JEE(MAIN) – 2018 TEST PAPER WITH SOLUTIONS (HELD ON SUNDAY 08th APRIL, 2018)

PART B – MATHEMATICS

31. The integral

$$\int \frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$$

is equal to

(1) $\frac{-1}{3(1+\tan^3 x)}+C$ (2) $\frac{1}{1+\cot^3 x}+C$

(3) $\frac{-1}{1+\cot^3 x}+C$ (4) $\frac{1}{3(1+\tan^3 x)}+C$

(where C is a constant of integration)

Ans. **(1)**

Sol. Let $I = \int \frac{\sin^2 x\cos^2 x \quad dx}{[(\sin^2 x+\cos^2 x)(\sin^3 x+\cos^3 x)]^2}$

$= \int \frac{\sin^2 x\cos^2 x}{(\sin^3 x+\cos^3 x)^2}dx$

$= \int \frac{\tan^2 x\cdot\sec^2 x}{(1+\tan^3 x)^3}dx$

put $(1 + \tan^3 x) = t$

$3\tan^2 x\sec^2 x\, dx = dt$

$I = \frac{1}{3}\int \frac{dt}{t^2} = -\frac{1}{3t} + C$

Hence, $I = \frac{-1}{3(1+\tan^3 x)} + C$

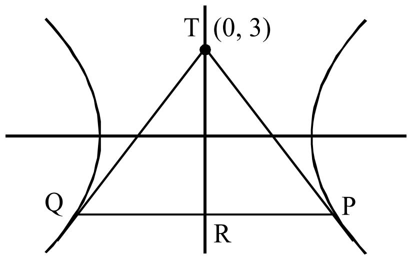
32. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the point P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is -

(1) $54\sqrt{3}$ (2) $60\sqrt{3}$ (3) $36\sqrt{5}$ (4) $45\sqrt{5}$

Ans. **(4)**

Sol. Equation PQ : chord of contact T = 0

$y = -12$



$\frac{x^2}{9}-\frac{y^2}{36}=1$

Area : $\frac{1}{2}$ PQ · TR

TR = 3 + 12 = 15, Point P $(3\sqrt{5},-12)$

$\Rightarrow$ PQ = $6\sqrt{5}$

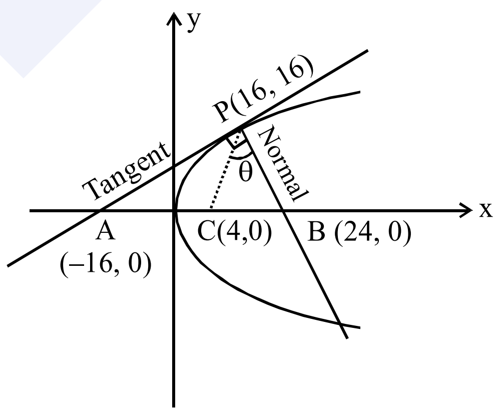
Area of $\Delta PTQ = \frac{1}{2}\cdot 15\cdot 6\sqrt{5} = 45\sqrt{5}$ sq. units

33. Tangent and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is-

(1) 2 (2) 3 (3) $\frac{4}{3}$ (4) $\frac{1}{2}$

Ans. **(1)**

Sol. Equation of tangent at P(16, 16) is $x - 2y + 16 = 0$



$m_{PC} = \frac{4}{3}$

$m_{PB} = -2$

Hence, $\tan\theta = \left|\frac{m_{PC}-m_{PB}}{1+m_{PC}.m_{PB}}\right|$

$\tan\theta = 2$

34. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b} = \hat{j}+\hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}.\vec{b} = 24$, then $|\vec{u}|^2$ is equal to-

(1) 315 (2) 256 (3) 84 (4) 336

Ans. **(4)**

Sol. $\vec{u} = \lambda(\vec{a}\times\vec{b})\times\vec{a}$

$= \lambda\{\vec{a}^2.\vec{b} - (\vec{a}.\vec{b})\vec{a}\}$

$= \lambda\{-4\hat{i} + 8\hat{j} + 16\hat{k}\}$

$\vec{u} = \lambda'\{-\hat{i} + 2\hat{j} + 4\hat{k}\}$

$\vec{u}.\vec{b} = 24$

$\Rightarrow \lambda' = 4$

$\vec{u} = -4\hat{i} + 8\hat{j} + 16\hat{k}$

$\vec{u}^2 = 336$

35. If $\alpha, \beta \in C$ are the distinct roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to-

(1) 0 (2) 1 (3) 2 (4) –1

Ans. (2)

