORMAL									
Que.	1	2	3	4	5	6	7	8	
Ans.	4	4	3	3	2	2	3	1	

MONOTONICITY						
Que.	1	2	3	4	5	
Ans.	4	3	2	2	2	

MAXIMA & MINIMA										
Que.	1	2	3	4	5	6	7	8	9	
Ans.	3	1	1	3	1	1	1	2	2	

DIFFERENTIAL EQUATION										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	2	1	1	2	2	4	2	2	2
Que.	11	12	13	14	15	16	17			
Ans.	1	4	3	3	2	4	1			

AREA UNDER THE CURVE										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	3	3	1	1	2	2	2	2	2	2
Que.	**11**	**12**	**13**							
Ans.	1	3	1							

MATRIX										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	1	3	4	1	2	4	2	4	4	1
Que.	**11**	**12**	**13**							
Ans.	4	3	3							

VECTOR										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	1	4	2	4	3	2	3	2	2	4
Que.	**11**	**12**	**13**	**14**	**15**	**16**				
Ans.	3	3	1	3	3	4				

3D										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	3	2	2	2	4	3	2	3	2,4	1
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	4	3	1	2	3	2	2	2	3	1
Que.	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	2	1	2	4	2	3	1	4	1	3
Que.	**31**	**32**	**33**							
Ans.	1	1	2							

PARABOLA										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	3	3	4	1,2,3,4	3	4	3	1	1	3
Que.	**11**	**12**	**13**	**14**	**15**	**16**				
Ans.	1	3	2	3	3	4				

ELLIPSE										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	
Ans.	3	2	3	2	3	1	3	4	4	

HYPERBOLA										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	2	1	3	4	1	4	3	1	3	1
Que.	**11**	**12**								
Ans.	3	1								

COMPLEX NUMBER										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	2	1	Bonus	4	4	4	2	1	1	1
Que.	**11**	**12**	**13**	**14**	**15**	**16**				
Ans.	1	3	3	2	4	3				

PROBABILITY										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	2	2	3	2	1	2	3	2	4	3
Que.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans.	2	4	4	3	1	3	2	4	2	2

STATISTICS										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	2	2	1	2	4	4	3	3	1	2
Que.	**11**	**12**	**13**	**14**						
Ans.	1	1	4	2						

REASONING										
Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.	1	1	4	4	1	3	1	2	4	4
Que.	**11**	**12**	**13**	**14**	**15**					
Ans.	2	4	2	3	4					

MATHEMATICAL INDUCTION		
Que.	**1**	
Ans.	4	

IMPORTANT NOTES

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