SCORE JEE (Advanced)

HOME ASSIGNMENT # 05

(SOLUTION)

MECHANICAL WAVE

EXERCISE # (O)

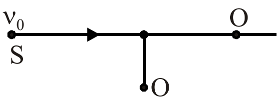
1. **Ans. (C)**

Sol. Since the two diametrical points are fixed, there will be nodes at these points. For the fundamental note $\frac{\lambda}{2} = \pi R = \frac{\pi D}{2}$ and all the integral multiples of the fundamental will be produced. Hence for the nth harmonic, the frequency $1 = n\left[\frac{v}{\lambda}\right] = n\left[\frac{v}{\pi D}\right]$

2. **Ans. (A)**

Sol. Replace : $x \to \left[x - v\left(t - t_0\right)\right]$

3. **Ans. (B, C)**

Sol. 

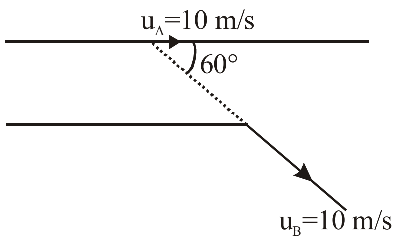
When source is nearest than frequency heard.

$$f' = \frac{fC}{C - V_s \cos\theta}$$

after crossing this point frequency obtained corresponding to wave emitted from this point is $f' = f_0$

4. **Ans. (B,D)**

At the bend, when the second train B passes the bend and moves at 60° to the first train *A*.



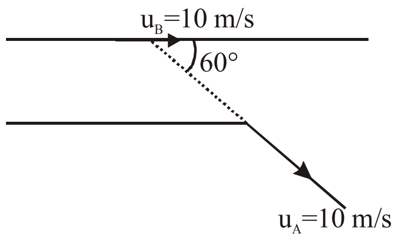
The apparent frequency heard by the passenger on train B, is given by $n_a = \left[\frac{V - u_B}{V + u_A \cos 60°}\right] n$

Given V = 300 m/s, $u_A = u_B$ = 36 km/h = 10 m/s.

Thus $n_a = \left[\frac{300-10}{300+5}\right]$1 kHz$=\left(\frac{290}{305}\right)(1000) = 950.82$ Hz

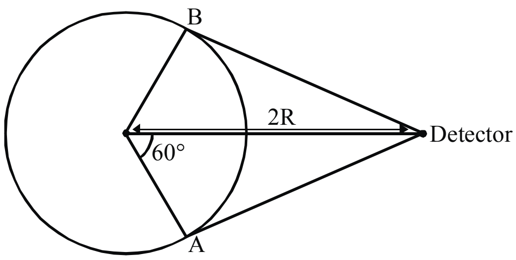
The passenger on train A is in the same train as source and so will always hear the source frequency of 1 kHz.

If the train *A* turns on the bend on the second track while the passenger on train *B* hears the sound while moving straight on the first track, the apparent frequency heard by him will be



$$n_a = \left[\frac{V - u_B \cos 60°}{V + u_A}\right] n = \left[\frac{300-5}{300+10}\right] kHz = \left(\frac{305}{310}\right) kHz = 983.87 \text{ Hz} < 1 \text{ kHz}$$

5. Ans. (A,B,C,D)



Sound emitted by source at A will result in maximum frequency while sound emitted by source at B will result in minimum frequency.

Time taken by sound to reach from A to detector = $\frac{2R\sin 60°}{330}$

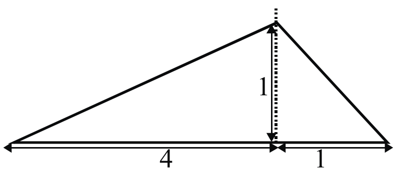
Angle travelled by source by this = $\frac{330\pi}{6\sqrt{3}R} \times \frac{2R\sin 60°}{330} = \frac{\pi}{6}$

$$f_{max} = \left(\frac{330}{330 - \frac{330\pi}{6\sqrt{3}}}\right) f \; ; \; f_{min} = \left(\frac{330}{330 + \frac{330\pi}{6\sqrt{3}}}\right) f$$

6. Ans. (A, C, D)

7. Ans. (AC)

The shape of the string will be



Area = $\frac{1}{2} \times 5 \times 1 = 2.5 \text{ cm}^2$

Wave velocity = $5\sqrt{\frac{5 \times 1000}{50}}$ m/s = 10 m/s

Thus the part with slope $\frac{1}{4}$ will be present at x = 13 & t = 0.015.

$$v_P = \frac{1}{4}\frac{dx}{dt} = \frac{-1}{4} v_{wave} = -250 \text{ cm/sec}$$

8. Ans. (CD)

Equation of SHM of particle who is at antinode is $y=2A\sin\left(\frac{2\pi}{T}\right)t$ at time $t = \frac{T}{8}$

$y = 2A\sin\frac{\pi}{4} = \sqrt{2}A$

Displacement of particle at note is always zero.

9. Ans. (B)

10. Ans. (B)

11. Ans. (B)

12. Ans. (B)

13. Ans. (A)

14. Ans. (C)

15. Ans. (C)

16. Ans. (A)

$f = \frac{C - V_R}{C + V_R} f_0 ; \frac{\Delta f}{f_0} = \frac{2V_R}{C + V_R} \approx \frac{2V_R}{C} \Rightarrow C = \frac{2 \times 0.1}{600} \times 5 \times 10^6 \approx 1700$ m/s

17. Ans. (D)

$f' = \frac{C - V_R}{C + V_R} f_0 ; \frac{\Delta f}{f_0} = \frac{2V_R}{C + V_R}$

velocity is 4 times $(A_1v_1 = A_2v_2) \Rightarrow \Delta f = 4\Delta f_0 = 2400$ Hz

18. Ans. (A)

$\frac{1}{2}\frac{dm}{dt}(v_2^2 - v_1^2) = \frac{1}{2} \times 1.5 \times 10^3 \times 0.1 \times 10^{-4} \times 0.1\ [(4 \times 0.1)^2 - (0.1)^2]$

$= 0.75 \times 10^{-5}\ [15] = 11.25 \times 10^{-5} = 1.125 \times 10^{-4}$ W]

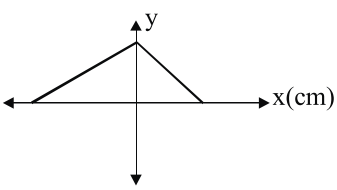
19. Ans. (B)

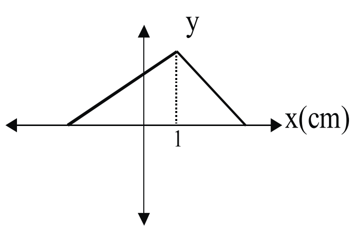
20. Ans. (A)

21. Ans. (C)

Solution 19 to 21

At t =0;





velocity of wave = 10 m/s. Thus pulse get shifted by (10×10^{-3})m

Wave equation; obtained by replacing x by (x–vt)

Pulse reaches x=20m at t = 2 sec

First half of the wave reaches at t = 1.999 sec; V = particle = $\frac{dx}{dt} = v_{wave} = 10$ m/s

Second half of the wave reaches at t=2 & leaves at t = 2.004 sec; $V = \frac{1}{4}\frac{dx}{dt} = -\frac{5}{2}$

22. Ans. (B)

23. Ans. (C)

24. Ans. (D)

Sol.(i) $v_1 = \frac{w}{k} = \frac{4}{2} = 2m/s$

$v_2 = \frac{10}{4} = 2.5\,m/s$

$v_3 = \frac{12}{6} = 2m/s$

$v_4 = \frac{16}{8} = 2m/s$

$v_5 = \frac{20}{10} = 2\ m/s$

$v = \sqrt{\frac{T}{\mu}}$ so T_2 is diff.

(ii) $E_1 = \frac{1}{2}\mu v A^2 w^2 T$

$\frac{1}{2}\mu v A^2 \times 4\pi f' t = 2\mu v \times A^2 \pi^2 f$

$E_1 = 2\mu v \times 2^2 \pi \times 4$

$E_2 = 2\mu v \times 3^2 \times \pi \times 10$

$E_3 = 2\mu v \times 2.5^2 \times \pi \times 12$

$E_4 = 2\mu v \times 1^2 \times \pi \times 16$

$E_5 = 2\mu v \times 4^2 \times \pi \times 20$

So E_1 & E_4 are same

(iii) slope BA cos (kx – wt)

$slope_1 = 2 \times 2 \cos(2 \times 8 - 4 \times 4)$

$slope_2 = 3 \times 4 \cos(6 \times 8 - 10 \times 4)$

$slope_3 = 2.5 \times 6 \cos(6 \times 8 - 12 \times 4)$

$slope_4 = 1 \times 8 \cos(8 \times 8 - 16 \times 4)$

$slope_5 = 4 \times 10 \cos(10 \times 8 - 20 \times 4)$

$slope_5$ is max.