Sol. α, β are roots of $x^2 - x + 1 = 0$

$\therefore \alpha = -\omega$ and $\beta = -\omega^2$

where ω is non-real cube root of unity

so, $a^{101} + \beta^{107}$

$\Rightarrow (-\omega)^{101} + (-\omega^2)^{107}$

$\Rightarrow -[\omega^2 + \omega]$

$\Rightarrow -[-1] = 1$

(As $1 + \omega + \omega^2 = 0$ & $\omega^3 = 1$)

36. Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$ and α, β $(\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha$, $x = \beta$ and $y = 0$ is-

(1) $\frac{1}{2}(\sqrt{3}+1)$ (2) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

(3) $\frac{1}{2}(\sqrt{2}-1)$ (4) $\frac{1}{2}(\sqrt{3}-1)$

Ans. (4)

Sol. $18x^2 - 9\pi x + p^2 = 0$; $gof(x) = \cos x$

$(3x - \pi)(6x - \pi) = 0$

$\alpha = \frac{\pi}{6}$, $\beta = \frac{\pi}{3}$

$A = \int_{\pi/6}^{\pi/3} \cos x \, dx$

$A = \frac{\sqrt{3}-1}{2}$

37. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$, $(x > 1)$ is-

(1) 0 (2) 1 (3) 2 (4) –1

Ans. (3)

Sol. using $(x + a)^5 + (x - a)^5$

$= 2[{}^5C_0\, x^5 + {}^5C_2\, x^3 \cdot a^2 + {}^5C_4 x \cdot a^4]$

$\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$

$= 2[{}^5C_0\, x^5 + {}^5C_2\, x^3(x^3 - 1) + {}^5C_4 x(x^3 - 1)^2]$

$\Rightarrow 2[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x]$

considering odd degree terms,

$2[x^5 + 5x^7 - 10x^3 + 5x]$

$\therefore$ Sum of coefficients of odd terms is 2

38. Let $a_1, a_2, a_3, \ldots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140m$, then m is equal to-

(1) 68 (2) 34 (3) 33 (4) 66

Ans. (2)

Sol. $\sum_{K=0}^{12} a_{4k+1} = 416$

$\Rightarrow \frac{13}{2}[2a_1 + 48d] = 416$

$\Rightarrow a_1 + 24d = 32$ ..... (1)

$a_9 + a_{43} = 66 \Rightarrow 2a_1 + 50d = 66$ ..... (2)

$2a_1 + 48d = 64$ by ...... (1)

$d = 1$ and $a_1 = 8$

$\Rightarrow 140m = \sum_{r=1}^{17} a_r^2 = \sum_{r=1}^{17}[8 + (r-1).1]^2$

$\Rightarrow 140m = \sum_{r=1}^{17}(r+7)^2$

$\Rightarrow 140m = \sum_{r=1}^{24} r^2 - \sum_{r=1}^{7} r^2$

$\Rightarrow 140m = \frac{24.25.49}{6} - \frac{7.8.15}{6}$

$\Rightarrow 140\ m = \frac{7.8.5}{6}[105 - 3]$

$\Rightarrow 140\ m = 280.17 \Rightarrow m = 34$

39. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, \ldots, x_9$ is-

(1) 4 (2) 2 (3) 3 (4) 9

Ans. (2)

Sol. Given $\sum_{i=1}^{9}(x_i - 5) = 9 \Rightarrow \Sigma x_i = 54$ ..... (1)

Also, $\sum_{i=1}^{9}(x_i - 5)^2 = 45$

$\Rightarrow \Sigma x_i^2 - 10.\Sigma x_i + 9(25) = 45$

$\Rightarrow \Sigma x_i^2$ 360 (using (1)) ..... (2)

