SCORE JEE (Advanced)
HOME ASSIGNMENT # 04

ERROR

EXERCISE # (O)

1. **Ans. (A,C)**
VSD = 1.1 mm and 1 MSD = 1.0 mm
Least count = LC = MSD ~ VSD = 0.1 mm
For (A) : Zero error = – (No. of divisions of vernier scale coincident) × LC = –7 × 0.1 = – 0.7 mm
For (C) : Reading = (No. of divisions coincident on main scale) × MSD
– (No. of divisions coincident on vernier scale) LC– zero error = 1.22 cm
2. **Ans. (B)**
3. **Ans. (D)**
Backlash error is random error
4. **Ans. (A)**
From screw gauge z.e. = + 0.20 mm
Dia of chalk = 3.5 – 0.2 = 3.3 mm = 0.33 cm
For vernier callipers, z.e. = – 0.05 cm
Thus it must show 0.33 – 0.05 = 0.28 cm
so (a) is correct
5. **Ans. (C)**
Radiation correction refers to correction introduced in the Joule's calorimeter due to radiation losses.
End correction corresponds to the meter bridge.
Backlash error is present in the apparatus involving screw system.
Index error corresponds to the optical bench.
6. **Ans. (C)**

$a + b = 14 \pm 0.14 \Rightarrow \%\text{ error} = \frac{0.14}{14} \times 100 = 1\%$

$a - b = 2 \pm 0.14 \Rightarrow \%\text{ error} = \frac{0.14}{2} \times 100 = 7\%$

$a \times b = 48 \pm 0.96 \Rightarrow \%\text{ error} = \frac{0.96}{48} \times 100 = 2\%$

order of % error x < z < y
7. **Ans. (A)**
8. **Ans. (D)**
9. **Ans. (A)**
10. **Ans. (A)**
11. **Ans. (A) – q, (B) – q, (C) – r, (D) – q,**

EXERCISE # (S)

1. Ans. 6

Least count = $\frac{1\text{mm}}{10}$ = 0.1 mm; Zero error = –(10–6) × 0.1 = – 0.4 mm;

Reading = 6 + 5 × (0.1) – (–0.4) = 7.2 mm

2. Ans. 18.18

$$\frac{T'-T}{T}\times 100=\left(\sqrt{\frac{\ell' g}{\ell g'}}-1\right)\times 100=\left(\sqrt{\frac{1.69}{1.21}}-1\right)\times 100=\left(\frac{1.3}{1.1}-1\right)\times 100=\frac{0.2}{1.1}\times 100=\frac{200}{11}=18.18\%$$

3. Ans. 3

$$V=\pi r^2 h=\frac{\pi D^2 h}{4}\Rightarrow\frac{\Delta V}{V}=\frac{2\Delta D}{D}+\frac{\Delta h}{h}\quad\frac{\Delta V}{V}\times 100=\left[2\times\left(\frac{0.01}{2.00}\right)+\left(\frac{0.1}{5.0}\right)\right]\times 100=3\%$$

4. Ans. 6

$$R=\frac{V}{I}=\frac{10^3}{1.43}=700\Omega=7\times 10^{-1}k\Omega$$

FLUID MECHANICS

EXERCISE # (O)

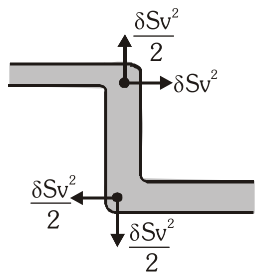
1. Ans. (A,B)

Total displacement of mercury against atmospheric process= $\left[\ell+\frac{\ell}{2}+\ell\right]$

$PE_{initial}=\left[\left(\rho S\frac{\ell}{2}\right)\frac{\ell}{4}\right]\times 2+0$ {Assuming zero at ground level}; $PE_{\max}\left[\left(\rho S\frac{\ell}{2}\right)\times 2+\rho S\ell\right]\ell$

2. Ans. (A)

The force has been exerted by liquid on the tube due to change in momentum at the corners i.e., when liquid is taking turn from A to B and from B to C.As corss-section area at A is half of that of B and C, so velocity of liquid flow at B and C is half to that of velocity at A. Let velocity of flow of liquid at A be v and cross section area at A be S, the velocity of flow of liquid at B and C would be $\frac{v}{2}$ [from continuity equation]



and corss section area at B and C would be 2S.

Due to flow of liquid, it is exerting a force per unit time of ρSv^2 on the tube, where ρ is the density of liquid, S is cross section areaand v is velocity of flow of liquid. The force exerted by liquid on tube is shown in the figure. Which clearly shows that a net force is acting on tube due to flowing liquid towards right and a clockwise torque sets in.

3. Ans. (D)

$F_b=(2\rho gh+\rho gh)a^2=3\rho gha^2$; $F_w=\left[\frac{\rho gh}{2}\right]ah+(2\rho gh)ah=\frac{5}{2}\rho gha^2$

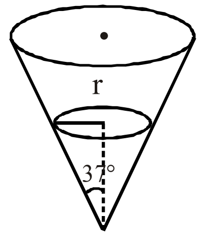
[here h=a]; $\frac{F_b}{F_w}=\frac{3}{5/2}=\left(\frac{6}{5}\right)$

4. Ans. (D)

$F=\frac{\Delta p}{\Delta t}=\frac{m(v_1+v_2)}{\Delta t}=V\rho(v_1+v_2)$ where $\frac{m}{\Delta t}=V=$ volume/second.

5. Ans. (D)

6. Ans. (C)



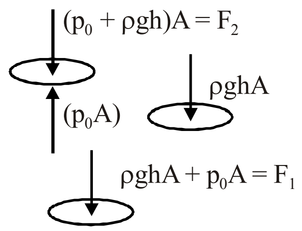
$\frac{r}{8} = \tan 37° = \frac{3}{4}$

r = 6m

$F = (P_0 + h\rho g)\,\pi r^2 = (10^5 + 10 \times 800 \times 10) \times \pi \times 36$

$1.8 \times 36 \times \pi \times 10^5 = 2 \times 10^7$

7. Ans. (C)



$F_2 = \rho ghp$; $F_1 = \rho ghp + p_0A$

8. Ans. (A,C)

$\rho V_A g = (m_{ice} + m_A) g$ and $\rho V_B g = (m_{ice} + m_B) g \Rightarrow \frac{V_A}{V_B} = \frac{m_{ice} + m_A}{m_{ice} + m_B} = \frac{0.9 \times 980 + 4.9 \times 20}{0.9 \times 980 + 1.9 \times 20} = \frac{49}{46}$

Also A will start sinking first.

9. Ans. (C)

10. Ans. (A)

As the cork moves up, the force due to buoyancy remains constant. As its speed increases, the retarding force due to viscosity increases, being proportional to the speed. Thus the acceleration gradually decreases. The acceleration is variable, and hence the relation between velocity and time is not linear.

11. Ans. (D)

B = mg

$(\rho_w) v_{sub} g = (\rho_B) v_0 g \Rightarrow v_{sub} = \frac{\rho_0}{\rho_w} v_0$

$\frac{v_{sub}}{v_0}$ depends only on the density of block & liquid.