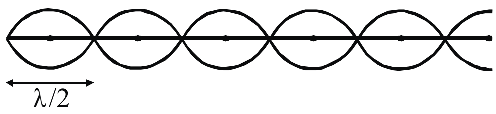
25. Ans. (A)

26. Ans. (B)

27. Ans. (B)

Solution

Speed of wave $v = \sqrt{\dfrac{y}{\rho}} = 4 \times 10^3$



$l = \dfrac{5\lambda}{2} + \dfrac{\lambda}{4} \Rightarrow \lambda = \dfrac{4\ell}{11}$

Frequency $\nu = \dfrac{v}{\lambda} = \dfrac{4 \times 10^3}{\dfrac{4}{11} \times 1} = 11 \times 10^3 Hz$; Wave Number $K = \dfrac{2\pi}{\lambda} = \dfrac{11\pi}{2}$

(i) Equation of standing wave in the rod

S = A coskx sin(ωt +ϕ) where A = 4 × 10^{-6} m $\because$ at x =0, t =0

$\Rightarrow$S=A $\Rightarrow$ A =A $\Rightarrow$ cosk(0) sinϕ $\Rightarrow$ sinϕ =1 $\Rightarrow$ ϕ= $\dfrac{\pi}{2}$

$$S = 4 \times 10^{-6} \cos\left(\frac{11\pi}{2}x\right)\cos\left(22\pi \times 10^3 t\right)$$

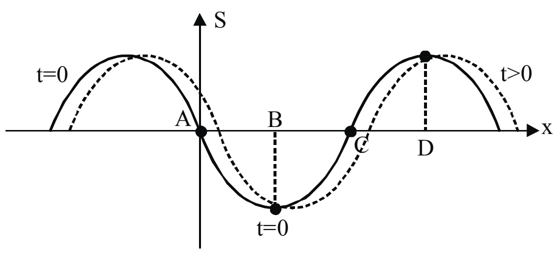
(ii) Strain $= \dfrac{ds}{dx} = -22\pi \times 10^{-6} \sin\left(\dfrac{11\pi}{2}x\right)\cos\left(22\pi \times 10^3 t\right)$ $\because stress = Y \times strain$

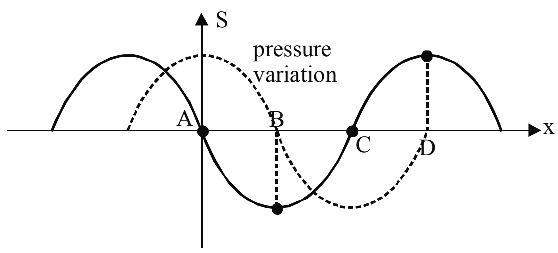
$$\Rightarrow stress = 140.8 \times 10^4 \cos\left(22\pi \times 10^3 t\right)\sin\left(\frac{11\pi}{2}x + \pi\right)$$

(iii) Strain at t = 1s and x = $\dfrac{\ell}{2} = \dfrac{1}{2}m$

$$\left|\frac{ds}{dx}\right|_{x=\frac{\ell}{2}}^{t=1} = 22\pi \times 10^{-6} \times \sin\left(\frac{11\pi}{4}\right) = 11\sqrt{2}\pi \times 10^{-6}$$

28. Ans. (A) → (P,Q,S,T) ; (B) → (R) ; (C) → (Q,T) ; (D) → (R)





29. Ans. (A) → (P, Q) ; (B) → (R, S) ; (C) → (T) ; (D) → (PST)

• For standing wave all particles have same direction of motion.

• For longitudinal wave : Velocity of particle is along the propagation of wave.

30. Ans. (A) → (Q,T) ; (B) → (Q, R) ; (C) → (S,T) ; (D) →(Q)

Time taken by wave to travel from Ranvir to Akshay

$$T_A = \frac{d}{C+\frac{v}{2}+v}\ ;\ T_B = \frac{d}{C+\frac{v}{2}-v}\ ;\ T_C = \frac{d}{C+\frac{v}{2}-v}\ ;\ T_D = \frac{d}{C+\frac{v}{2}+v}$$

Time taken by wave to travel from Akshay to Ranvir

$$T_A = \frac{d}{C-\frac{v}{2}-v}\ ;\ T_B = \frac{d}{C-\frac{v}{2}-v}\ ;\ T_C = \frac{d}{C-\frac{v}{2}+v}\ ;\ T_D = \frac{d}{C-\frac{v}{2}+v}$$

Time will not be same for any case

Wavelength observed by Akshay

$$\lambda_A = \frac{C+\frac{v}{2}+v}{f}\ ;\ \lambda_B = \frac{C+\frac{v}{2}+v}{f}\ ;\lambda_C = \frac{C+\frac{v}{2}-v}{f}\ ;\ \lambda_D = \frac{C+\frac{v}{2}-v}{f}$$

Wavelength observed by Ranvir

$$\lambda_A = \frac{C-\frac{v}{2}-v}{f}\ ;\ \lambda_B = \frac{C-\frac{v}{2}+v}{f}\ ;\ \lambda_C = \frac{C-\frac{v}{2}+v}{f}\ ;\ \lambda_D = \frac{C-\frac{v}{2}-v}{f}$$

Frequency observed by Akshay

$$f_A = f\ ;f_B = f\left[\frac{C+\frac{v}{2}-v}{C+\frac{v}{2}+v}\right];\ f_C = f\ ;\ f_D = f\left[\frac{C+\frac{v}{2}+v}{C+\frac{v}{2}-v}\right]$$

Frequency observed by Ranvir

$$f_A = f\ ;\ f_B = f\left[\frac{C-\frac{v}{2}-v}{C-\frac{v}{2}+v}\right]\ ;\ f_C = f\ ;\ f_D = f\left[\frac{C-\frac{v}{2}+v}{C-\frac{v}{2}-v}\right]$$

31. Ans. (A) → (P, Q) → (B) → (P,S) ; (C) → (P,R)

EXERCISE # (S)

1. $f' = \left(\frac{v+\omega}{v+\omega-v_s}\right)f$

$= \left(\frac{1200+40}{1200+40-40}\right)580$

≈ 599 Hz.

For echo to be heard by the driver, the source is to be considered at the hill having frequency 599 Hz.

$f' = \left(\frac{1200-40+40}{1200-40}\right)599$

t_1(wave to reach the hill) = $\frac{1}{1200+40} = \frac{1}{1240}$ hr.s

in the above duration train moves = $\frac{40}{1240} = \frac{1}{31}$ km.

Now the distance between train and hill = $1 - \frac{1}{31} = \frac{30}{31}$ km

After this instant echo will be heard

$40t + (1200 - 40)t = 30/31$

$t = \frac{1}{1240}$ hrs.

The distance travelled by the train in this time

$= 40 \times \frac{1}{1240} = \frac{1}{31}$ km

Distance from the hill = $1 - \frac{2}{31} = 0.935$ km.

2. Ans. (a) Along a straight in xy plane through origin in at 30° with x-axis, (b) 1m, (c) 4π

3. Ans. 300

$$P = T\left(\frac{\partial y}{\partial t}\right)\left(\frac{\partial y}{\partial x}\right);\ y = A\sin(\omega t - kx + \phi)$$

$$\therefore \left|P_{avg}\right| = \frac{1}{2}TA^2\omega K = \frac{1}{2}\sqrt{T\mu}A^2\omega^2 = \frac{1}{2}\sqrt{T\mu}A^2(2\pi f)^2 \Rightarrow f = 300Hz$$

4. Ans. 6

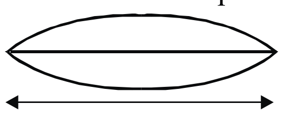
(a) Given $\dfrac{P_{Fe}}{P_{Al}} = \dfrac{\sqrt{\dfrac{T}{\rho_{Al}}}}{\sqrt{\dfrac{T}{\rho_{Fe}}}} = \sqrt{\dfrac{7.5}{2.7}} = \dfrac{5}{3}$ Here fifth harmonic of F_e = third harmonic of A*l* wire.