As, variance = $\frac{\Sigma x_i^2}{9} - \left(\frac{\Sigma x_i}{9}\right)^2$

$= \frac{360}{9} - \left(\frac{54}{9}\right)^2$

= 40 – 36

= 4

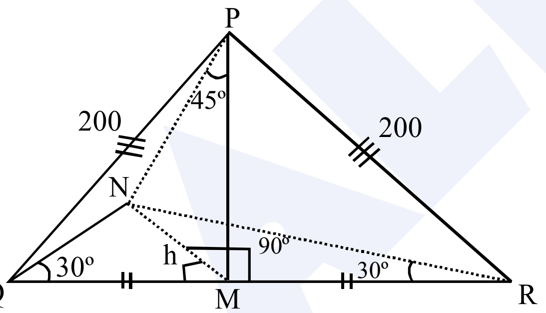
Hence standard deviation is 2 (As standard deviation = $\sqrt{\text{variance}}$

40. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P,Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is-

(1) 50 (2) $100\sqrt{3}$ (3) $50\sqrt{2}$ (4) 100

Ans. **(4)**

Sol.



Let height of tower MN is 'h'

In Δ QMN

$\frac{MN}{QM}$ = tan 30°

$\therefore$ QM = $\sqrt{3}$ h = MR ......(1)

Now in Δ MNP

MN = PM ......(2)

In Δ PMQ

$MP = \sqrt{(200)^2 - (\sqrt{3}h)^2}$

$\therefore$ From (2)

$\sqrt{(200)^2 - (\sqrt{3}h)^2} = h \Rightarrow h = 100$ m

41. Two sets A and B are as under

A = {(a, b) ∈ R × R : |a – 5| < 1 and |b – 5| < 1};

B = {(a, b) ∈ R × R : $4(a - 6)^2 + 9(b - 5)^2 \le 36$}. Then :-

(1) A ⊂ B

(2) A ∩ B = φ (an empty set)

(3) neither A ⊂ B nor B ⊂ A

(4) B ⊂ A

Ans. **(1)**

Sol. A = {(a, b) ∈ R × R : |a – 5| < 1, |b – 5| < 1}

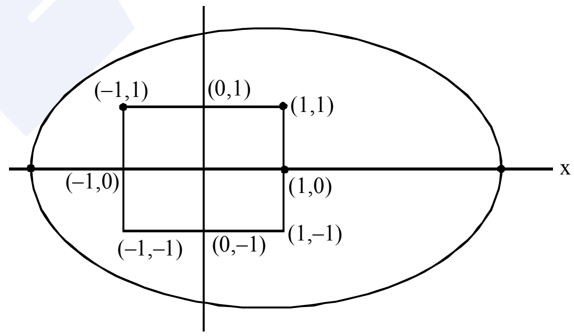
Let a – 5 = x, b – 5 = y

Set A contains all points inside |x| < 1, |y| < 1

B = {(a, b) ∈ R × R : $4(a - 6)^2 + 9(b - 5)^2 \le$ 36}

Set B contains all points inside or on

$$\frac{(x-1)^2}{9} + \frac{y^2}{4} = 1$$



(± 1, ± 1) lies inside the ellipse ⇒ A ⊂ B

42. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is-

(1) less than 500

(2) at least 500 but less than 750

(3) at least 750 but less than 1000

(4) at least 1000

Ans. **(4)**

Sol. Number of ways = $\binom{6}{4}\binom{3}{1}$ 4!

= 15 × 3 × 24

= 1080

43. Let $f(x) = x^2 + \frac{1}{x^2}$ and

$g(x) = x - \frac{1}{x}, x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$,

then the local minimum value of h(x) is :

(1) –3 (2) $-2\sqrt{2}$ (3) $2\sqrt{2}$ (4) 3

Ans. (3)

Sol. $h(x) = \frac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}} = \left(x - \frac{1}{x}\right) + \frac{2}{x - \frac{1}{x}}$

when $x - \frac{1}{x} < 0 \Rightarrow x - \frac{1}{x} + \frac{2}{x - \frac{1}{x}} \le -2\sqrt{2}$

so $-2\sqrt{2}$ will be local maximum value

when $x - \frac{1}{x} > 0 \Rightarrow x - \frac{1}{x} + \frac{2}{x - \frac{1}{x}} \ge 2\sqrt{2}$

so $2\sqrt{2}$ will be local minimum value

44. For each $t \in R$, let [t] be the greatest integer less than or equal to t. Then

$\lim_{x \to 0+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right]\right)$

(1) is equal to 15.

