# TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

**(Held On Thursday 10th JANUARY, 2019) TIME : 2 : 30 AM To 5 : 30 PM**

## MATHEMATICS

**1.** Let $z = \left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$. If R(z) and I[z] respectively denote the real and imaginary parts of z, then :

(1) R(z) > 0 and I(z) > 0

(2) R(z) < 0 and I(z) > 0

(3) R(z) = –3

(4) I(z) = 0

**Ans. (4)**

**2.** Let $a_1, a_2, a_3, ...., a_{10}$ be in G.P. with $a_i > 0$ for i = 1,2,...., 10 and S be the set of pairs (r,k), r k∈N (the set of natural numbers) for which

$$\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$$

Then the number of elements in S, is :

(1) Infinitely many (2) 4

(3) 10 (4) 2

**Ans. (1)**

**3.** The positive value of λ for which the co-efficient of $x^2$ in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is :

(1) $\sqrt{5}$ (2) 4

(3) $2\sqrt{2}$ (4) 3

**Ans. (2)**

**4.** The value of $\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\cdot.....\cdot\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$ is :

(1) $\frac{1}{256}$ (2) $\frac{1}{2}$

(3) $\frac{1}{512}$ (4) $\frac{1}{1024}$

**Ans. (3)**

**5.** The value of $\int_{-\pi/2}^{\pi/2}\frac{dx}{[x]+[\sin x]+4}$, where [t] denotes the greatest integer less than or equal to t, is :

(1) $\frac{1}{12}(7\pi+5)$ (2) $\frac{3}{10}(4\pi-3)$

(3) $\frac{1}{12}(7\pi-5)$ (4) $\frac{3}{20}(4\pi-3)$

**Ans. (4)**

**6.** If the probability of hitting a target by a shooter, in any shot, is 1/3, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater than $\frac{5}{6}$, is :

(1) 6 (2) 5

(3) 4 (4) 3

**Ans. (2)**

**7.** If mean and standard deviation of 5 observations $x_1$, $x_2$, $x_3$,$x_4$,$x_5$ are 10 and 3, respectively, then the variance of 6 observations $x_1$,$x_2$, ....,$x_5$ and –50 is equal to :

(1) 582.5 (2) 507.5

(3) 586.5 (4) 509.5

**Ans. (2)**

**8.** The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is :

(1) $2\sqrt{11}$ (2) $3\sqrt{2}$

(3) $6\sqrt{3}$ (4) $8\sqrt{2}$

**Ans. (3)**

**9.** Let $A=\begin{bmatrix}2 & b & 1\\ b & b^2+1 & b\\ 1 & b & 2\end{bmatrix}$ where $b > 0$. Then the minimum value of $\frac{\det(A)}{b}$ is :

(1) $\sqrt{3}$ (2) $-\sqrt{3}$

(3) $-2\sqrt{3}$ (4) $2\sqrt{3}$

**Ans. (4)**

**10.** The tangent to the curve, $y = xe^{x^2}$ passing through the point (1,e) also passes through the point :

(1) $\left(\frac{4}{3},2e\right)$ (2) (2,3e)

(3) $\left(\frac{5}{3},2e\right)$ (4) (3,6e)

**Ans. (1)**

**11.** The number of values of $\theta\in(0,\pi)$ for which the system of linear equations

$x + 3y + 7z = 0$

$-x + 4y + 7z = 0$

$(\sin 3\theta)x + (\cos 2\theta)\, y + 2z = 0$

has a non-trivial solution, is :

(1) One (2) Three

(3) Four (4) Two

**Ans. (4)**

**12.** If $\int_0^x f(t)dt = x^2 + \int_x^1 t^2 f(t)dt$, then f'(1/2) is :

(1) $\frac{6}{25}$ (2) $\frac{24}{25}$

(3) $\frac{18}{25}$ (4) $\frac{4}{5}$

**Ans. (2)**

**13.** Let $f : (-1,1)\to R$ be a function defined by $f(x) = \max\left\{-|x|,-\sqrt{1-x^2}\right\}$. If K be the set of all points at which f is not differentiable, then K has exactly :

(1) Three elements (2) One element

(3) Five elements (4) Two elements

**Ans. (1)**

**14.** Let $S=\left\{(x,y)\in R^2 : \frac{y^2}{1+r}-\frac{x^2}{1-r}=1\right\}$, where $r \neq \pm 1$. Then S represents :

(1) A hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, where $0 < r < 1$.

(2) An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, where $r > 1$

(3) A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when $0 < r < 1$.