(b) Using $P_{Fe} = 5$; $f = \dfrac{5}{2\times1}\sqrt{\dfrac{75\pi\times4}{3.14\times10^{-6}\times7.5\times10^{3}}}$ = 500 Hz

5. Ans. 375

$\rho = \dfrac{m}{A\ell} \Rightarrow \ell = \dfrac{m}{\rho A}$ and $\mu = \dfrac{m}{\ell} \Rightarrow v = \sqrt{\dfrac{T}{\mu}}$

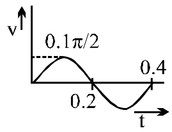
fundamental frequency



$\ell=\lambda/2$

$f = \dfrac{v}{\lambda}$

6. Ans. (a) max speed = 4.48 m/s, max acceleration = 8.0×10^{3} m/s^{2} ;
(b) max speed = 3.14 m/s, max acceleration = 5.6×10^{3} m/s^{2}]

7. Ans. (a) $v = \dfrac{w}{k} = \dfrac{1}{2}$ m/s, $\dfrac{3}{2}$ m/s...... (b) 

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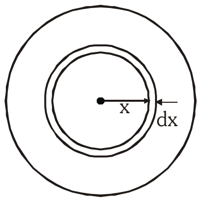
RIGID BODY DYNAMICS

EXERCISE # (O)

1. Ans. (C, D)

2. Ans. (A, C)

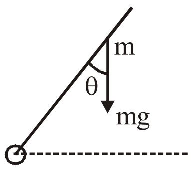
$$d\tau = \left[\frac{\omega x}{h}\eta 2\pi x dx\right]x = \frac{2\pi\omega\eta}{h}x^3 dx$$

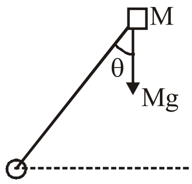


$$\therefore \tau = \int d\tau = \frac{\pi\omega R^4\eta}{2h} \Rightarrow P = \vec{\tau}.\vec{\omega} = \frac{\pi\omega^2 R^4\eta}{2h}$$

3. Ans. (D)

Sol.





$$\alpha = \frac{mg\frac{\ell}{2}\sin\theta}{m\frac{\ell^2}{3}} = \frac{3g\sin\theta}{2\ell} \qquad \alpha \simeq \frac{Mg\ell\sin\theta}{M\ell^2} = \frac{g\sin\theta}{\ell}$$

Since angular acceleration for the weighted stick is less, it will hit the flour later.

4. Ans. (C)

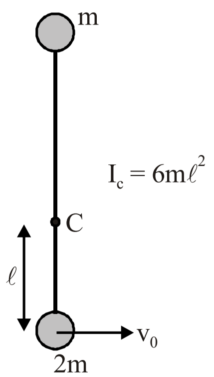
5. Ans. (B, C)

For (B) : $2mv_0\ell = 6m\ell^2\omega$

$$\therefore \omega = \frac{v_0}{3\ell}$$

$$T = 2m\omega^2\ell = \frac{2mv_0^2}{9\ell}$$

For (C) : $I_A = 2m(3\ell)^2$; $I_B = m(3\ell)^2$



6. Ans. (D)

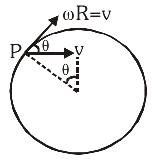
For the semicricular lamina of mass m, the moment of inertia about an axis through C is $I_C = \frac{1}{2}mr^2$.

Let I_{CM} = moment of inertia about an axis through its centre of mass.

$$I_C = I_{CM} + md^2 = \frac{1}{2}mr^2 - m\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{1}{2} - \frac{16}{9\pi^2}\right)mr^2$$

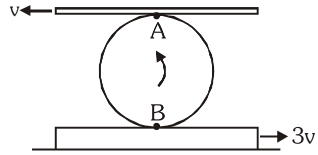
7. Ans. (C)

8. Ans. (B)



$v_p = \sqrt{v^2 + v^2 + 2v^2\cos\theta} = \sqrt{2v^2(1+\cos\theta)}$ = 2v cos θ/2

9. Ans. (A,C)



$v_A = \omega_0 R - v_0 = v$

$\omega_0 R - v_0 = v$ ....(i)

$v_B = \omega_0 R + v_0 = 3v$ ...(ii)

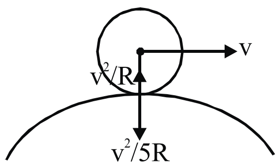
from equation (i) & (ii) $2\omega_0 R = 4V \Rightarrow \omega_0 R = 2v$

$\omega_0 = \frac{2v}{R}$ from (1) $v_0 = v$

10. Ans. (D)

As the sphere rolls up its speed is decreasing and while rolling down its speed is increasing. Hence the acceleration of its centre of mass is down the incline and is thus always negative.

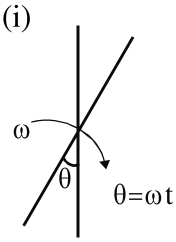
11. Ans. (A)



net acceleration $\frac{v^2}{R} - \frac{v^2}{5R} = \frac{4v^2}{5R}$

12. Ans. (C)

$(dS)^2 = (dx)^2 + (dy)^2 \Rightarrow dS = \sqrt{\left(\frac{dy}{dx}\right)^2 + 1}\; dx$ ...(i)



Velocity of B after time t.

$\frac{dx}{dt} = v_x = \left(\frac{\omega\ell}{2} - \frac{\omega\ell}{2}\cos\omega t\right)$ ...(ii)

$\frac{dy}{dt} = v_y = \left(\frac{\omega\ell}{2}\sin\omega t\right)$ ...(iii)

From (ii) and (iii) $\Rightarrow \frac{dy}{dx} = \frac{\sin\omega t}{1-\cos\omega t}$

Now $dS = \sqrt{\left(\frac{\sin\omega t}{1-\cos\omega t}\right)^2 + 1}$ dx ; $dx = v_x dt$

$$dS = \sqrt{\left(\frac{\sin\omega t}{1-\cos\omega t}\right)^2 + 1}\; \frac{\omega\ell}{2}(1-\cos\omega t)\,dt$$

$$dS = \frac{\omega\ell}{2}\sqrt{(\sin\omega t)^2 (1-\cos\omega t)^2} \text{ dt}$$

$$dS = \frac{\omega\ell}{2}\sqrt{\sin^2\omega t + \cos^2\omega t + 1 - 2\cos\omega t}$$

$$\Rightarrow dS = \frac{\omega\ell}{2}\left(2\sin\frac{\omega t}{2}\right) \Rightarrow \int_0^S dS = \omega\ell\int_0^{T=\frac{2\pi}{\omega}}\sin\frac{\omega t}{2}\;dt = \frac{\ell\omega}{\left(\frac{\omega}{2}\right)}\left[\cos\frac{\omega t}{2}\right]_0^{\frac{2\pi}{\omega}} = 4\ell$$

13. Ans. (A, B, C, D)

Sol. For any point on the surface of paraboloid, $(x^2 + z^2) = 4ay$

(A) $I = \sum_{i=1}^{3} m(x_i^2 + z_i^2)$ (distance of m_i from y-axis is $\sqrt{x_i^2 + z_i^2}$) $= 4ma\ (y_1 + y_2 + y_3)$.

(B) $mg\ (x_1 + x_2 + x_3) = mg(y_1 + y_2 + y_3)$

(C) $mg\ x_1 = \frac{1}{2}mv_1^2 \Rightarrow v_1 = \sqrt{2gy_1}$

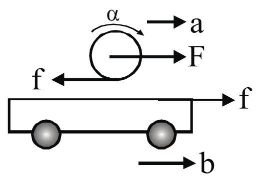
(D) Distance y-mass m_p from y-axis, $r_i = \sqrt{x_i^2 + z_i^2} = \sqrt{4ay_i}$

$KE = \frac{1}{2}m\omega^2\ (r_1^2 + r_2^2 + r_3^2) = \frac{1}{2}m\omega^2\ 4a\ (y_1 + y_2 + y_3)$

14. Ans. (B)

for trolley : $f = Mb$...(i)

For disc : $F - f = Ma$...(ii)



$fR = \frac{MR^2}{2}\alpha$ ...(iii)

For pure rolling $a - \alpha R = b$ ....(iv)

Solving (i), (ii), (iii) & (iv) $a = 3b$

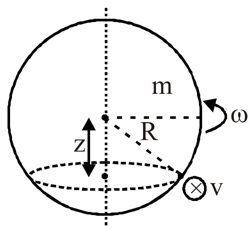
15. Ans. (B)

Since normal is impulsive friction will also be impulsive and it will reduce ω and give some horizontal velocity to C.M. $v \le \omega r$ friction cannot act when there is no tendency of relative motion.