12. Ans. (C)

Here $A_0v_0 = A_1v_1$

So Force exerted $= -\frac{\Delta p}{\Delta t} = -\left(\frac{\rho A_1 v_1^2 - \rho A_0 v_0^2}{1}\right) = \rho A_0 v_0^2 \left[1 - \frac{A_1}{A_0}\left(\frac{v_1}{v_0}\right)^2\right] = \rho A v_0^2 \left[1 - \frac{A_0}{A_1}\right]$

13. Ans. (A)

$P_{open} + \frac{1}{2}\rho v^2 = P_{closed} \Rightarrow v = \sqrt{\frac{2(P_{closed} - P_{open})}{\rho}} = \sqrt{\frac{2 \times (3.5 - 3) \times 10^5}{10^3}}$ = 10 m/s

14. Ans. (D)

$$\frac{1}{2}\rho v^2 = \rho gh + \frac{Mg}{A} \Rightarrow v = \sqrt{2gh + \frac{2Mg}{\rho A}} = \sqrt{2\times 10\times 6 + \frac{2\times 50\times 10}{10^3\times 1}} = \sqrt{120+1} = \sqrt{121} = 11 m/s$$

15. Ans. (D)

As vessel is falling freely under gravity, the pressure at all points within the liquid remains the same as the atmospheric pressure.

If we apply Bernoulli's theorem just inside and outside the hole, then

$$P_{inside} + \frac{\rho v_{inside}^2}{2} + \rho g_{eff} y = p_{outside} + \frac{\rho v_{outside}^2}{2} + \rho g_{eff} y$$

$v_{inside} = 0$, $p_{inside} = p_{outside} = p_0$ [atmospheric pressure]

Therefore, $v_{outside} = 0$. i.e., no water comes out.

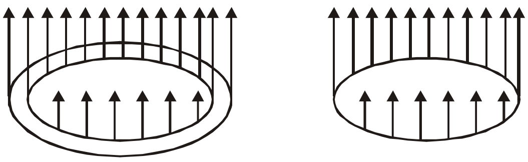
16. Ans. (B)

$$Av = Q \Rightarrow v = \frac{Q}{\pi r^2}; F = \frac{\Delta p}{\Delta t} = \text{mass flowing / sec} \times \text{velocity} = \rho V/\sec \times v = \rho Q v = \rho Q \frac{Q}{\pi r^2} = \frac{\rho Q^2}{\pi r^2}$$

The direction of force of reaction is upward, moment of force about Q = $\frac{\rho Q^2}{\pi r^2} L$

17. Ans. (C)

Ring have double surface than that of disk.



18. Ans. (A,C)

$E = T\ 4\pi r^2 \Rightarrow \frac{4}{3}\pi R^3 = n \times \frac{4\pi}{3} r^3 \Rightarrow n = \frac{R^3}{r^3} \Rightarrow R = n^{1/3} r$

Surface energy of big drop $E' = T4\pi R^2 = T4\pi n^{2/3} r^2 = En^{2/3}$

Energy released = $nE - E' = nE - n^{2/3}E = E(n - n^{2/3})$

19. Ans. (D)

20. Ans. (C)

Net force exerted by liquid on styrofoam is buoyout force = $\rho_w g \frac{\ell^2 L}{2}$

21. Ans. (D)

Arrange pressure on slant surface $P_{avg} = \frac{(P_0 + \rho_w g h_0) + \left(P_0 + \rho_w g h_0 - \frac{l}{\sqrt{2}}\right)}{2} = \left(P_0 + \rho_w g h_0 - \frac{l}{2\sqrt{2}}\right)$

Force on any one of the slant face = $\left[P_0 + \rho_w g(h_0 - \frac{\ell}{2\sqrt{2}})\right] L\ell$

22. Ans. (B)

Balancing force in vertical

$(M_{styrofoam} + m)g + F_{surface\ Tension} = F_{bouyancy}$

23. Ans. B

24. Ans. D

25. Ans. (B)

The volume of liquid should remain unchanged

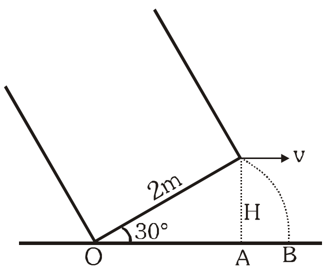
Hence, $2 \times 2 \times 2 = \frac{1}{2}\left[x + x + \frac{2}{\sqrt{3}}\right] \times 2$

$\therefore \quad x \approx 1.42\text{m}$

Now h = x sin 60° = 1.23 m

$\therefore v = \sqrt{2gh} = \sqrt{2 \times 10 \times 1.23} = 4.96\text{m/s}$

26. Ans. (C)



H = 2 sin 30° = 1m $\therefore t = \sqrt{\frac{2H}{g}} = \frac{1}{\sqrt{5}}\text{s}$

27. Ans. (D)

$OA = 2\cos 30° - \sqrt{3}\text{m} \quad \Rightarrow \quad AB = vt = \frac{(4.96)}{\sqrt{5}}\text{s}$

$\therefore$ OB = 3.95 m

28. Ans. (C)

Sol. Bernaullies equation given $\frac{F}{A} + P_0 = \frac{1}{2}\rho v^2 + P_0$, $v = \sqrt{\frac{2F}{A\rho}}$

29. Ans. (A)

Sol. Volume flow rate = av $\therefore$ av × t = V $\therefore$ t= $\frac{V}{a\sqrt{\frac{2F}{A\rho}}} = \frac{V}{a}\sqrt{\frac{A\rho}{2F}}$

30. Ans. (B)

Sol. Work energy theorem given W=$\frac{1}{2}mv^2 = \frac{1}{2}[\rho V]v^2 = \frac{1}{2}[\rho V] \times \frac{V^2}{a^2t^2} = \frac{\rho V^3}{2a^2t^2}$

31. Ans. (C)

Out of four options, (C) corresponds to the above phenomena.

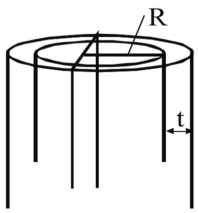
32. Ans. (B)

$$B = -\frac{dP}{\left(\frac{dV}{V}\right)} \Rightarrow dP = 2.2 \times 10^{8}\,Pa$$

33. Ans. (D)

Balancing, force per unit length, 2P.R = $2(\sigma t)$

⇒ for lesser σ, R should decrease & thickness should increase.