(2) is equal to 120.

(3) does not exist (in R).

(4) is equal to 0.

Ans. (2)

Sol. $\lim_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right]\right)$

$\lim_{x \to 0^+} x\left(\frac{1+2+3+.....+15}{x}\right) - \left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + ... + \left\{\frac{15}{x}\right\}\right)$

$\because \ 0 \le \left\{\frac{r}{x}\right\} < 1$

$0 \le x\left\{\frac{r}{x}\right\} < x$

$\lim_{x \to 0^+} x\left(\frac{1+2+3+......+15}{x}\right) = \frac{15.16}{2} = 120$

45. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is :

(1) $\frac{\pi}{2}$ (2) 4π (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{8}$

Ans. (3)

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x} dx$ ..... (i)

using property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^{-x}} dx$ ..... (ii)

adding (i) and (ii)

$2I = \int_{-\pi/2}^{\pi/2} \sin^2 x\, dx$

$\Rightarrow 2I = 2.\int_0^{\pi/2} \sin^2 x\, dx$

$\Rightarrow 2I = 2 \times \frac{\pi}{4} \Rightarrow I = \frac{\pi}{4}$

46. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is:

(1) $\frac{2}{5}$ (2) $\frac{1}{5}$ (3) $\frac{3}{4}$ (4) $\frac{3}{10}$

Ans. (1)

Sol. Let R_i be the event of drawing read ball in i^{th} draw and B_i be the event of drawing black ball in i^{th} draw.

Now, In bag there are 4R and 6B balls

$\therefore \ P(R_1) = \frac{4}{10}$ and $P(B_1) = \frac{6}{10}$

Now according to given information

$P\left(\frac{R_2}{R_1}\right) = \frac{6}{12}$ and $P\left(\frac{R_2}{B_1}\right) = \frac{4}{12}$

Required probabiflity

$= P(R_1).\,P\left(\frac{R_2}{R_1}\right) + P(B_1).\,P\left(\frac{R_2}{B_1}\right)$

$= \frac{4}{10}\times\frac{6}{12}+\frac{6}{10}\times\frac{4}{12}$

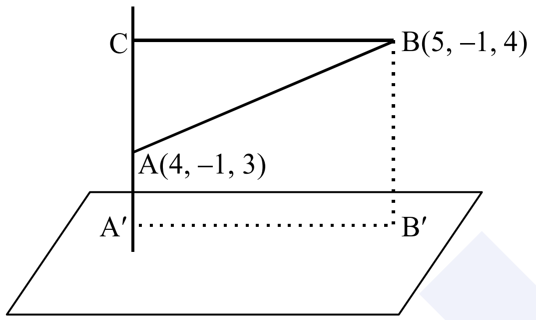
$= \frac{2}{5}$

47. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, x + y + z = 7 is :

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) $\sqrt{\frac{2}{3}}$ (4) $\frac{2}{\sqrt{3}}$

Ans. (3)

Sol.



$AC = \overrightarrow{AB}.\widehat{AC}$

$= (\hat{i}+\hat{k}).\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

$A'B' = BC = \sqrt{AB^2 - AC^2} = \sqrt{2-\frac{4}{3}} = \sqrt{\frac{2}{3}}$

Length of projection $= \sqrt{\frac{2}{3}}$

48. If sum of all the solutions of the equation $8\cos x\cdot\left(\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right) = 1$ in $[0, \pi]$ is $k\pi$, then k is equal to :

(1) $\frac{13}{9}$ (2) $\frac{8}{9}$ (3) $\frac{20}{9}$ (4) $\frac{2}{3}$

Ans. (1)