16. Ans. (C)

Sol. $m(\omega R)\ R = mv\sqrt{R^2 - z^2}$ .... (i)

$mgz = \frac{1}{2}mv^2 - \frac{1}{2}m(\omega R)^2$ .... (ii)



On solving (1) and (2)

$2g\ (R^2 - z^2) = \omega^2R^2z$

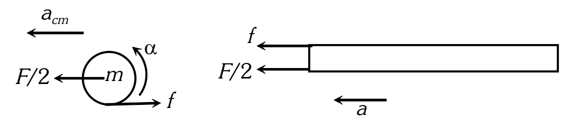
$\therefore\ z \simeq \frac{2g}{\omega^2}$

17. Ans. (C)

$\frac{F}{2} - f = m\,a_{cm}$ … (i) $\quad fR = \frac{1}{2}mR^2\alpha$ … (ii) $\quad \frac{F}{2} + f = 2ma$ … (iii)

For pure rolling $a_{cm} - R\alpha = a$ ...(iv)

On solving $f = \frac{F}{14}$, $a_{cm} = \frac{3F}{7m}$, $a = \frac{2F}{7m}$

$\therefore$ 

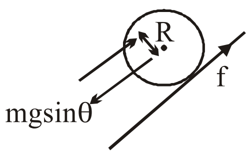
18. **Ans. (D)**

19. **Ans. (B)**

20. **Ans. (D)**

$F = mg\sin\theta - f$

$\tau = 0 = Fh - fR \quad \Rightarrow \quad f = \dfrac{Fh}{R}$



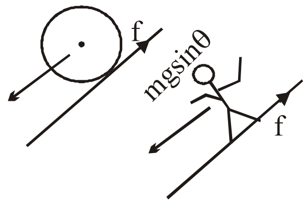
$F = mg\sin\theta - \dfrac{Fh}{R}$

$F = \dfrac{mg\sin\theta}{1+\dfrac{h}{R}}$

F_{min} when 'h' is max = R

21. **Ans. (A)**

$a_s = \dfrac{g\sin\theta}{1+\dfrac{2}{5}\dfrac{mR^2}{mR^2}} = \dfrac{5}{7}g\sin\theta$



$ma_{sisyphus} = mg\sin\theta - f$

$f = \dfrac{2}{7}mg\sin\theta$ f is backwards

$mg\sin\theta - f_{stone} = ma_{stone}$

$f_{stone} = \dfrac{2}{7}mg\sin\theta$

22. **Ans. (A)**

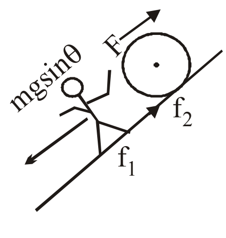
$F_{min} \Rightarrow F$ at R , $\alpha = 0$, $a = 0$

$mg\sin\theta + F = f_1$ $\rightarrow$ sisyphus

$F + f_2 = mg\sin\theta$ $\rightarrow$ stone

$FR = f_2R \rightarrow$ Stone

$\Rightarrow F = f_2 \quad \Rightarrow F = \dfrac{mg\sin\theta}{2} = f_2 \le \mu\, mg\cos\theta$



$\mu \le \tan\theta/2$

for sisyphus,

$mg\sin\theta \;\; F = f_1 \le \mu\, mg\cos\theta$

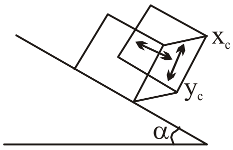
$\dfrac{3mg\sin\theta}{2} \le \mu\, mg\cos\theta \quad \Rightarrow \quad \mu \ge 3/2\tan\theta$

For stone to roll as well as sisyphus to not slip $\mu \ge 3/2\tan\theta$.

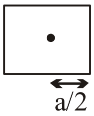
23. **Ans. (A)**

24. **Ans. (A)**

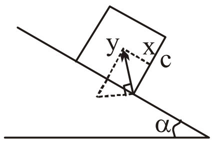
25. **Ans. (B)**

Sol. (i) 

at toppling cm is just a box base line.



$$x_c = \frac{\frac{a}{2}\times m + \frac{a}{2}\times m + 0\times m}{3m} = \frac{a}{3}$$



$$y_{cm} = \frac{a}{2}$$

$$\tan(90° - \alpha) = \frac{y_c}{x_c} = \frac{3}{2} \Rightarrow \tan\alpha = \frac{2}{3}$$

(ii) for it to not slide $3mg\sin\alpha \le 3\mu\, mg\cos\alpha$

$$\frac{2}{3}\tan\alpha \le \mu$$

$\Rightarrow$ $\mu \ge 0.66$

(iii) $a_n = g\sin\alpha$

$\Rightarrow$ $a = \frac{1}{2} g\sin\alpha t^2$ $\quad\Rightarrow$ $t = \sqrt{\frac{2a}{g\sin\alpha}}$

26. Ans. (D)

27. Ans. (D)

28. Ans. (B)

29. Ans. (A)

30. Ans. (A) → (Q) ; (B) → (P) ; (C) → (S) ; (D) → (R)

31. Ans. (A) → (Q) ; (B) → (R) ; (C) → (P) ; (D) → (S)

For (A) : Acceleration of 1 w.r.t. centre of mass =

$$r\alpha\hat{i} - \omega^2 r\hat{j} \Rightarrow \vec{a}_1 = r\alpha\hat{i} - \omega^2 r\hat{j} + R\alpha\hat{i} = (R+r)\alpha\hat{i} - \omega^2 r\hat{j}$$

For (B) : $\vec{a}_2 = -r\alpha\hat{j} - \omega^2 r\hat{i} + R\alpha\hat{i} = (R\alpha - \omega^2 r)\hat{i} - r\alpha\hat{j}$

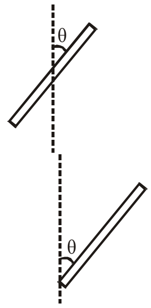
For (C) : $\vec{a}_3 = -r\alpha\hat{i} + \omega^2 r\hat{j} + R\alpha\hat{i} = (R\alpha - r\alpha)\hat{i} + \omega^2 r\hat{j}$

For (D) : $\vec{a}_4 = r\alpha\hat{j} + \omega^2 r\hat{i} + R\alpha\hat{i} = (R\alpha + \omega^2 r)\hat{i} + r\alpha\hat{j}$

32. Ans. (A) → (P,T) ; (B) → (P,R) ; (C) → (P,Q,S) ; (D) → (R)

EXERCISE # (S)

1. Ans. 3



$$I = \frac{ML^2}{12}\sin^2\theta \Rightarrow 1 = \frac{ML^2}{12}\sin\left(\frac{\pi}{2}\right) \Rightarrow ML^2 = 12$$

$$I = \frac{ML^2}{3}\sin^2\theta, \text{ at } \theta = \frac{\pi}{3}, I = \frac{ML^2}{3}\left(\frac{3}{4}\right) = \frac{ML^2}{4} = \frac{12}{4} = 3 \text{ kg-m}^2$$

2. Ans. t = 1.5 sec

3. Ans. Loss in P.E. = Gain in K.E.

$$mg\frac{\ell}{3} + mg\left(\frac{2\ell}{3}\right) + mg\ell = \frac{1}{2}\left(m\left(\frac{\ell}{3}\right)^2 + m\left(\frac{2\ell}{3}\right)^2 + m\ell^2\right)\omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{36g}{14\ell}} \Rightarrow v_B = \omega\ell_B = \frac{2\ell}{3}\sqrt{\frac{36g}{14\ell}} = \sqrt{\frac{8g\ell}{7}}$$

4. Ans. 1

$$\frac{dL}{dt} = \frac{dm}{dt}(v_1 - v_2)R = 200 \times (5–2.5) \times 2 = 1000 \text{ J}$$

5. Ans. 4

$$mv_0 \cdot 2R = \left[\frac{3}{2}mR^2 + m(2R)^2\right]\omega \;;\; \omega = \frac{4v_0}{11R} = 4 \text{ rad/s}$$

6. Ans. 0500

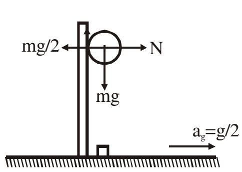
Sol. (a) The net torque that the rope exerts on the capstan, and hence the net torque that the capstan exerts on the rope, is the difference between the forces of the ends of the rope times the radius of the capstan.

The capstan is doing work on the rope at a rate $P = \tau\omega = F_{net}\, r\, \frac{2\pi \text{ rad}}{T}$ = (520 N) (5.0×10^{-2} m) $\frac{2\pi \text{ rad}}{(0.90 \text{ s})}$ = 182 W, or 180 W to two figures. A larger number of turns might increase the force, but for given forces, the torque is independent of the number of turns.

(b) $\frac{\Delta T}{t} = \frac{Q/t}{mc} = \frac{P}{mc} = \frac{(182 \text{ W})}{(6.00 \text{ kg})(470 \text{ J/kg.k})}$ = 0.064 C°/s.