34. Ans. (A) PQ (B) PQ (C) PRST (D) PRST

If $dgh_1 > P \Rightarrow$ height of liquid will not cross h_1 $dgh_1 < P \Rightarrow$ height of liquid will cross h_1

35. Ans. (A) → (PT); (B) → (PS) ; (C) →(QT); (D) →(R)

EXERCISE # (S)

1. **Ans. 5**

Force due to piston = 50 × 0.5 = 25 N

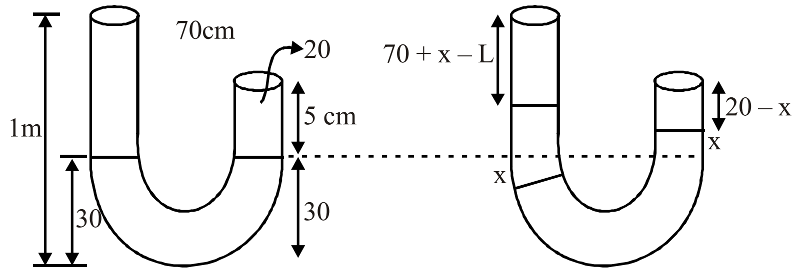
There due to fluid = (ρah) A=A (1000 × 5 × 0.5) = 2500 Pa × 0.01 m^2 = 25 N

There on the surface AB = 50N = 5 decanewton

2. **Ans.** zero

3. **Ans. 6**

$2x \times \frac{\rho}{2} g = L \times \rho g \quad \Rightarrow x = L$



frequency same ⇒ λ = same; $20 - x = \frac{\lambda}{4}$; $70 = \frac{5\lambda}{4}$; 70 = 100 – 5x; 5x = 30 ; x = 6 cm

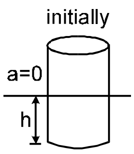
4. **Ans.** $\sqrt{\frac{18g}{19a}}$

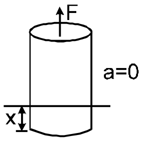
5. **Ans. 3**

6. **Ans. 5**

$KA = (mg - B) \Rightarrow A = \frac{(mg-B)}{K}$ where B is buoyancy force.

7. **Ans. 5**

Applying Newton's law vertical direction  $mg = F_B$

$\Rightarrow dSH \times g = \rho_w Sh \times g \quad \Rightarrow \quad h = \frac{dH}{\rho}$ 

Now when force F is applied, for minimum work a = 0 (∵ for a = 0, F is minimum)

F – mg + ρSxg = 0; F = mg – ρSxg

$W = \int F dx = \int (mg - \rho Sxg) dx = mg \int dx - \rho Sg \int x dx$

$$W = mgh - \frac{\rho S g h^2}{2} = (\rho Sh)\,gh - \frac{\rho S g h^2}{2} = \frac{\rho S g h^2}{2} = \frac{\rho S g}{2}\left(\frac{dH}{\rho}\right)^2 = \frac{Sgd^2H^2}{2\rho}$$

[here $S = 4\ m^2$, $H = 1$ m, $d = 500\ kg/m^3$, and $\rho = 1000\ kg/m^3$]

8. Ans. 040

$$mg = \rho_w \frac{v}{3} g = \rho_A \frac{v}{2} g \text{ ...(i)}$$

$$\rho_{mixture} = \frac{\rho_w v_0 + \rho_A v_0}{2v_0} = \frac{\rho_w + \rho_A}{2}$$

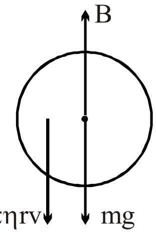
$$mg = \left[\frac{\rho_w + \rho_A}{2}\right] \times \left[\frac{v}{n}\right] g \text{ ...(ii)}$$

From equation (i) & (ii) $\rho_w \frac{v}{3} g = \frac{vg}{2n}\left[\rho_w + \frac{2\rho_w}{3}\right]; \frac{1}{3} = \frac{1}{2n}\left[\frac{5}{3}\right] \Rightarrow n = \frac{5}{2}$

9. Ans. 2

$$F = \frac{\eta A v}{d}; d = \frac{\eta A v}{F} = \frac{0.01 \times 10^{-1} \times 1^2 \times 4}{0.002} = 2\text{m}$$

10. Ans. 4



$$1000 \times \frac{4}{3}\pi r^3 g - 500 \times \frac{4}{3}\pi r^3 g - 6\pi\eta r v = 500 \times \frac{4}{3}\pi r^3 a$$

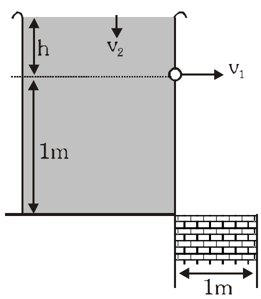
11. Ans. 5

For minimum velocity at orifice

$$\Rightarrow (v_{min}) \times \sqrt{\frac{2h}{g}} = 1 \Rightarrow v_{min} = \sqrt{5}\ m/s$$

This will give us minimum height for this velocity

$$\Rightarrow \sqrt{2gh_{min}} = \sqrt{5} \Rightarrow h_{min} = \frac{1}{4} m = 0.25 m$$



$v_1 = \sqrt{2gh}$ & $v_2 = -dh/dt$

By equation of continuity $av_1 = Av_2$

$$\Rightarrow a\sqrt{2gh} = A\left(-\frac{dh}{dt}\right) \Rightarrow \int_{0.81}^{0.25} \frac{dh}{\sqrt{h}} = \int_0^t \frac{\left(a\sqrt{2g}\right)}{A} dt$$

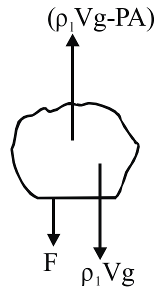
$\Rightarrow t = 125$ sec $= (5)^3 = (A)^3 \Rightarrow \alpha = 5$

12. Ans. 6

Applying Bernoulli's equation up and down the plate $\frac{1}{2}\rho v^2 = \frac{mg}{A}$

$$v = \sqrt{\frac{2mg}{\rho A}} = \sqrt{\frac{2mg \times 18}{\rho A \times 18}} = 6\left(\sqrt{\frac{mg}{18\rho A}}\right) = x\sqrt{\frac{mg}{18\rho A}} \Rightarrow x = 6$$

13. Ans. 3



We have $\rho_\ell 2g = 1500g \Rightarrow \rho_\ell = 750\ \text{kg/m}^3$

The area joined by glue is not exposed to the liquid.

P is the pressure at glued part = $\rho_\ell gH$

$\therefore$ For not breaking $\rho_\ell Vg - \rho_\ell gHA = F + \rho_1 gV \Rightarrow H = 3$ m

14. Ans. 100

$T_{\max} = 7 \times 10^6 \times 10^{-6} N = 7N$

$\therefore F_{B_{\min}} = (15 - 7) = 8N$

$\therefore$ Water level its be lowered = 2 cm

$\therefore$ Area occupied by water level at the top = 100 cm^2

$\therefore$ 2t = 100 × 2 $\Rightarrow$ t = 100 sec

15. Ans. 108

From continuity equation; $A_1v_1 = A_2v_2 \Rightarrow 4v_1 = v_2$...(i)

From Bernoulli's equation $\frac{\rho v_1^2}{2} = \frac{\rho v_2^2}{2} - \rho gh$ ...(ii)

