TEST PAPER OF JEE(MAIN) EXAMINATION – 2019

(Held On Wednesday 09th JANUARY, 2019) TIME : 02 : 30 PM To 05 : 30 PM

MATHEMATICS

1. Let f be a differentiable function from R to R such that $|f(x)-f(y)| \le 2|x-y|^{\frac{3}{2}}$, for all x, y ε R. If $f(0) = 1$ then $\int_0^1 f^2(x)dx$ is equal to

(1) 0 (2) $\frac{1}{2}$ (3) 2 (4) 1

Ans. (4)

2. If $\int_0^{\frac{\pi}{3}} \frac{\tan\theta}{\sqrt{2k\sec\theta}}d\theta = 1-\frac{1}{\sqrt{2}}, (k>0)$, then the value of k is :

(1) 2 (2) $\frac{1}{2}$ (3) 4 (4) 1

Ans. (1)

3. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is

(1) 12 (2) 15 (3) 10 (4) 14

Ans. (2)

4. For each xεR, let [x] be the greatest integer less than or equal to x. Then

$\lim\limits_{x\to 0^-} \frac{x([x]+|x|)\sin[x]}{|x|}$ is equal to

(1) – sin1 (2) 0 (3) 1 (4) sin1

Ans. (1)

5. If both the roots of the quadratic equation $x^2 – mx + 4 = 0$ are real and distinct and they lie in the interval [1,5], then m lies in the interval:

(1) (4,5) (2) (3,4) (3) (5,6) (4) (–5,–4)

Ans. (Bonus)

6. If

$$A=\begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$$

Then A is-

(1) Invertible only if $t=\frac{\pi}{2}$

(2) not invertible for any tεR

(3) invertible for all tεR

(4) invertible only if t=π

Ans. (3)

7. The area of the region

$A=\left[(x,y): 0 \le y \le x|x|+1 \text{ and } -1 \le x \le 1\right]$

in sq. units, is :

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) 2 (4) $\frac{4}{3}$

Ans. (3)

8. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6iz_0^{81} - 3iz_0^{93}$, then arg z is equal to:

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) 0 (4) $\frac{\pi}{6}$

Ans. (1)

9. Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a} + \vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to:

(1) $\sqrt{22}$ (2) 4 (3) $\sqrt{32}$ (4) 6

Ans. (4)

10. Let A(4,–4) and B(9,6) be points on the parabola, $y^2 + 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB, is:

(1) $31\frac{3}{4}$ (2) 32 (3) $30\frac{1}{2}$ (4) $31\frac{1}{4}$

Ans. (4)

11. The logical statement

$[\sim(\sim p \vee q) \vee (p \wedge r) \wedge (\sim q \wedge r)]$ is equivalent to:

(1) $(p \wedge r) \wedge \sim q$ (2) $(\sim p \wedge \sim q) \wedge r$

(3) $\sim p \vee r$ (4) $(p \wedge \sim q) \vee r$

Ans. (1)

12. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added to the urn; the original ball is not returned to the urn. Now, a second ball is drawn at random from it. The probability that the second ball is red, is :

(1) $\frac{26}{49}$ (2) $\frac{32}{49}$ (3) $\frac{27}{49}$ (4) $\frac{21}{49}$

Ans. (2)

13. If $0 \le x < \frac{\pi}{2}$, then the number of values of x for which $\sin x - \sin 2x + \sin 3x = 0$, is

(1) 2 (2) 1 (3) 3 (4) 4

Ans. (1)

14. The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is:

(1) $x + 2y - 2z = 0$ (2) $x - 2y + z = 0$

(3) $5x + 2y - 4z = 0$ (4) $3x + 2y - 3z = 0$

Ans. (2)

15. Let the equations of two sides of a triangle be $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$. If the orthocentre of this triangle is at (1,1), then the equation of its third side is :

(1) $122y - 26x - 1675 = 0$

(2) $26x + 61y + 1675 = 0$

(3) $122y + 26x + 1675 = 0$

(4) $26x - 122y - 1675 = 0$

Ans. (4)

16. If $x = 3\tan t$ and $y = 3\sec t$, then the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$, is:

(1) $\frac{3}{2\sqrt{2}}$ (2) $\frac{1}{3\sqrt{2}}$ (3) $\frac{1}{6}$ (4) $\frac{1}{6\sqrt{2}}$