7. Ans. (i) $\frac{M\omega_0}{M + 2qt}$, (ii) $q\omega_0 R^2$

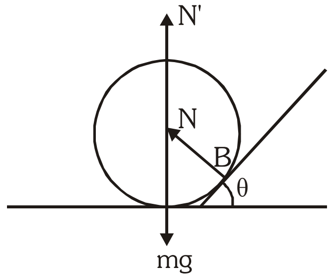
8. Ans. 8



$mg - f_s = m\,(a_{cm})_{rel}$ ...(1)

$N - \frac{mg}{2} = m\,(0)$; $N = \frac{mg}{2}$; $f_s \times R = \frac{2}{5}mR^2\alpha$

9. Ans. 44



Initial angular momentum of the solid sphere about

$B = I\omega + mv_{cm}r \Rightarrow \frac{MR^2}{2}\frac{v_C}{R} + mv_C R\cos\theta$

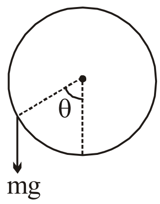
Final angular momentum of the solid sphere at $B = mv'^2 R + \frac{MR^2}{2}\frac{v'}{R}$

Angular momentum about the B will remain conservation $L_i = L_F$

Angular momentum about the B will remain conservation

10. Ans. (i) $t = \frac{6a\pi}{\sqrt{3}\,v_0}$ **; (ii)** $s = \frac{a}{\sqrt{3}}\sqrt{1+\left(2\pi+\sqrt{3}\right)^2}$

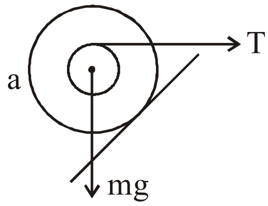
11. Ans. 5



$\tau = mgR\sin\theta = -I\frac{d^2\theta}{dt^2}$; $\frac{d^2\theta}{dt^2} = -\frac{mgR\theta}{MR^2}$;

$T = 2\pi\sqrt{\frac{MR}{mg}}$ solving for m we get m = 5 gm.

12. Ans. 2



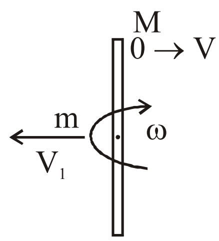
mg sinθ – T cosθ = ma ∴ a = 2m/s²

13. Ans. 8

$a_C = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3}g\sin\theta$; $a_P = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{g\sin\theta}{2}$

$a_{rel} = \frac{g\sin\theta}{6} = \frac{10\times\frac{3}{5}}{6} = 1$; $s_{rel} = \frac{1}{2} \times 1 \times 16 = 8$ m

14. Ans. 9



(i) $L_i = L_f$; $\frac{ml^2}{12}\omega = MV \times \frac{l}{2} \Rightarrow V = \frac{m\omega l}{6M}$

(ii) $P_i = P_f$; $0 = MV - mV_1 \Rightarrow V = \frac{mV_1}{M} \Rightarrow \frac{\omega l}{6} = V_1$

(iii) $e = 1 = \frac{V-(-V_1)}{\omega\frac{l}{2}-0} \Rightarrow V + V_1 = \frac{\omega l}{2}$; $\frac{m\omega l}{6M} + \frac{\omega l}{6} = \frac{\omega l}{2}$; $\left(\frac{m}{M}+1\right) = 3$

$\Rightarrow \frac{m}{M} = 2 \Rightarrow M = \frac{m}{2} = 9$ kg.

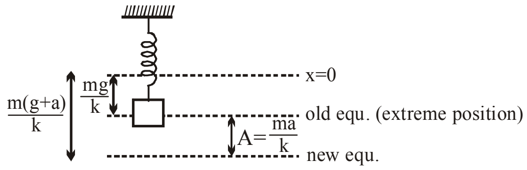
SIMPLE HARMONIC MOTION

EXERCISE # (O)

1. **Ans. (A, C)**

2. **Ans. (A, B, D)**

Sol. (A) Isochronous system $\therefore$ $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(B) 

(C) $v_{max} = \omega A = \sqrt{\frac{k}{m}}\ \frac{ma}{k} = \sqrt{\frac{m}{k}}\ a$

3. **Ans. (A, D)**

Sol. (A) kx = 3 mg ; equ^n.

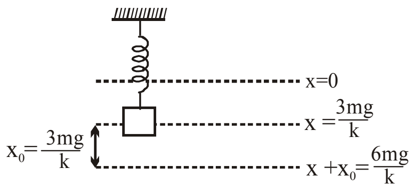
$k(x + x_0) - T - mg = ma$ .....(1)

$T - 2\ mg = 2\ ma$ .....(2)

On solving,

$T = \frac{2m}{3}\frac{k(x+x_0)}{}$ $\therefore$ for $x_0 = \frac{3mg}{k}$

T = 4 mg



(D) If $x_0 = \frac{3mg}{k}$

If $x_0 > \frac{3mg}{k}$ then string will become lack when 'm' comes to rest at top most extreme possible.]

4. **Ans. (D)**

Velocity of object = 2Aω cos(ωt)

Velocity of mirror = $-A\omega\cos\left(\omega t - \frac{\pi}{3}\right)$; $\vec{V}_{IM} = -\vec{V}_{OM}$

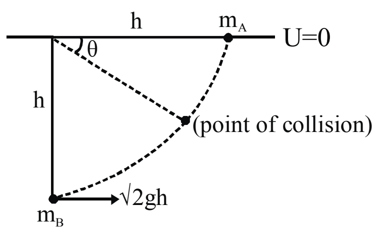
Velocity of object wr.t. mirror = $2A\omega\cos(\omega t) + A\omega\cos\left(\omega t - \frac{\pi}{3}\right)$

Velocity of image = $-\left[2A\omega\cos(\omega t) + A\omega\cos\left(\omega t - \frac{\pi}{3}\right)\right] - A\omega\cos\left(\omega t - \frac{\pi}{3}\right) = 0$

5. **Ans. (C)**

Since their initial mechanical energy is same

$m_A gh\sin\theta = \frac{1}{2}m_A v_A^2$

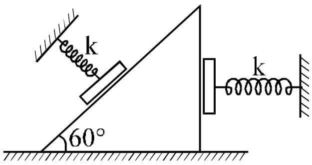


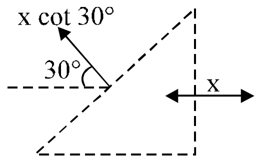
$$-mgh+\frac{1}{2}m_B\left(\sqrt{2gh}\right)^2=\frac{1}{2}m_Bv_B^2-mgh\sin\theta$$

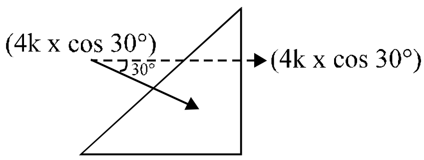
$KE_A = KE_B$

6. Ans. (B)

Sol.







$F = 4kx\cos^2 30°$ $\Rightarrow \vec{a} = -\left(\frac{k3}{M^4}\right)x4\ \vec{x}$ $\Rightarrow \infty^2 = \frac{3k}{M}, \infty = \sqrt{\frac{3k}{M}}$

$\Rightarrow T = \frac{2\pi}{\infty} = \frac{2\pi}{\sqrt{3k/M}}$ $\Rightarrow T = \frac{2\pi}{\sqrt{3k}}\sqrt{M}$ $\Rightarrow t_1 = \frac{T_1}{2} = \frac{\pi}{\sqrt{3k}}\sqrt{M}$

$\Rightarrow t_2 = \frac{T_2}{2} = \pi\sqrt{\frac{M}{k}}$ $\Rightarrow$ time period $= t_1 + t_2 = \left[\frac{\pi\sqrt{M}}{\sqrt{3k}} + \pi\sqrt{\frac{M}{k}}\right]$

$\Rightarrow$ time period $= t_1 + t_2 = \pi\sqrt{\frac{M}{k}}\left[1+\frac{1}{\sqrt{3}}\right]$ Ans.

7. Ans. (A)

8. Ans. (C)

9. Ans. (B, C)

$E=Ax^2 + Bv^2$

Velocity is maximum, when x= 0 $v_{\max} = \sqrt{\frac{E}{B}}$; Time period $= \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{A/B}} = 2\pi\sqrt{\frac{B}{A}}$

10. Ans. (A,B,D)

Power $= \frac{dE}{dt} \Rightarrow \frac{(u_p+u_K)dl}{dt} = \left(u_p+u_k\right)\sqrt{\frac{T}{\mu}}$

11. Ans. (A,B,D)

$$v_{rms} = \sqrt{\frac{v_0^2\int_0^T \cos^2\left(\omega t+\phi\right)dt}{\int_0^T dT}} = \frac{v_0}{\sqrt{2}}$$

12. Ans. (A,B)

For (A) : $T = \frac{T_1}{2}+\frac{T_2}{2} = \pi\sqrt{\frac{m}{k}} + \pi\sqrt{\frac{m}{4k}} = \frac{3\pi}{2}\sqrt{\frac{m}{k}}$

For (B) $\frac{1}{2}k(A)^2 = \frac{1}{2}4k(x)^2$

For (C) : Not possible $[x = \frac{-A}{2}]$

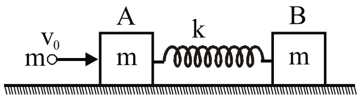
For (D) : TE = $\frac{1}{2}kA^2$

13. Ans. (A,B,C,D)

During the collision imples is not transferred to B **:** $mv_0 = 2mv' \Rightarrow v' = \frac{v_0}{2}$

Just after collision $\Rightarrow v_A = \frac{v_0}{2}$ and $v_B = 0$

and at the time of maximum compression



$v_B = mv_0 = 3mv'' \Rightarrow v'' = \frac{v_0}{3}$

$$\frac{1}{2}kx_0^2 = \frac{1}{2}2m\left(\frac{v_0}{2}\right)^2 - \frac{1}{2}3m\left(\frac{v_0}{3}\right)^2 \Rightarrow x_0 = v_0\sqrt{\frac{m}{6k}}$$

Loss in KE $= \frac{1}{2}mv_0^2 - \frac{1}{2}(2m)\left(\frac{v_0}{2}\right)^2 = \frac{mv_0^2}{4}$

14. Ans. (A, B)

Since time period of oscillation is indpenedent of mass of bob, thus remains same.