Sol. $8\cos x\left(\cos^2\frac{\pi}{6}-\sin^2 x-\frac{1}{2}\right)=1$

$\Rightarrow 8\cos x\left(\frac{1}{4}-(1-\cos^2 x)\right) = 1$

$\Rightarrow 8\cos x\left(\cos^2 x-\frac{3}{4}\right) = 1$

$\Rightarrow 2\cos 3x = 1 \Rightarrow \cos 3x = \frac{1}{2}$

$\therefore\ 3x + 2n\pi \pm \frac{\pi}{3},\ n \in I$

$\Rightarrow x = \frac{2n\pi}{3}\pm\frac{\pi}{9}$

In $x \in [0, \pi] : x = \frac{\pi}{9}, \frac{2\pi}{3}+\frac{\pi}{9}, \frac{2\pi}{3}-\frac{\pi}{9}$ only

$\text{sum} = \frac{13\pi}{9}$

49. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is :

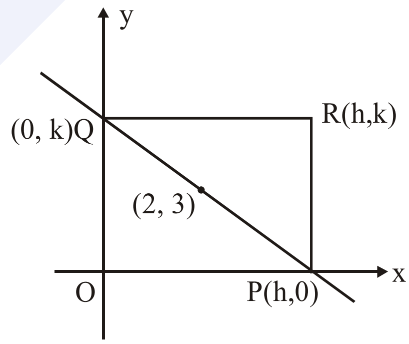
(1) 2x + 3y = xy (2) 3x + 2y = xy

(3) 3x + 2y = 6xy (4) 3x + 2y = 6

Ans. (2)

Sol. Equation of PQ is

$\frac{x}{h}+\frac{y}{k}=1$



passes through (2, 3) so $\frac{2}{h}+\frac{3}{k}=1$

So locus $\frac{2}{x}+\frac{3}{y}=1$

3x + 2y = xy

50. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$1^2 + 2\cdot2^2 + 3^2 + 2\cdot4^2 + 5^2 + 2\cdot6^2 + \ldots\ldots$

If B – 2A = 100λ, then λ is equal to :

(1) 248 (2) 464 (3) 496 (4) 232

Ans. (1)

Sol. $B - 2A = \sum_{r=1}^{40} T_r - 2\sum_{r=1}^{20} T_r$

$= \sum_{r=21}^{40} T_r - \sum_{r=1}^{20} T_r$

$$B - 2A = (21^2 + 2.22^2 + 23^2 + 2.24^2 + \ldots\ldots + 40^2) - (1^2 + 2.2^2 + 3^2 + 2.4^2 \ldots\ldots + 20^2)$$

$$= 20[22 + 2.24 + 26 + 2.28 + \ldots\ldots\ldots + 60]$$

$$= 20\left[\underbrace{(22+24+26.....60)}_{20\text{ terms}} + \underbrace{(24+28+......+60)}_{10\text{ terms}}\right]$$

$$= 20\left[\frac{20}{2}(22+60)+\frac{10}{2}(24+60)\right]$$

$$= 10[20.82 + 10.84]$$

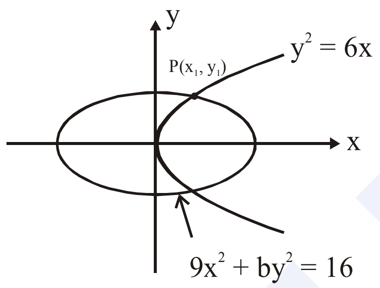
$$= 100[164 + 84] = 100.248$$

51. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is :

(1) $\frac{7}{2}$ (2) 4 (3) $\frac{9}{2}$ (4) 6

Ans. **(3)**

Sol. Let curve intersect each other at point $P(x_1, y_1)$



$$y_1^2 = 6x_1 \qquad \ldots\ldots \text{(i)}$$

$$\& \quad 9x_1^2 + by_1^2 = 16 \qquad \ldots\ldots \text{(ii)}$$

Now differentiate both curve and get slope of tangent to both curve at point $P(x_1, y_1)$

$$\therefore \left(\frac{dy}{dx}\right)_{(x,y)} = m_1 = \frac{3}{y_1} \qquad \ldots\ldots \text{(iii)}$$

$$\& \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = m_2 = -\frac{9x_1}{by_1} \qquad \ldots\ldots \text{(iv)}$$

$$\because \; m_1m_2 = -1 \Rightarrow \frac{27x_1}{by_1^2} = 1$$

$$\therefore \text{ from equation (i)} \quad b = \frac{9}{2}$$

52. Let the orthocentre and centroid of a triangle be A(–3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is:

(1) $2\sqrt{10}$ (2) $3\sqrt{\frac{5}{2}}$ (3) $\frac{3\sqrt{5}}{2}$ (4) $\sqrt{10}$