Solving (i) & (ii); $v_1 = \frac{1}{5}$ m/s ; $v_2 = \frac{4}{5}$ m/s

Total volume wasted = $\pi\left(\frac{6 \times 10^{-3}}{2}\right)^2 v_1 \times t = \pi \times 9 \times 10^{-6} \times \frac{1}{5} \times 60 \times 1000$ = 108 π mL/minute

PARTICLE DYNAMICS

EXERCISE # (O)

1. Ans. (A)

$a = g\cos\theta$; $L_{AB} = 2R\cos\theta$; $\frac{1}{2}at^2 = L_{AB}$

2. Ans. (B, D)

for A : $y_A = 10\,t$; $x_A = \int_0^t 0.5 \times 10t\,dt = \frac{5t^2}{2}$

for B : $y_B = 20\,t$; $x_B = \int_0^t 0.5 \times 20t\,dt = 5t^2$

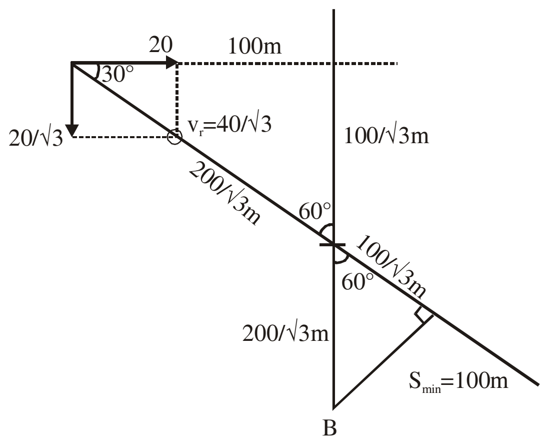
$x_B = x_A + 250$

Difference of height of tower = $y_B = y_A$

3. Ans. (A)

Time of travel from A to B, $t = \sqrt{\frac{2h}{g}}$ In this time, the particle must make an integral number of rotations say n. $\Rightarrow 2\pi rn = ut \Rightarrow 2\pi rn = u\sqrt{\frac{2h}{g}} \Rightarrow n = \frac{u}{2\pi r}\sqrt{\frac{2h}{g}}$

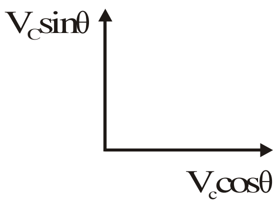
4. Ans. (B,C)



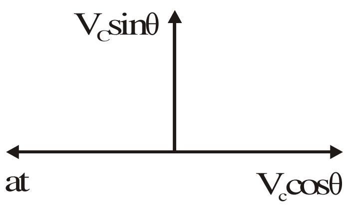
$$t = \frac{\left(\frac{200}{\sqrt{3}}\right) + \left(\frac{100}{\sqrt{3}}\right)}{\left(40/\sqrt{3}\right)} = 7.5\ \text{s}$$

5. Ans. (A,D)

Time is independent of plane



Velocity w.r.t. earths



Velocity w.r.t. cart

* Vertical component same so height will also same.

6. Ans. (A, B, C)

$\vec{v}_{a/p} = 8\hat{i} + 6\hat{j}$

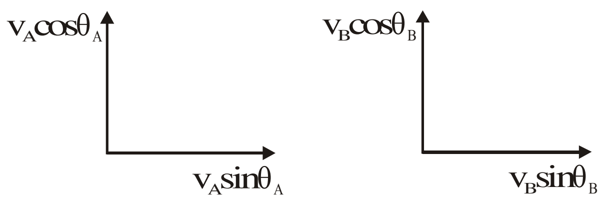
$\vec{v}_{p/g} = 10\hat{i}$

$\vec{v}_{a/g} = 18\hat{i} + 6\hat{j}$

$\vec{r} = (18t)\hat{i} + (6t)\hat{j}$

$\tan\theta = \dfrac{6}{18}$

7. Ans. (A, B, D)



8. Ans. (A)

9. Ans. (A)

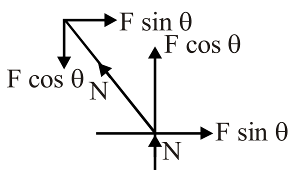
Sol. $N = F\cos\theta$

$F\sin\theta = \mu N$

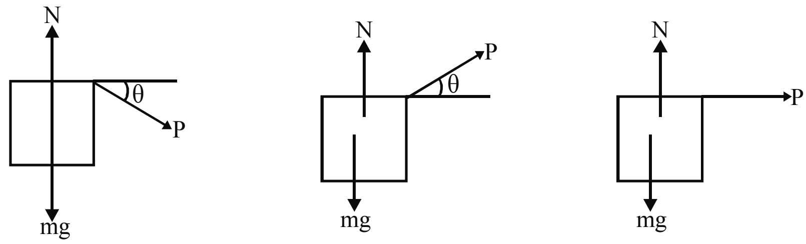
$\dfrac{\sin\theta}{\cos\theta} = \mu$

$\tan\theta = 0.9$

$\theta = \tan^{-1}(0.9)$



10. Ans. (A,B,C)



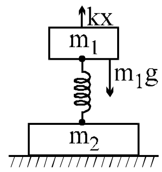
N = mg + Psinθ	N = mg – Psinθ	N = mg
$f_{max} = \mu N$	$f_{max} = \mu N$	$f_{max} = \mu N$
Friction may be static = Pcosθ	Friction may be static = Pcosθ	Friction may be static

11. Ans. (B)

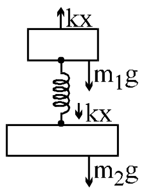
12. Ans. (A)

Sol. Initially $m_1g = kx$

When support is removed, spring force does not change.



New FBD

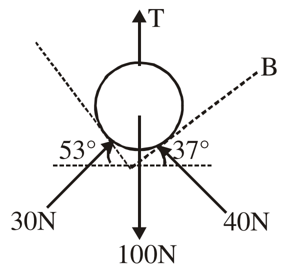


For m_1 : $m_1g - kx = m_1a_1$ $\Rightarrow a_1 = 0$

For m_2 : $m_2g + kx = m_2a_2$ $\Rightarrow a_2 = \dfrac{(m_1 + m_2)g}{m_2}\downarrow$

13. Ans. (B)

Resultant of 30 N & 40 N is 50 N in vertical direction. Hence T = 50 N



14. Ans. (A,C,D)

For (A) : $F_{min} = mg = \left(\int dm\right)g = \int(\lambda d\ell)\,g = \int_0^{\ell}\left(\lambda_0 e^{2x}dx\right)g = \dfrac{\lambda_0}{2}\left(e^{2\ell}-1\right)g$

For (B) : Lower half is heavier so, $T > \dfrac{F}{2}$

For (C) : $F - mg = ma \Rightarrow F = m(g+a) = \lambda_0\left(e^{2\ell}-1\right)g$ and $a = g$

15. **Ans. (C)**

Normal from ground on both ball will be same for horizontale quilibrium from F.B.D. of m it can be said tension in rod will be compressive in nature.