Due to collision, K.E. at the mean position increases $\Rightarrow$ amplitude increases.

15. Ans. (A, B)

$T = 2\pi\sqrt{\frac{\mu}{k}}$ Now, $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}$ etc.

16. Ans. (A)

Sol. $\frac{2c}{mR^2} = \frac{k}{m}$

$R = \sqrt{\frac{2c}{k}}$

17. Ans. (A)

Sol. In CM frame both the masses execute SHM with $\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{2k}{m}}$

Initially particles are at extreme distance $= L_0 + (L - L_0)\cos\sqrt{\frac{2k}{m}}\,t$

18. Ans. (B)

19. Ans. (B)

Elastic string never gets slacked, so there is always a restoring force $\Rightarrow$motion is oscillatory.

20. Ans. (D)

21. Ans. (C)

Sol. $mg = F_b$

$a^3 \times 0.4\,\rho g = a^2 h\rho g$

$h = \frac{2}{5}a$

$m = a^3 \times 0.4\rho$

$F_{net} = a^2 x\rho g$

$T = 2\pi\sqrt{\frac{a^3 \times 0.4\rho}{a^2\rho g}} \Rightarrow T = 2\pi\sqrt{\frac{2a}{5g}}$

22. Ans. (B)

Sol. Displacement must be less than submergence depth of cube.

23. Ans. (A)

R = radius of gyration about mass centre = $\sqrt{\frac{2R}{5}}$

ℓ= distance between point of suspension and mass centre = L +R

$$\ell_{eq} = \frac{k^2}{\ell} + \ell = \frac{2}{5}\frac{R^2}{(L+R)} + (L+R)$$

24. Ans. (C)

When the bob is hollow $\ell_{eq} = \frac{2R^2}{3(\ell+R)} + (L+R) \Rightarrow T_1 = 2\pi\sqrt{\frac{\ell_{eq}}{g}} = 2\pi\sqrt{\frac{\frac{2R^2}{3(L+R)} + (L+R)}{g}}$ ...(i)

when filled with water, total energy

$$E = \frac{1}{2}m_w\omega^2(L+R)^2 + \frac{1}{2}M\omega^2(L+R)^2 + \frac{1}{2}\times\frac{2}{3}M\omega^2R^2 + (M_w + M)g(L+R)(1-\cos\theta)$$

$$\frac{dE}{dt} = 0 \Rightarrow \frac{d^2\theta}{dt^2} + \left[\frac{g(L+R)}{\frac{2}{3}\frac{MR^2}{(M+M_w)} + (L+R)^2}\right]\theta = 0 \Rightarrow T_2 = 2\pi\sqrt{\frac{\frac{2}{3}\left(\frac{M}{M+M_w}\right)\frac{R^2}{L+R} + (L+R)}{g}}$$

Therefore $T_2 < T_1 \Rightarrow \frac{T_1}{T_2} > 1$

OR

Water will not rotate therefore system has more translation KE, hence more average speed for same amplitude. So time period will decrease.

25. Ans. (B)

When water freezes it rotates $T_3 = 2\pi\sqrt{\frac{\frac{2R^2}{5(L+R)} + (L+R)}{g}} \Rightarrow T_3 < T_2$

OR

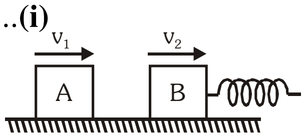
Ice will rotate therefore system has less translation KE, hence for same amplitude average speed is less. So time period will increase.

26. Ans. (C)

$T = 2\pi\sqrt{\frac{m}{K}} \Rightarrow m = \frac{T^2K}{4\pi^2} = \frac{0.2 \times 0.2 \times 1000}{4 \times 10} = 1\text{kg}$

27. Ans. (A)

Immediately after the collision, suppose velocities of the blocks are v_1 and v_2 as shown $\frac{1}{2}$ (velocity of approach) = velocity of separation. $\Rightarrow 5 = v_2 - v_1$ ...**(i)**



Using principal of conservation of momentum for the collision

$2 = 0.2\, v_1 + v_2 \Rightarrow 10 = v_1 + 5v_2$ ...**(ii)**

On solving $v_2 = 2.5$ m/s and $v_1 = -2.5$ m/s.

Hence block A moves leftward after the collision with speed 2.5 m/s.

And the block B moves towards right with speed 2.5 m/s.

The maximum velocity of B = 2.5 = $\omega A \Rightarrow A = v\sqrt{\frac{m}{k}} = 2.5\sqrt{\frac{1}{1000}}\text{m} = 2.5\sqrt{10}\text{cm}$

28. Ans. (B)

Time of flight = $\sqrt{\frac{2h}{g}} = 1\text{s} \Rightarrow d = (2.5 \text{ m/s}) \times 1\text{s} = 2.5$ m

29. Ans. (A) → (P,Q,R,S,T) ; (B) → (P,Q,R,S,T) ; (C) → (P,Q,R,S) ; (D) → (P,Q,R,S)

(A) For equilibrium : $2mg = kx$; $x = \frac{2mg}{k}$; $A = \frac{2mg}{k}$

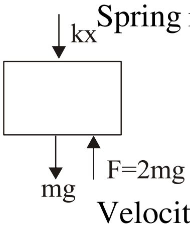
$v_{max} = A\omega = \frac{2mg}{k}\sqrt{\frac{k}{m}} = 2g\sqrt{\frac{m}{k}}$; $a_{max} = \omega^2 A = \frac{k}{m} \times \frac{2mg}{k} = 2g$

(B) Same as (A)

(C) Initially : $mg = kx_0$; $3mg = kx$; $A = \frac{2mg}{k}$

Spring will not acquire natural length.

(D) $mg = kx_0$



Spring is compressed by $\frac{mg}{k}$ in equilibrium.

$kx = mg$

$A = \frac{2mg}{k}$

$x = \frac{mg}{k}$

Velocity at natural length $v = \omega\sqrt{A^2 - x^2}$

$x = \frac{mg}{k}$

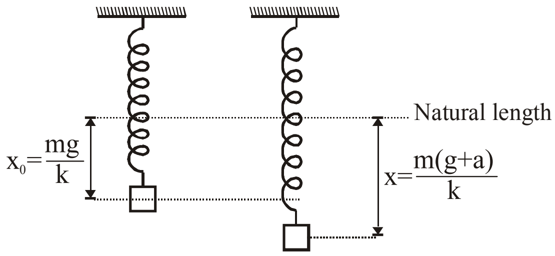
EXERCISE # (S)

1. **Ans.** $\pi\sqrt{\frac{4ML}{3(KL+2Mg)}} + \pi\sqrt{\frac{2L}{3g}}$

2. **Ans. (a)** $\frac{1}{2\pi}\sqrt{\frac{2C}{MR^2}}$ **, (b)** $\theta_{new} = \frac{\theta_0}{\sqrt{3}}$ **,** $f' = \frac{1}{2\pi}\sqrt{\frac{2C}{3MR^2}}$

3. **Ans. 5**

From SHM :



$$A = \frac{ma}{K} = \frac{3(1)}{1200}m = \frac{3}{12}cm \Rightarrow A = \frac{1}{4}cm$$

$$v_{max} = \omega A = \sqrt{\frac{k}{m}}A = \sqrt{\frac{1200}{3}} \times \frac{1}{4}cm = 20 \times \frac{1}{4} = 5cm/s$$

From Energy conservation :

$W_{gravity} + W_{spring} + W_{Pseudo} = \Delta KE$

$m(g+a)(x-x_0) - \frac{1}{2}k(x^2 - x_0^2) = \frac{1}{2}mv^2$

4. **Ans. 8**

$$\frac{1}{2}kA^2 = U_{max} - U_{min} = \frac{1}{2}mv_{max}^2 \Rightarrow U_{min} = 2J$$

Potential energy at $\frac{A}{2}$ is $= U_{min} + \frac{1}{2}k\left(\frac{A}{2}\right)^2 = 2 + \frac{8}{4} = 4J$

5. **Ans. 8**

$\omega = \sqrt{\frac{k}{m}} = \sqrt{40}$ when spring breaks new $\omega = \sqrt{20}$