Ans. **(2)**

Sol. Orthocentre A(–3, 5) centroid B(3, 3) and AB $= \sqrt{40} = 2\sqrt{10}$

Centroid divides orthocentre and circumcentre in ratio 2 :1

$\therefore$ AB : BC = 2 : 1

Now $AB = \frac{2}{3}AC$

$$AC = \frac{3}{2}.AB = \frac{3}{2}\left(2\sqrt{10}\right)$$

$$AC = 3\sqrt{10}$$

Radius of circle with AC as diametre is

$$\frac{3}{2}\sqrt{10} = 3\sqrt{\frac{5}{2}}$$

53. Let $S = \{t \in R : f(x) = |x - \pi|\cdot(e^{|x|} - 1)\sin|x|$ is not differentiable at $t\}$. Then the set S is equal to:

(1) {0} (2) {π}

(3) {0, π} (4) φ (an empty set)

Ans. **(4)**

Sol. $f(x) = |x - \pi|\,(e^{|x|} - 1)\sin|x|$

we check differentiability at $x = \pi$ & $x = 0$

<u>at $x = \pi$</u>

$$\text{R.H.D} = \lim_{h\to 0^+}\frac{|\pi+h-\pi|(e^{|\pi+h|}-1)\sin|\pi+h|-0}{h}$$

$$= 0$$

$$\text{L.H.D} = \lim_{h\to 0^+}\frac{|\pi-h-\pi|(e^{|\pi-h|}-1)\sin|\pi-h|-0}{-h}$$

$$= 0$$

$\because$ RHD = LHD so function is differentiable at $x = \pi$

<u>at $x = 0$</u>

$$\text{R.H.D} = \lim_{h\to 0^+}\frac{|h-\pi|(e^{|h|}-1)\sin|h|-0}{h} = 0$$

$$\text{L.H.D} = \lim_{h\to 0^+}\frac{|-h-\pi|(e^{|-h|}-1)\sin|-h|-0}{-h} = 0$$

$\because$ RHD = LHD so function is differentiable at x= 0

set S is empty set, φ

54. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$,

then the ordered pair (A, B) is equal to :

(1) (–4, 3) (2) (–4, 5)

(3) (4, 5) (4) (–4, –5)

Ans. **(2)**

Sol. $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$

Put x = 0 $\Rightarrow \begin{vmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{vmatrix} = A^3 \Rightarrow A = -$

4

$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (Bx - 4)(x + 4)^2$

$\begin{vmatrix} 1-\frac{4}{x} & 2 & 2 \\ 2 & 1-\frac{4}{x} & 2 \\ 2 & 2 & 1-\frac{4}{x} \end{vmatrix} = \left(B-\frac{4}{x}\right)\left(1+\frac{4}{x}\right)^2$

Put $x \to \infty \Rightarrow \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = B \Rightarrow B = 5$

ordered pair (A, B) is (–4, 5)

55. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to :

(1) p (2) q

(3) ~q (4) ~p

Ans. **(4)**

Sol. $\sim(p \vee q) \vee (\sim p \wedge q)$

$(\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$\Rightarrow \sim p \wedge (\sim q \vee q)$

$\Rightarrow \sim p \wedge t \equiv \sim p$

56. If the system of linear equations

x + ky + 3z = 0

3x + ky – 2z = 0

2x + 4y – 3z = 0

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to :

(1) 10 (2) – 30

(3) 30 (4) –10

Ans. **(1)**

Sol. For non zero solution $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$

$\Rightarrow$ k = 11

Now equations

x + 11y + 3z = 0 .....(1)

3x + 11y – 2z = 0 .....(2)

2x + 4y – 3z = 0 .....(3)

on equation (1) + (3) we get 3x + 15y = 0

$\Rightarrow$ x = – 5y

Now put x = – 5y in equation (1)

we get – 5y + 11y + 3z = 0

$\Rightarrow$ z = – 2y

$\frac{xz}{y^2} = \frac{(-5y)(-2y)}{y^2} = 10$

57. Let S = {x ∈ R : x ≥ 0 and $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$}. Then S :

(1) contains exactly one element.