Ans. (4)

17. If $x = \sin^{-1}(\sin 10)$ and $y = \cos^{-1}(\cos 10)$, then $y - x$ is equal to:

(1) π (2) 7π (3) 0 (4) 10

Ans. (1)

18. If the lines $x = ay + b$, $z = cy + d$ and $x = a'z + b'$, $y = c'z + d'$ are perpendicular, then:

(1) $cc' + a + a' = 0$

(2) $aa' + c + c' = 0$

(3) $ab' + bc' + 1 = 0$

(4) $bb' + cc' + 1 = 0$

Ans. (2)

19. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is :

(1) 2 (2) 5 (3) 3 (4) 4

Ans. (3)

20. A hyperbola has its centre at the origin, passes through the point (4,2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is :

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{3}{2}$ (3) $\sqrt{3}$ (4) 2

Ans. (1)

21. Let $A = \{x \in R : x \text{ is not a positive integer}\}$

Define a function $f : A \to R$ as $f(x) = \frac{2x}{x-1}$ then f is

(1) injective but not surjective

(2) not injective

(3) surjective but not injective

(4) neither injective nor surjective

Ans. (1)

22. If $f(x)=\int\frac{5x^8+7x^6}{\left(x^2+1+2x^7\right)^2}dx,(x\geq 0)$ and $f(0) = 0$, then the value of $f(1)$ is :

(1) $-\frac{1}{2}$ (2) $\frac{1}{2}$ (3) $-\frac{1}{4}$ (4) $\frac{1}{4}$

Ans. (4)

23. If the circles $x^2 + y^2 - 16x-20y + 164 = r^2$ and $(x-4)^2 + (y - 7)^2 = 36$ intersect at two distinct points, then:

(1) $0 < r < 1$ (2) $1 < r < 11$

(3) $r > 11$ (4) $r = 11$

Ans. (2)

24. Let S be the set of all triangles in the xy-plane, each having one vertex at the origin and the other two vertices lie on coordinate axes with integral coordinates. If each triangle in S has area 50sq. units, then the number of elements in the set S is :

(1) 9 (2) 18 (3) 32 (4) 36

Ans. (4)

25. The sum of the follwing series

$$1+6+\frac{9\left(1^2+2^2+3^2\right)}{7}+\frac{12\left(1^2+2^2+3^2+4^2\right)}{9}$$

$$+\frac{15\left(1^2+2^2+....+5^2\right)}{11}+....\text{ up to 15 terms, is:}$$

(1) 7820 (2) 7830 (3) 7520 (4) 7510

Ans. (1)

26. Let a, b and c be the 7^{th}, 11^{th} and 13^{th} terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to:

(1) $\frac{1}{2}$ (2) 4 (3) 2 (4) $\frac{7}{13}$

Ans. (2)

27. If the system of linear equations

$x-4y+7z = g$

$3y - 5z = h$

$-2x + 5y - 9z = k$

is consistent, then :

(1) $g + h + k = 0$

(2) $2g + h + k = 0$

(3) $g + h + 2k = 0$

(4) $g + 2h + k = 0$

Ans. (2)

28. Let $f:[0,1]\to R$ be such that $f(xy) = f(x).f(y)$ for all $x,y,\varepsilon[0,1]$, and $f(0)\neq 0$. If $y = y(x)$ satisfies the differential equation, $\frac{dy}{dx}=f(x)$ with $y(0) = 1$, then $y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)$ is equal to

(1) 4 (2) 3 (3) 5 (4) 2

Ans. (2)

29. A data consists of n observations:

$x_1, x_2, ..., x_n$. If $\sum_{i=1}^{n}(x_i+1)^2=9n$ and $\sum_{i=1}^{n}(x_i-1)^2=5n$, then the standard deviation of this data is :

(1) 5 (2) $\sqrt{5}$ (3) $\sqrt{7}$ (4) 2

Ans. (2)

30. The number of natural numbers less than 7,000 which can be formed by using the digits 0,1,3,7,9 (repitition of digits allowed) is equal to :

(1) 250 (2) 374 (3) 372 (4) 375

Ans. (2)