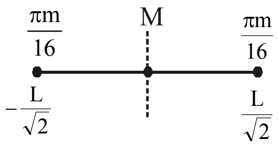
Horizontal component along the inclines must vanish :

$T\cos(45°-\theta) = mg\cos 45°$; $T\cos(45°+\theta) = 3mg\cos 45° \Rightarrow 3 = \frac{\cos(45°+\theta)}{\cos(45°-\theta)}$

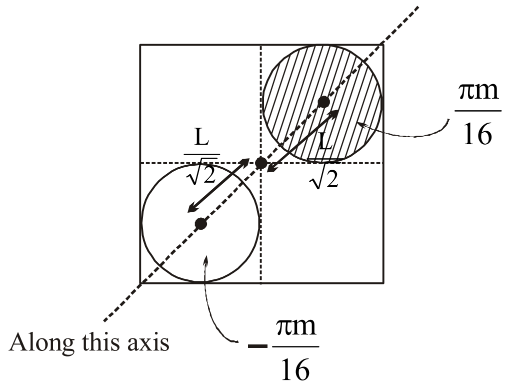
Where T is the tension of the bar, θ is the angle of the bar to the horizontal plug in the numbers from the choices to solve the equation.

16. **Ans. (D)**

Sol.



$$X_{Cr} = \frac{\frac{M\pi\left(\frac{L}{2}\right)^2}{4L^2} \times \left(L\hat{i} + L\hat{j}\right)}{M}$$



17. **Ans. (D)**

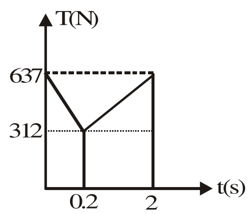
Centre of mass should be at $R\hat{i}$

18. **Ans. (B,D)**

Velocity and acceleration are w.r.t. sphere.

From string constraint, $a_{tangential}$ of m = a & $v_{tangential}$ of m = v $\quad \therefore a_P = \sqrt{a_t^2 + \left(\frac{v^2}{R}\right)^2}$

19. **Ans. (C)**



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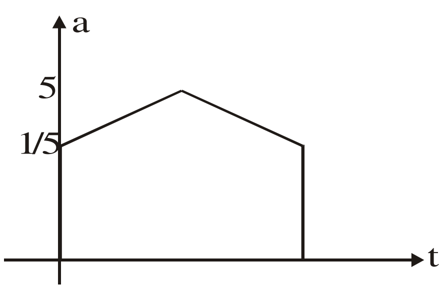


Area of a v/s t graph will give change in velocity.

20. **Ans. (A)**

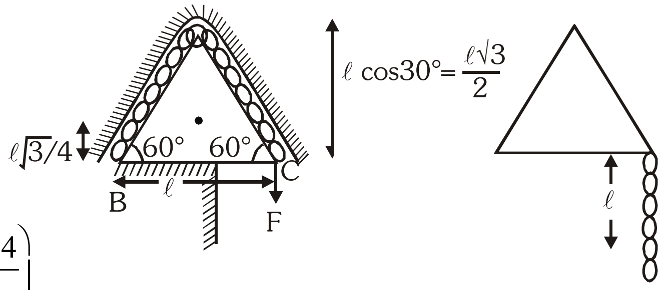
$\mu kd = mg$

21. **Ans. (D)**

Both centre of mass are considing.

22. Ans. (D)

Change in position of CM in vertical direction



$(\Delta h) = \left(\ell \dfrac{\cos 30^\circ}{2}\right) + \ell$

$\therefore$ Work done by gravity

$= + mg\Delta h = + mg \left(\dfrac{\ell\sqrt{3}}{4} + \ell\right) = + mg\ell \left(\dfrac{\sqrt{3}+4}{4}\right)$

23. Ans. (D)

$mgh = \dfrac{1}{2} mV^2$

$\mu mgl = \dfrac{1}{2} \times \dfrac{mM}{(m+M)} V^2$

24. Ans. (A)

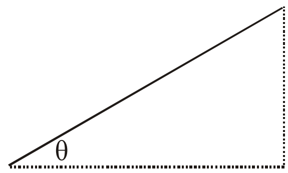
$a = \dfrac{mg}{(2m+M)}; V = \sqrt{2aH}; t = \dfrac{d}{V}$

25. Ans. (C)

The speed at angle θ is given by conservation of energy

$\Rightarrow \dfrac{1}{2} mv^2 = mgh \Rightarrow \dfrac{1}{2} mv^2 = mg\ R\cos\theta \Rightarrow\ v = \sqrt{2gR\cos\theta}$ R will cancel out in the final answer.

So we have the following setup of projectile motion.



$v = \sqrt{2gR\cos\theta}$

$v_x = v \cos\theta$

$v_y = v \sin\theta$

The time of flight is twice the time to get to the top $\Rightarrow t = 2\left(\dfrac{v_y}{g}\right)$

$\Rightarrow x = v_x t = v_x \left(\dfrac{2v_y}{g}\right) = \dfrac{2v_x v_y}{g} = \dfrac{2v_x v_y}{g} = \dfrac{2(v\cos\theta)(v\sin\theta)}{g} = \dfrac{2v^2 \sin\theta\cos\theta}{g} = \dfrac{2(2gR\cos\theta)\sin\theta\cos\theta}{g}$

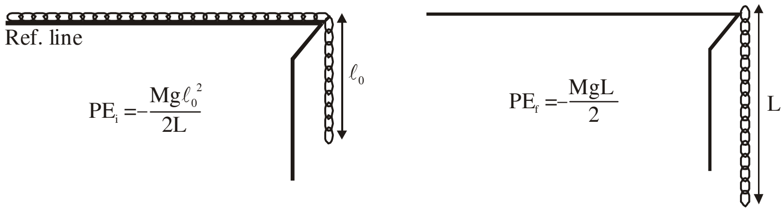
$= 4R\cos^2\theta\sin\theta$

Maximise this $\Rightarrow$ take the derivative and equate it to zero

$\dfrac{dx}{d\theta} = -8R\cos\theta\sin^2\theta + 4R\cos^3\theta = 0 \Rightarrow 2\sin^2\theta = \cos^2\theta \Rightarrow \tan^2\theta = \dfrac{1}{2} \Rightarrow \tan\theta = \dfrac{1}{\sqrt{2}}$

26. Ans. (C)

Potential energy of centre of mass is equal to potential energy of a system.



Loss in potential energy = gain in kinetic energy $-\frac{Mg}{2}\left[\frac{\ell_0^2}{L} - L\right] = \frac{1}{2}MV^2 \Rightarrow V = \sqrt{\frac{g}{L}\left(L^2 - \ell_0^2\right)}$

27. Ans. (B,D)

We use P.E. only for internal conservative forces.