(2) contains exactly two elements.

(3) contains exactly four elements.

(4) is an empty set.

Ans. **(2)**

Sol. Case-I : x ∈ [0, 9]

$2(3 - \sqrt{x}) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 8\sqrt{x} + 12 = 0 \Rightarrow \sqrt{x} = 4, 2$

$x = 16, 4 \Rightarrow x = 4$

↑ rejected (x = 16)

Case-II : x ∈ [9, ∞]

$2(\sqrt{x} - 3) + x - 6\sqrt{x} + 6 = 0$

$x - 4\sqrt{x} = 0 \Rightarrow x = 16, 0$

↑ rejected (x = 0)

So x = 4, 16

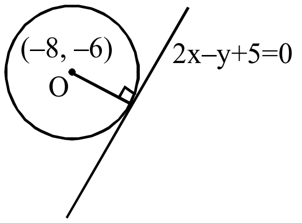
58. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is :

(1) 185 (2) 85

(3) 95 (4) 195

Ans. **(3)**

Sol. Equation of tangent at (1, 7) to $x^2 = y - 6$ is $2x - y + 5 = 0$.



Now, perpendicular from centre O(–8, –6) to $2x - y + 5 = 0$ should be equal to radius of the circle

$$\left|\frac{-16+6+5}{\sqrt{5}}\right| = \sqrt{64+36-C}$$

$$\sqrt{5} = \sqrt{100-C}$$

then, C = 95

59. Let y = y(x) be the solution of the differential equation $\sin x\frac{dy}{dx} + y\cos x = 4x$, $x \in (0, \pi)$.

If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $\frac{-8}{9\sqrt{3}}\pi^2$ (2) $-\frac{8}{9}\pi^2$

(3) $-\frac{4}{9}\pi^2$ (4) $\frac{4}{9\sqrt{3}}\pi^2$

Ans. **(2)**

Sol. $\sin x\,dy + y\cos x\,dx = 4x\,dx$

$\Rightarrow d(y.\sin x) = 4x\,dx$

Integrate we get

$\Rightarrow y.\sin x = 2x^2 + C$

$\Rightarrow$ passes through $\left(\frac{\pi}{2},0\right) \Rightarrow 0 = \frac{\pi^2}{2}+C$

$\Rightarrow C = -\frac{\pi^2}{2}$

$\Rightarrow y\sin x = 2x^2 - \frac{\pi^2}{2}$ is the solution

$$y\left(\frac{\pi}{6}\right)=\left(2.\frac{\pi^2}{36}-\frac{\pi^2}{2}\right)2 = -\frac{8\pi^2}{9}$$

60. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2 is :

(1) $\frac{1}{3\sqrt{2}}$ (2) $\frac{1}{2\sqrt{2}}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{4\sqrt{2}}$

Ans. **(1)**

60. Plane passes through line of intersectuion of first two planes is

$(2x - 2y + 3z - 2) + \lambda(x - y + z + 1) = 0$

$x(\lambda + 2) - y(2 + \lambda) + z(\lambda + 3) + (\lambda - 2) = 0$ ..... (1)

is having infinite number of solution with $x + 2y - z - 3 = 0$ and $3x - y + 2z - 1 = 0$ then

$$\begin{vmatrix} (\lambda+2) & -(\lambda+2) & (\lambda+3) \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

Solving $\lambda = 5$

$7x - 7y + 8z + 3 = 0$

perpendicular distance from (0, 0, 0)

is $\frac{3}{\sqrt{162}} = \frac{1}{3\sqrt{2}}$