Equilibrium position of original system $(2k)\,x_0 = mg$ or $x_0 = \frac{1}{4}$ m

New equilibrium is at $kx = mg$; $x = \frac{1}{2}$ m thus $v_{max} = A\omega = \left(\sqrt{40}\right) \times \left(\frac{1}{4}\right)$

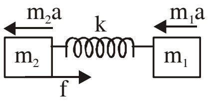
$$\frac{\sqrt{10}}{2} = \sqrt{20}\left[A^2 - \left(\frac{1}{4}\right)^2\right]^{1/2};\ \frac{10}{4} = 20\left[A^2 - \frac{1}{16}\right];\ \frac{1}{8} + A^2 = A'^2;\ A' = \frac{1}{2}n = 50\text{ cm}$$

6. **Ans. (a)** $\frac{1}{2}kx^2 - 2mgR$, **(b)** $3mg = \frac{mv_f^2}{R}$, **(c)** $\sqrt{\frac{7mgR}{k}}$

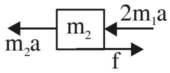
7. **Ans: T =** $2\pi\sqrt{\frac{m\ell^2}{mg\ell + 2Kb^2}}$

8. **Ans. 2**

viewed from frame of plank



[initial condition]



[at the time of maximum compression]

$\because 2m_1a + m_2a = 8N$ and $f_{max} = \mu N = 10N$ $\Rightarrow$Friction is static

kA = m_1a $\Rightarrow A = \frac{m_1a}{k} = 2cm$

9. **Ans. 9**

$$\tau = \left(mg\frac{\ell}{2} + mg\ell\right)\sin\theta;\ I = \frac{m\ell^2}{3} + m\ell^2\ \therefore \alpha = \frac{\tau}{I} = \frac{3mg\ell\theta}{2\left[\frac{4m\ell^2}{3}\right]} = \frac{9}{8}\frac{g\theta}{\ell};\ \omega^2 = \frac{9g}{8\ell} = 9$$

10. **Ans. 4**

$$\frac{kq^2}{2a^2} = mg \Rightarrow\ -mga\left[\sin\theta - \frac{\sin\theta/2}{2\cos^2\theta/2}\right] = ma^2\alpha$$

$$\Rightarrow -\frac{g}{a}\left[\frac{2\sin\theta}{2} - \frac{\cos\theta}{2} - \frac{\sin\theta/2}{2\cos^2\theta/2}\right] = \alpha \Rightarrow\ \frac{\sin\theta}{2}\frac{g}{a}\left[2\cos\frac{\theta}{2} - \frac{1}{\cos^2\theta/2}\right] = \alpha$$

for small θ, $\cos\frac{\theta}{2} = 1$; $\sin\frac{\theta}{2} = \frac{\theta}{2}$; $\frac{-g}{a}\frac{\theta}{2}\times\left(1-\frac{1}{2}\right) = \alpha$; $\alpha = \frac{3g}{4a}\theta$; $\alpha = -\omega^2\theta$;

$$T = 2\theta\sqrt{\frac{4a}{3g}} = 2\pi\sqrt{\frac{4\times3}{3\times\pi^2}} = 4\sec$$

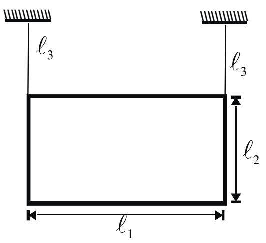
11. **Ans. 050**

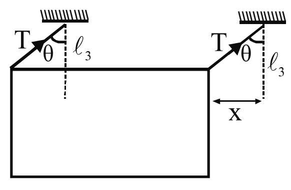
The motion of the plate is translational motion.

For small angle θ, 2T = mg

2T sinθ =ma

$$\therefore mg\left(\frac{-x}{\ell}\right) = ma \Rightarrow a = -\frac{g}{\ell}x = -\omega^2x \Rightarrow \omega = \sqrt{\frac{g}{\ell}}$$



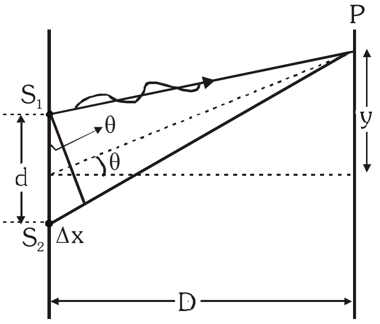


[Refer Ex. Q. 26 Chapter 12 (SHM) HC Verma Part-I]

12. **Ans: (a) T =** $\frac{m_1m_2g}{m_1+m_2}$ **; (b)** $\frac{2m_2g}{k}$ **; (c)** $T_{min} = \frac{m_1m_2g}{m_1+m_2}; T_{max} = \frac{m_2g(m_1+2m_2)}{(m_1+m_2)}$

WAVE OPTICS

EXERCISE # (O)



1. **Ans. (A,B)**

For microwaves, c=fλ $\Rightarrow \lambda = \frac{c}{f} = \frac{3\times10^8}{10^6} = 300\text{m}$

Also, Δx=dsinθ, $\Rightarrow \phi = \frac{2\pi}{\lambda}\ \Delta x = \frac{2\pi}{\lambda}(d\sin\theta) = \frac{2\pi}{300}(150\sin\theta) = \pi\sin\theta$

So, $I = I_1 + I_2 + 2\left(\sqrt{I_1I_2}\right)\cos\phi$

with $I_1=I_2$ and $\phi=\pi\sin\theta$, above equation reduces to $I_R = 2I_1[1+\cos(\pi\sin\theta)] = 4I_1\cos^2\left(\frac{\pi\sin\theta}{2}\right)$

As I_R will be maximum when $\cos^2[(\pi\sin\theta)/2]$ is maximum,
i.e., equal to 1, so $(I_R)_{max}=4I_1=I_0$ and hence $I=I_0\cos^2[(\pi\sin\theta)/2]$

If θ=0°, $I=I_0\cos0° = I_0$ If θ=30°, $I=I_0\cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$ If θ=90°, $I=I_0\cos^2\left(\frac{\pi}{2}\right)=0$

2. **Ans. (B, C)**

(i) Path difference in air is $\Delta x = \frac{d^2}{D} + (\mu_2-\mu_3)t + \frac{\mu_3 yd}{2D}$

for position of central maxima Δx =0 $\Rightarrow y = \frac{2}{9}mm$

(ii) Thickness of the slab so that central maxima forms at point P

$$\Rightarrow t(\mu_2-\mu_3) + \frac{d^2}{D} = 0 \Rightarrow t = \frac{(2\times10^{-3})^2}{0.6} = \frac{20}{3}\times10^{-6}m$$

3. **Ans. (A,B,C,D)**

4. **Ans. (C)**

$I_p = \frac{I_{max}}{2}[1+\cos\phi] = \frac{I_{max}}{2}\left(1+\cos\frac{2\pi y}{\beta}\right)$; where $\beta = \frac{D\lambda}{d}$

First maxima is observed at P, i.e., $\cos\left(\frac{2\pi y}{\beta}\right)$=1. As D increases β will increase and the value of $\cos\left(\frac{2\pi y}{\beta}\right)$ should be negative. Hence, the ratio $\frac{I_P}{I_{max}}$ starts decreasing but starts increasing again as $\cos\left(\frac{2\pi y}{\beta}\right)$ again starts becoming positive.

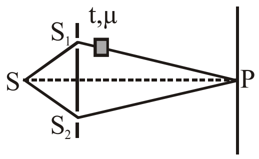
5. Ans. (B)

Δx at P :

$\Delta x = (S_1P - t)_{air} + t_{medium} - S_2P_{air}$

$= [S_1P - S_2P + \mu t - t]_{air}$

$= (\mu - 1)t$



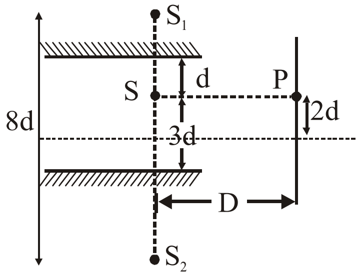
Earlier, Δx at $P = S_1P - S_2P = 0$

So, change in optical path due to insertion of slab $= (\mu - 1)t$

For intensity to be zero at P, we have $\Delta x = \frac{(2n-1)\lambda}{2}$ [n = 1, 2, ...]

$\Rightarrow (\mu - 1)\, t = t = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, ...$

6. Ans. (C)



$\frac{\lambda D}{8d} = d$

7. Ans. (A)

8. Ans. (D)

If A_1 and A_2 are the amplitudes of the narrow and wide slits respectively, then $A_1 = E_0$ and $A_2 = 2E_0$ where E_0 is the amplitude of the electric field vector due to the narrow slit.

At the central maximum ($\theta = \beta = 0$) the two amplitudes, being in phase, add up and the intensity is $I_m = (A_1 + A_2)^2 = (3E_0)^2 = 9E_0^2 = 9I_0$ where I_0 is the intensity due to the narrow slit along.