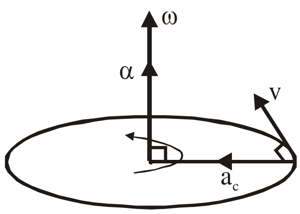
28. Ans. **B**

$$mg\left(\frac{h}{\cos\theta} - h\right) = \frac{1}{2}mv^2$$

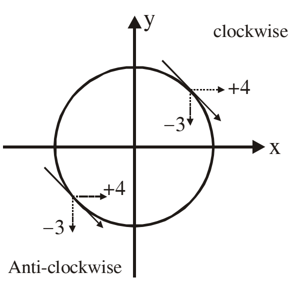
29. Ans. (C)

Energy conservation $\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}mu^2 + \frac{1}{2}kx^2 \Rightarrow v_1^2 + v_2^2 = 4.4\ (\text{m/s})^2$

30. Ans.(A,C,D)



31. Ans.(A,C)



32. Ans. (D)

$$\frac{dy}{dx} = \tan\theta = x$$

tangential acceleration = $g\sin\theta$

33. Ans.(B)

34. Ans.(D)

Total number of revolution = Area of graph

$$= \frac{1}{2}\times 10\times\frac{1200}{60}+10\times\frac{1200}{60}+\frac{1}{2}\times 20\times\frac{1200}{60}=\frac{1}{2}\times 10\times 20+10\times 20+\frac{1}{2}\times 20\times 20$$

= 100 + 200 + 200 = 500

35. Ans.(C)

Since speed is increasing so a_C will increases so, angle with radial decreases.

36. Ans. (B)

As when they collide $\quad Vt + \frac{1}{2}\left(\frac{72V^2}{25\pi R}\right)t^2 - \pi R = Vt \therefore t = \frac{5\pi R}{6V}$

37. Ans.(A,C,D)

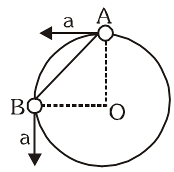
at t = 0

$$V_{b/g} = V\hat{i} - V\hat{j}$$

$$T = \frac{mv^2}{\ell}$$

38. Ans.(C)

Sol. Both rings will have equal acceleration as shown



Mg – Tcos45° = ma

T sin 45° = ma

a = g/2

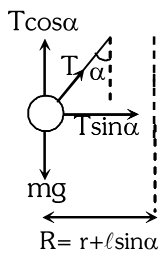
39. Ans.(C)

$$T\sin\alpha = m\omega^2 R$$

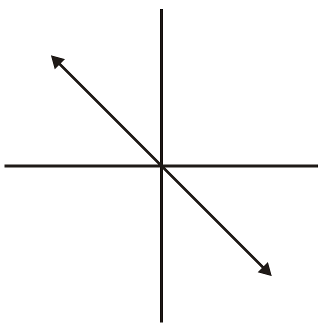
$$T\cos\alpha = mg$$

$$\tan\alpha = \frac{\omega^2 R}{g}$$

$$\omega = \sqrt{\frac{g\tan\alpha}{R}} = \sqrt{\frac{g\tan\alpha}{r+l\sin\alpha}}$$



40. Ans. (D)



41. Ans. (D)

42. Ans. (C)

43. **Ans. (C)**

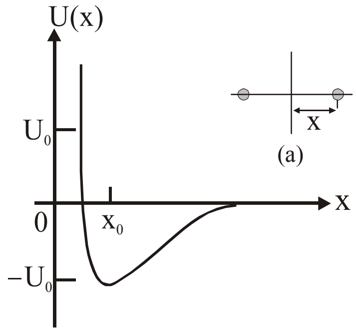
Solution (41 to 43)

By Eq. $F_x = -\frac{\partial U}{\partial x}$, the force is the negative of the derivative of U with respect to x :

$$F_x = \frac{\partial U}{\partial x} = -U_0\left(-\frac{2}{b}e^{-2(x-x_0)/b} + \frac{2}{b}e^{-(x-x_0)/b}\right)$$

The force is zero if $-\frac{2}{b}e^{-2(x-x_0)/b} + \frac{2}{b}e^{-(x-x_0)/b} = 0$

which happens when $x = x_0$.



As total energy is given by $E = U(x) + \frac{1}{2}mv^2$

The regions x > a and x < a' are forbidden regions; the region a' ≤ x ≤ a is permitted. The particle is said to be in a bound orbit. The motion is periodic, i.e., it repeats again and again whenever the particle returns to its starting point.

$$F_x = -\frac{\partial U}{\partial x}$$

This gives $$F_x = \frac{2U_0}{b}\left(e^{-2(x-x_0)/b} - e^{-(x-x_0)/b}\right)$$

In the vicinity of the equilibrium point, the motion of the atom will be simple harmonic. The "spring" constant for motion is

$$k = -\left.\frac{dF_x}{dx}\right|_{x=x_0} = -\frac{2U_0}{b}\left(-\frac{2}{b} - \frac{1}{b}\right) = \frac{6U_0}{b^2}$$

44. **Ans. (B)**

45. **Ans. (A)**

46. Ans. (A)

Sol. (17 to 19)

$U(x) = 2x^2 - 9x^2 + 12x \Rightarrow U'(x) = 6x^2 - 18x + 12 \Rightarrow U''(x) = 12x - 18$

at equilibrium $U'(x) = 0 \Rightarrow x = 2,1$

for minima $U''(x) > 0$ and for maxima $U''(x) < 0$

$U''(1) = -6$; $U''(2) = 6$ $\Rightarrow$ Maxima is at x = 1 & minima is at x=2

for stable equilibrium x_0 -2

• for oscillatory motion $E < U(1) \Rightarrow E < 2 - 9 + 12 < 5J$

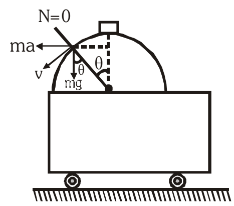
• $F = U'(x) = -6(x^2 - 3x + 2) = -6(x-2)(x-1)$

Replaced x = X + 2 ⇒F = – 6X ⇒ $m\omega^2 = 6 \Rightarrow 1.5\ \omega^2 = 6 \Rightarrow \omega = 2$ rad/sec

$\because T = \dfrac{2\pi}{\omega} \Rightarrow T = \pi$ sec

47. Ans. (A)

$\because$ N = 0 $\therefore$ $mg\cos\theta - ma\sin\theta = \dfrac{mv^2}{R}$ and from work energy theorem W=ΔKE



ma (Rsinθ) + mgR (1–cosθ) = $\dfrac{1}{2}mv^2$

$\Rightarrow \dfrac{2}{3}\sin\theta + 2 - 3\cos\theta = 0 \Rightarrow \theta = 37°$

48. Ans. (D)

Required height = h + Rcosθ= $\dfrac{4R}{3} + \dfrac{4R}{5} = \dfrac{32R}{15}$

49. Ans. (C)

By using s = ut + $\dfrac{1}{2}at^2$: Initial vertical velocity of mass = $v\sin\theta = \left(\sqrt{\dfrac{2Rg}{3}}\right)\left(\dfrac{3}{5}\right) = \sqrt{\dfrac{6Rg}{25}}$

$\dfrac{4R}{3} + \dfrac{4R}{5} = \left(\sqrt{\dfrac{6Rg}{25}}\right)t + \dfrac{g}{2}t^2 \Rightarrow t^2 + 2\sqrt{\dfrac{6R}{25g}} - \dfrac{64R}{15g} = 0 \Rightarrow t = \dfrac{-2\sqrt{\dfrac{6R}{25g}} \pm \sqrt{4\left(\dfrac{6R}{25g}\right) + \dfrac{4\times 64R}{15g}}}{2}$