(4) An ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when $r > 1$

**Ans. (4)**

**15.** If $\sum_{r=0}^{25}\left\{{}^{50}C_r \cdot {}^{50-r}C_{25-r}\right\} = K\left({}^{50}C_{25}\right)$, then K is equal to :

(1) $2^{25} - 1$ (2) $(25)^2$ (3) $2^{25}$ (4) $2^{24}$

**Ans. (3)**

**16.** Let N be the set of natural numbers and two functions f and g be defined as $f,g : N\to N$ such that : $f(n)=\begin{cases}\frac{n+1}{2} & \text{if } n \text{ is odd}\\ \frac{n}{2} & \text{if } n \text{ is even}\end{cases}$

and $g(n) = n-(-1)^n$. The fog is :

(1) Both one-one and onto

(2) One-one but not onto

(3) Neither one-one nor onto

(4) onto but not one-one

**Ans. (4)**

**17.** The values of $\lambda$ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is :

(1) 2 (2) $\frac{4}{9}$

(3) $\frac{15}{8}$ (4) 1

**Ans. (1)**

**18.** Two vertices of a triangle are (0,2) and (4,3). If its orthocentre is at the origin, then its third vertex lies in which quadrant ?

(1) Fourth

(2) Second

(3) Third

(4) First

**Ans. (2)**

**19.** Two sides of a parallelogram are along the lines, $x + y = 3$ and $x - y + 3 = 0$. If its diagonals intersect at (2,4), then one of its vertex is :

(1) (2,6) (2) (2,1)

(3) (3,5) (4) (3,6)

**Ans. (4)**

**20.** Let $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of $\lambda$ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is :

(1) –3 (2) 4

(3) 3 (4) –4

**Ans. (4)**

**21.** The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is :

(1) $\frac{22}{23}$ (2) $\frac{23}{22}$ (3) $\frac{21}{19}$ (4) $\frac{19}{21}$

**Ans. (3)**

**22.** With the usual notation, in $\Delta ABC$, if $\angle A + \angle B = 120^\circ$, $a = \sqrt{3} + 1$ and $b = \sqrt{3} - 1$, then the ratio $\angle A : \angle B$, is :

(1) 7 : 1 (2) 5 : 3

(3) 9 : 7 (4) 3 : 1

**Ans. (1)**

**23.** The plane which bisects the line segment joining the points (–3,–3,4) and (3,7,6) at right angles, passes through which one of the following points ?

(1) (4, –1,7) (2) (4,1,–2)

(3) (–2,3,5) (4) (2,1,3)

**Ans. (2)**

**24.** Consider the following three statements :

P : 5 is a prime number.

Q : 7 is a factor of 192.

R : L.C.M. of 5 and 7 is 35.

Then the truth value of which one of the following statements is true ?

(1) $(P \wedge Q) \vee (\sim R)$

(2) $(\sim P) \wedge (\sim Q \wedge R)$

(3) $(\sim P) \vee (Q \wedge R)$

(4) $P \vee (\sim Q \wedge R)$

**Ans. (4)**

**25.** On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane, $x + y + z = 2$ ?

(1) $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$

(2) $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$

(3) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$

(4) $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$

**Ans. (3)**

**26.** Let f be a differentiable function such that

$f'(x)=7-\frac{3}{4}\frac{f(x)}{x},(x>0)$ and $f(1) \neq 4$.

Then $\lim\limits_{x\to 0^+} x f\left(\frac{1}{x}\right)$:

(1) Exists and equals 4

(2) Does not exist

(3) Exist and equals 0

(4) Exists and equals $\frac{4}{7}$

**Ans. (1)**

**27.** A helicopter is flying along the curve given by $y - x^{3/2} = 7$, $(x \geq 0)$. A soldier positioned at the point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is :

(1) $\frac{1}{2}$ (2) $\frac{1}{3}\sqrt{\frac{7}{3}}$

(3) $\frac{1}{6}\sqrt{\frac{7}{3}}$ (4) $\frac{\sqrt{5}}{6}$

**Ans. (3)**

**28.** If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then f(x) is equal to :

(1) $-4x^3 - 1$ (2) $4x^3 + 1$

(3) $-2x^3 - 1$ (4) $-2x^3 + 1$

**Ans. (1)**

**29.** The curve amongst the family of curves, represented by the differential equation, $(x^2 - y^2)dx + 2xy\, dy = 0$ which passes through (1,1) is :

(1) A circle with centre on the y-axis

(2) A circle with centre on the x-axis

(3) An ellipse with major axis along the y-axis

(4) A hyperbola with transverse axis along the x-axis

**Ans. (2)**

**30.** If the area of an equilateral triangle inscribed in the circle, $x^2 + y^2 + 10x + 12y + c = 0$ is $27\sqrt{3}$ sq. units then c is equal to :

(1) 20 (2) 25

(3) 13 (4) –25

**Ans. (2)**