At an angle θ to the central maximum, putting $\beta = \frac{\pi d \sin\theta}{\lambda}$, the phase difference between the coherent waves is ϕ (=2β) and the resultant intensity is given by

$I_\theta = \frac{I_m}{9}\, [1 + (2)^2 + 2(1)(2)\cos\phi] = I_0\, [1 + 4(1 + \cos 2\beta)] = I_0\, [1 + 8\cos^2\beta]$

where $I_0 = E_0^2$ is the intensity due to narrow slit alone.

9. Ans. (C)

$\Delta\phi = 2n\pi \Rightarrow \frac{\pi}{2} + \frac{2\pi}{\lambda} d \sin\theta = 2n\pi;\ \frac{2\pi}{\lambda} d\sin\theta = \left(2n - \frac{1}{2}\right)\pi$

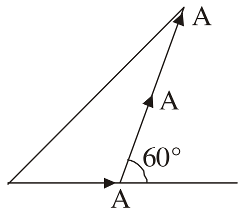
$\sin\theta = \left(2n - \frac{1}{2}\right)\frac{\lambda}{2d} = \frac{1}{2} \times \frac{\lambda}{2 \times 3\lambda} = \frac{1}{12} \Rightarrow \frac{y}{\sqrt{(100\lambda)^2}} = \frac{1}{12}$

$144y^2 = (100\lambda)^2;\ y \approx \frac{100\lambda}{12} = \frac{25\lambda}{3}$

10. Ans. (B)

$$\Delta x_{12} = \frac{yd}{D} = \frac{0.7\times10^{-3}\times0.7\times10^{-3}}{2\times0.3}$$

$$\Delta\phi_{12} = \frac{2\pi}{7\times10^{-7}}\times\Delta x = \frac{2\pi}{7\times10^{-7}}\times7\times10^{-7}\times\frac{0.7}{0.6} = \frac{7\pi}{3} = 2\pi+\frac{\pi}{3}$$

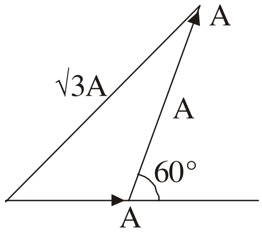


Similarly, $\Delta\phi_{23} = 2\pi+\frac{\pi}{3}$

$$A_r^2 = (2A)^2 + A^2 + 4A^2\times\frac{1}{2}$$

$I = 7\,I_0 \Rightarrow I_{res} = 7\,I_0$

11. Ans. (C)

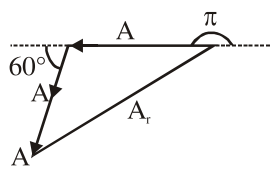


If only S_3 is covered $I = \frac{3I_0}{7} I_{res}$

12. Ans. (A)

$$\Delta x = (1.25-1)\times1.4\times10^{-6} = \frac{0.7}{2}\times10^{-6} = \frac{7\times10^{-7}}{2} = \frac{\lambda}{2} \Rightarrow \Delta\phi = \pi$$

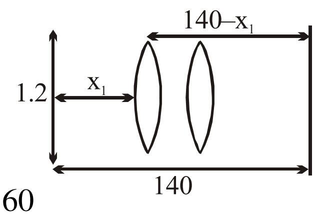
$$\Rightarrow \Delta\phi_{12} = \pi+\frac{\pi}{3}$$



$$\Delta\phi_{23} = \pi+\frac{\pi}{3}$$

$$A_r^2 = A^2 + 4A^2 - 4A^2\times\frac{1}{2} = 3A^2 \Rightarrow 3I_0/7$$

13. Ans. (B)



$h_0 = \sqrt{h_1h_2} = \sqrt{1.6\times0.9}$ = 1.2 mm; h =1.6 ; h_0 = 1.2

$m_1 = \frac{4}{3} = \frac{140-x_1}{x_1} \Rightarrow x_1 = 80 \Rightarrow \frac{1}{60}-\frac{1}{-80} = \frac{1}{f} \Rightarrow f = \frac{240}{7}$cm and $x_2 = 60$

14. Ans. (D)

$(x_2 - x_1) = 20$ cm

15. Ans.(D)

$h_0 = \sqrt{h_1h_2} = \sqrt{1.6\times0.9}$ = 1.2 mm = 2aδ

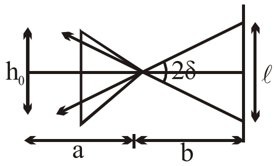
1.2 = 2aA (μ – 1); $A = \frac{0.05}{180}\times\pi$; a = 30 cm ⇒ μ ≈ 1.76

16. Ans. (A)

d = 1.2 mm; D = 140 cm ∴ $\beta = \frac{\lambda D}{d} = 0.7$mm

17. Ans. (C)

$1 = 2\delta b$



$$\frac{44}{7}n = \frac{2\delta b}{\beta} = \frac{h_0 b}{a\beta} \Rightarrow \frac{44}{7}$$

18. Ans. (A) → (P,R,S); (B) →(P,Q,S); (C)→ (P, Q, S)

$y_1 = n_1\beta_1 = n_2\beta_2$ = LCM of β_1 and β_2

$2y_1 = 2n_1\beta_1 = 2n_2\beta_2$

Hence at this point both maxima again coincide

$$y_2\left(n_1 - \frac{1}{2}\right)\beta_1 = \left(n_2 - \frac{1}{2}\right)\beta_2 \; ; \; \frac{\beta_1}{\beta_2} = \frac{n_2 - \frac{1}{2}}{n_1 - \frac{1}{2}} \Rightarrow \frac{\beta_1}{\beta_2} = \frac{2n_1 - 1}{2n_1 - 1}$$

Which will have a solution. If $\frac{\beta_1}{\beta_2}$ expressed as a proper fraction will be of form $\frac{\text{odd}}{\text{odd}}$.

For (B and C) : $\frac{\beta_1}{\beta_2}$ is of form $\frac{\text{Odd}}{\text{even}}$. Hence no solution i.e. the two minima will never coincide.

For (A) $\frac{\beta_1}{\beta_2}$ is of form $\frac{\text{odd}}{\text{odd}}$. Hence at some finite y_2 the two minima will coincide.

At $2y_2$ the two maxima (and not minima) will coincide.

$\therefore$ $y = 3y_2$ is the next nearest point where minima coincide.

19. Ans. (A) → (P,R,S,T); (B) →(Q,R,S,T); (C) →(R); (D) →(P,R,S,T)

20. Ans. (A) → (R) ; (B) → (T) ; (C) → (Q,R) ; (D) → (P,S)

Path difference remains same on a circle for case D

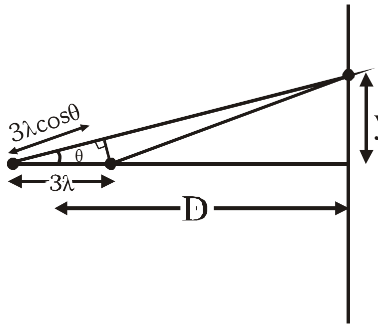
Shape of fringe pattern for pin hole is hyperbolic

Shape of fringe pattern for slit is straight line

Δx_{max} can't be greater than 'd' distance between the source in A, B & C & Δx_{min} can't be less than d-distance between the source in D.

EXERCISE # (S)

1. Ans. 7



Path difference = $3\lambda\cos\theta = 2\lambda \Rightarrow \cos\theta = \frac{2}{3}$; $y = D\tan\theta = \frac{D\sqrt{5}}{2} \Rightarrow m + n = 5 + 2 = 7$

2. Ans. 33

3. Ans. $4I_0$, $\theta = \sin^{-1}\left(\frac{2n+1}{8}\right)$ **n = 0, 1, 2, 3, (iii) 393.75 m**

4. Ans. (i) 6 mm, (ii) 50π/3

5. Ans. 2311

Sol. $I = 4I_0\cos^2\frac{\phi}{2}$

Case - 1, $\phi = 0 \Rightarrow I = 4I_0$

Case - 2, $I = \frac{3I}{4} = 4I_0\cos^2\frac{\phi}{2} \Rightarrow \cos^2\frac{\phi}{2} = \frac{3}{4}$

$\cos\frac{\phi}{2} = \frac{\pm\sqrt{3}}{2} \Rightarrow \frac{\phi}{2} = \frac{\pi}{6}$ $\qquad \phi = \frac{\pi}{3}$

Now, $\phi = \frac{(\mu-1)t\times 2\pi}{\lambda}$

$\frac{\pi}{3} = \frac{(\mu-1)t - 2\pi}{\lambda}$

$t = \frac{\lambda}{6(\mu-1)}$

$t = \frac{6933}{3} = 2311$ Å **Ans.**

6. Ans. (i) 0, (ii) $\frac{40}{27}\times 10^{-3}$ m downwards