$\Rightarrow t = -\sqrt{\dfrac{6R}{25g}} \pm \sqrt{\left(\dfrac{6R}{25g}\right)\left(\dfrac{169}{9}\right)} \Rightarrow t = -\sqrt{\dfrac{6R}{25g}} \pm \sqrt{\dfrac{6R}{25g}}\left(\dfrac{13}{3}\right) \Rightarrow t = 2\sqrt{\dfrac{2R}{3g}}$

50. Ans. (C)

51. Ans. (B)

Radial acceleration at $\theta_0 = \dfrac{v^2}{R} = \dfrac{\frac{5}{6}Rg}{R} = \dfrac{5}{6}g$

52. Ans. (A)

In presence of friction $\frac{1}{2}mv_0^2 + mgR(1 - \cos\theta) = \frac{1}{2}mv^2 + W_f$ where W_f = work done against friction

$\Rightarrow \cos\theta = \frac{2}{3} + \frac{v_0^2}{3Rg} - \frac{W_f}{mgR} \Rightarrow \cos\theta < \cos\theta_0 \Rightarrow \theta > \theta_0$

53. Ans. (B)

54. Ans. (B)

55. Ans. (B)

56. Ans. (D)

57. Ans. (A)

58. Ans. (D)

Acceleration changed the direction from A.

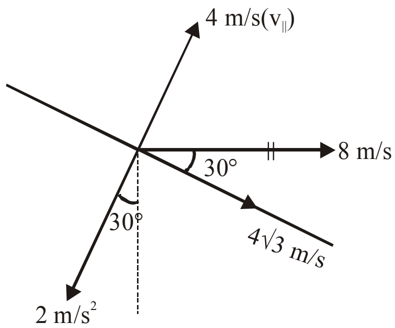
59. Ans. (D)

Initial velocity is non-zero.

60. Ans. (A)

s = ut + ½ at^2 at point a is velocity is zero, so t = $\sqrt{\frac{2s}{g}}$

61. Ans. (D)



The component 4√3 will not change with time to get 8 m/s again, ⊥ component should again be 4 m/s.

$\therefore t = \frac{2 \times 4}{2} = 4$ sec

62. Ans. (C)

a & v will be ⊥ if component of velocity parallel to acceleration ($v_{||}$) become zero.

$0 = 4 - 2 \times t$

t = 2sec

63. Ans. (A)

In 4 seconds, displacement in the direction of accelreation is zero & for ⊥ direction

$S = 4\sqrt{3} \times 4 = 16\sqrt{3}$ m

64. Ans. (A)

$x_1 + x_2 = 10$

$x_1 = \frac{10}{3}, \quad x_2 = \frac{20}{3},$

$U_1 = \frac{1000}{9}$, $U_2 = \frac{2000}{9} \Rightarrow \frac{U_1}{U_2} = \frac{1}{2}$

65. Ans. (B)

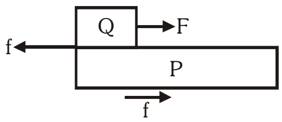
$F = K_1x_1 = \frac{200}{3}$ N

66. Ans. (B)

Spring force will not change instantly F = ma

$\Rightarrow a = \frac{40}{3} m/s^2$

67. Ans. (A)



$F - f = m_Q a = (2a)$; $f = m_P.a = 8a \Rightarrow f = 8N$

68. Ans. (C)

For some time both blocks will move with same constant acceleration. Then when friction force becomes (μmQ_g) then relative motion starts. So, Q will have greater acceleration than P. So, graph will be (C).

69. Ans. (A)

$F - f = m_Q a_Q \Rightarrow$ when relative motion does not start. $a_Q = a_P$

For limiting condition : $f_{max} = \mu m_Q g = 16N \Rightarrow a_P = 2\ m/s^2$, $a_Q = \left(\frac{F-f}{2}\right) = \left(\frac{5t-16}{2}\right) = 2$

$\Rightarrow$ t=4 sec $\Rightarrow$ v_P and v_Q are same till t=4 sec.

$F = (m_P + m_Q)\ a = 5t \Rightarrow a = \left(\frac{5}{10}\right)t \Rightarrow \frac{dv}{dt} = \frac{1}{2}t \Rightarrow \int_0^{v_1} dv = \int_0^4 \frac{1}{2} t dt = \left[\frac{t^2}{4}\right]_0^4 = 4m/s$

After 4 sec :– Equation of force balance :– $F - 16 = m_Q a_Q \Rightarrow \left(\frac{5t-16}{2}\right)dt = dv$

$\Rightarrow \int_{v_1}^{v_Q} = \left[\left(\frac{5t^2}{4} - 8t\right)\right]_4^6 \Rightarrow (v_Q - 4) = \frac{5(36)}{4} - 8(6) - \frac{5(16)}{4} + 8(4) \Rightarrow v_Q = 4 + 9 = 13\ m/s$

and for P $\Rightarrow v_P - v_1 = \left(\frac{16}{8}\right)$ (6–4) =4 $\Rightarrow v_P = 8$ m/s

70. Ans. A

$F_{max} = kx + \mu mg$; $F_{min} = kx - \mu mg$ $\therefore F_{max} - F_{min} = 2\ \mu mg \Rightarrow 2 = 2\ \mu\ 10$ $\therefore \mu = 0.1$

71. Ans. A

$F_{max} + F_{min} = 2$ kx. From graph $F_{max} + F_{min} = 5$ and x = 0.1 $\Rightarrow$ k = 25 N/m.

72. Ans.A

When x = 0.03 ; kx = 25 × 0.03 = 0.75 N, which is less than μ mg=0.1 × 10 = 1 N

$\therefore$ The block will be at rest, without applying forceF.

73. Ans. (D)

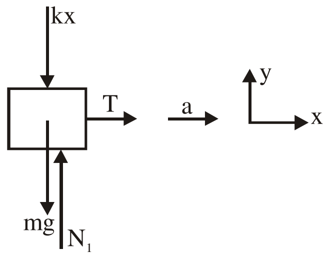
$\Sigma F_x = ma_x$

$\Sigma F_y = 0$

$kx + mg = N$

$kx = 46 - 40$

$k = \dfrac{6}{0.03} = 200$ N/m.



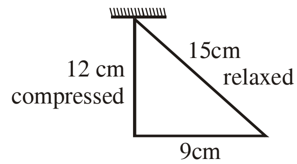
74. Ans. (A)

When $N_1 = 40$ N, kx = 0, since acceleration of B can never be more than 10 m/s^2, the string remains taut.

So $a_A = a_B = \dfrac{10N}{5kg} = 2$ m/s^2

75. Ans. (B)

As the block moves the spring pushes the block such that the force has a horizontal component. So acceleration of A & B will be more that what it would be if there was no spring. When no spring $a_A = a_B = 2$ m/s^2 always. Time taken is t : $\dfrac{9}{100} = \dfrac{1}{2} \times 2 \times t^2 \Rightarrow$ t = $\dfrac{3}{10}$ = 0.3 s.



