TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Wednesday 09th JANUARY, 2019) TIME : 9 : 30 AM To 12 : 30 PM

MATHEMATICS

1. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is :

(1) $\frac{14}{3}$ (2) $\frac{56}{3}$ (3) $\frac{8}{3}$ (4) $\frac{32}{3}$

Ans. (3)

2. The maximum volume (in cu. m) of the right circular cone having slant height 3m is :

(1) $3\sqrt{3}\ \pi$ (2) $6\ \pi$

(3) $2\sqrt{3}\ \pi$ (4) $\frac{4}{3}\ \pi$

Ans. (3)

3. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral

$$\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2(x^2-1)}{2\sin(x^2-1)+\sin 2(x^2-1)}}dx$$

is equal to :

(where c is a constant of integration)

(1) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$

(2) $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right|+c$

(3) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

(4) $\frac{1}{2}\log_e\left|\sec(x^2-1)\right|+c$

Ans. (1)

4. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to :

(1) 512 (2) –512 (3) –256 (4) 256

Ans. (3)

5. If $y = y(x)$ is the solution of the differential equation,

$x\frac{dy}{dx}+2y=x^2$ satisfying

$y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to :

(1) $\frac{7}{64}$ (2) $\frac{13}{16}$ (3) $\frac{49}{16}$ (4) $\frac{1}{4}$

Ans. (3)

6. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is:

(1) $2\sqrt{3}y = 12x + 1$

(2) $2\sqrt{3}y = -x - 12$

(3) $\sqrt{3}y = x + 3$

(4) $\sqrt{3}y = 3x + 1$

Ans. (3)

7. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is:

(1) 200 (2) 300 (3) 500 (4) 350

Ans. (2)

8. Three circles of radii a, b, c(a < b < c) touch each other externally. If they have x-axis as a common tangent, then :

(1) $\frac{1}{\sqrt{a}}=\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{c}}$

(2) a, b, c are in A. P.

(3) $\sqrt{a},\sqrt{b},\sqrt{c}$ are in A. P.

(4) $\frac{1}{\sqrt{b}}=\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{c}}$

Ans. (1)

9. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to :

(1) 14 (2) 6 (3) 4 (4) 8

Ans. (4)

10. Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it ?

(1) (4, –4) (2) $(5, 2\sqrt{6})$

(3) (8, 6) (4) $(6, 4\sqrt{2})$

Ans. (3)

11. The plane through the intersection of the planes x + y + z = 1 and 2x + 3y – z + 4 = 0 and parallel to y-axis also passes through the point :

(1) (–3, 0, –1) (2) (3, 3, –1)

(3) (3, 2, 1) (4) (–3, 1, 1)

Ans. (3)

12. If a, b and c be three distinct real numbers in G. P. and a + b + c = xb, then x cannot be :

(1) 4 (2) –3 (3) –2 (4) 2

Ans. (4)

13. Consider the set of all lines px + qy + r = 0 such that 3p + 2q + 4r = 0. Which one of the following statements is true ?

(1) The lines are all parallel.

(2) Each line passes through the origin.

(3) The lines are not concurrent

The lines are concurrent at the point

(4) $\left(\frac{3}{4}, \frac{1}{2}\right)$

Ans. (4)

14. The system of linear equations.

x + y + z = 2

2x + 3y + 2z = 5

$2x + 3y + (a^2 - 1)z = a + 1$

(1) has infinitely many solutions for a = 4

(2) is inconsistent when $|a| = \sqrt{3}$

(3) is inconsistent when a = 4

(4) has a unique solution for $|a| = \sqrt{3}$

Ans. (2)

15. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a}.\vec{c} = 4$, then $|\vec{c}|^2$ is equal to :-

(1) $\frac{19}{2}$ (2) 8 (3) $\frac{17}{2}$ (4) 9

Ans. (1)

16. Let $a_1, a_2, \ldots\ldots\ldots, a_{30}$ be an A. P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and S – 2T = 75, then a_{10} is equal to :

(1) 57 (2) 47 (3) 42 (4) 52

Ans. (4)

17. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is:

(1) 22 (2) 20 (3) 16 (4) 18

Ans. (2)

18. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P(X = 1) + P(X = 2) equals :

(1) 52/169 (2) 25/169

(3) 49/169 (4) 24/169

Ans. (2)

19. For $x \in R - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 \circ J \circ f_1)(x) = f_3(x)$ then J(x) is equal to :-

(1) $f_3(x)$ (2) $f_1(x)$

(3) $f_2(x)$ (4) $\frac{1}{x} f_3(x)$

Ans. (1)

20. Let

$$A = \left\{\theta \in \left(-\frac{\pi}{2}, \pi\right) : \frac{3+2i\sin\theta}{1-2i\sin\theta} \text{ is purely imaginary}\right\}$$

Then the sum of the elements in A is :

(1) $\frac{5\pi}{6}$ (2) $\frac{2\pi}{3}$ (3) $\frac{3\pi}{4}$ (4) π

Ans. (2)

21. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan\theta|$ is equal to :

(1) 4/9 (2) 7/17 (3) 8/17 (4) 8/15

Ans. (4)

22. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to :

(1) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (2) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

(3) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (4) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Ans. (1)

23. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval :

(1) (2, 3] (2) (3, ∞)

(3) (3/2, 2] (4) (1, 3/2]

Ans. (2)

24. The equation of the line passing through (–4, 3, 1), parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is:

(1) $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

(2) $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

(3) $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

(4) $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

Ans. (2)

25. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals :

(1) $13 - 4\cos^6\theta$

(2) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

(3) $13 - 4\cos^2\theta + 6\cos^4\theta$

(4) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

Ans. (1)

26. If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}\left(x > \frac{3}{4}\right)$ then x is equal to :

(1) $\frac{\sqrt{145}}{12}$ (2) $\frac{\sqrt{145}}{10}$

(3) $\frac{\sqrt{146}}{12}$ (4) $\frac{\sqrt{145}}{11}$

Ans. (1)

27. The value of $\int_0^{\pi} |\cos x|^3\, dx$

(1) 2/3 (2) 0 (3) –4/3 (4) 4/3

Ans. (4)

28. If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$, then the ordered pair $(\oplus, \odot)$ is:

(1) $(\wedge, \vee)$ (2) $(\vee, \vee)$

(3) $(\wedge, \wedge)$ (4) $(\vee, \wedge)$

Ans. (1)

29. $\lim_{y \to 0} \frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

(1) exists and equals $\frac{1}{4\sqrt{2}}$

(2) does not exist

(3) exists and equals $\frac{1}{2\sqrt{2}}$

(4) exists and equals $\frac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

Ans. (1)

30. Let f : R → R be a function defined as :

$$f(x) = \begin{cases} 5, & \text{if } x \le 1 \\ a+bx, & \text{if } 1 < x < 3 \\ b+5x, & \text{if } 3 \le x < 5 \\ 30, & \text{if } x \ge 5 \end{cases}$$

Then, f is :

(1) continuous if a = 5 and b = 5

(2) continuous if a = –5 and b = 10

(3) continuous if a = 0 and b = 5

(4) not continuous for any values of a and b

Ans. (4)