So time with spring will be less than 0.3 s.

76. Ans. (A)

77. Ans. (A)

78. Ans. (A)

79. Ans. (D)

80. Ans. (B)

81. Ans. (D)

82. Ans. (D)

83. Ans. (C)

84. Ans. (D)

85. Ans. (B)

86. Ans. (A)

87. Ans. (D)

88. Ans. (A) P,Q (B) P,Q,S (C) P,Q,R (D) P,Q,R

Sol. No slipping anywhere.

Net force is centripetal as v = constant

89. Ans. (A) PST (B) PR (C) PRST (D) P

90. Ans.(A) Q, (B) P,Q,T (C) P,T (D) P,R,S

(A) Friction along tangent only.

(B) Friction provide centripetal and tangential acceleration.

(C) Friction provide only centripetal acceleration.

(D) If speed is not equal to $\sqrt{rg\tan\theta}$ then friction is along the incline of road to avoid slipping & if speed is equal to $\sqrt{rg\tan\theta}$, no friction.

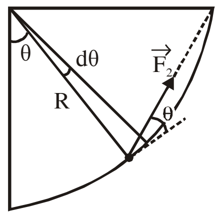
91. Ans. (A) P, Q, R,T ; (B) P, Q, R, S, T ; (C) P ; (D) Q, R, S, T

92. Ans. (A) Q,R (B) Q,R (C) P,R ; (D) Q,R,T

93. Ans. (A) T (B) P (C) R (D) Q

94. Ans. (A) →(R); (B) →(Q); (C) →(S); (D) →(R)

For (A) : Work done by $\vec{F}_1 = FR$



For (B) : $dW = \vec{F}.d\vec{s} = (FRd\theta)\cos\theta = FR\cos\theta d\theta$

$$W = \int_0^{\pi/4} FR\cos\theta d\theta = FR(\sin\theta)_0^{\pi/4} = \frac{FR}{\sqrt{2}}$$

For (C) : $W = \int \vec{F}.d\vec{s} = F\left(\frac{\pi R}{2}\right) = \frac{\pi FR}{2}$

For (D) : $W = \int \vec{F}.d\vec{s} = (F)(R) = FR$

95. Ans. (A)-P, (B)-S, (C)-R, (D)-Q

96. Ans. (A) RQS (B) PR (C) PR

97. Ans. (A) S (B) P (C) Q, R (D) T

98. Ans. (A) P, (B) Q,S,T, (C) R,S, (D) P,T

99. Ans. (A) PQRST, (B) PRT (C) PQT (D) PQST

For a body in equilibrium $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$ i.e. $\vec{F}_1 + \vec{F}_2 = -\vec{F}_3$

On equilibrium in case PRT $F_1^2 + F_2^2 = F_3^2$

Spring force does not change instantaneously but string force can change suddenly when gravity is reversed it acts upward. In case PQST net force becomes upward.

100. Ans. (A) PQRS (B) PQRS (C) PQRS (D) QS

For (A) : $F_{net_{external}} = 0$ $\Rightarrow a_{CM} = 0$
$\Rightarrow$ momentum is conserved
$\Rightarrow$ work done by external=0 $\Rightarrow$ Mechanical energy is conserved

Potential energy gets converted to K.E. $\Rightarrow$ KE $\neq$ constant

For (B) : $F_{net_{external}} = 0$ $\Rightarrow a_{CM} = 0$
$\Rightarrow$ momentum is conserved
$\Rightarrow$ work done by external=0 $\Rightarrow$ Mechanical energy is conserved

KE gets stored in the masses during collision as PE $\Rightarrow$ KE $\neq$ constant

For (C) : $F_{net_{external}} = 0$ $\Rightarrow a_{CM} = 0$
$\Rightarrow$ momentum is conserved
$\Rightarrow$ work done by external=0 $\Rightarrow$ Mechanical energy is conserved

KE gets stored in the masses during collision as PE $\Rightarrow$ KE $\neq$ constant

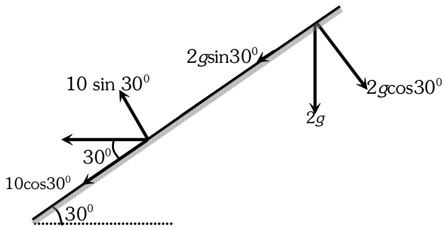
For (D) : $F_{net_{external}} \neq 0$ $\Rightarrow a_{CM} \neq 0$
$\Rightarrow$ momentum of system is not conserved

K.E. gets conserved in PE of spring $\Rightarrow$ KE $\neq$ constant.

EXERCISE # (S)

1. Ans. 3

Acceleration of ball w.r.t. plane

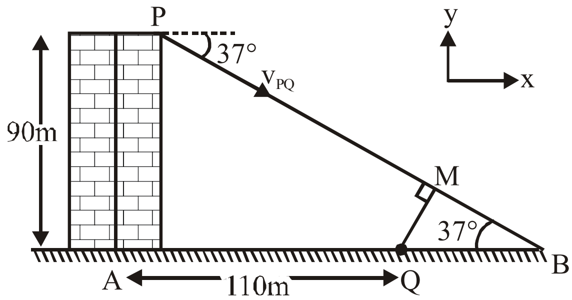


$a_{bp} = a_{bg} - a_{pg} = 2g$ (downward)

$s = ut + \frac{1}{2}at^2 \Rightarrow 0 = (10\sin 30^0)\, t - \frac{1}{2} \times (2g\cos 30^0)\, t^2$

$\Rightarrow t = 1/Ö3$

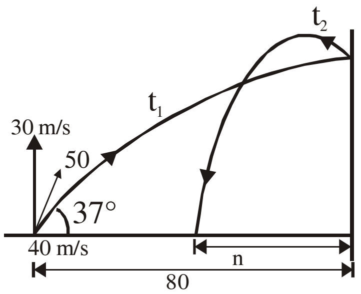
2. Ans. 6



$\vec{v}_{PQ} = \vec{v}_P - \vec{v}_Q = \left(\frac{10}{\sqrt{2}}\hat{i} + \frac{10}{\sqrt{2}}\hat{j}\right) - \left(-\frac{70}{\sqrt{2}}\hat{i} + \frac{70}{\sqrt{2}}\hat{j}\right) = 40\sqrt{2}\,\hat{i} - 30\sqrt{2}\,\hat{j}$

AB= $90 \times \frac{4}{3}$ = 120m Þ QB = 10m Þ QM =10 sin37° = $10 \times \frac{3}{5}$ =6m

3. Ans. 140



After first collision : $v_y = 30 - gt$; $v_x = -20\hat{i}$; $t_1 = \frac{80}{40} = 2\,\text{sec}$; $t_2 = T - t_1 = 4$ sec

Before second collision : $v_x = -20\hat{i}$; x = 20 × 4 = – 80 m; $v_y = 10 - 10\,(t_2) = -30\,\hat{j}$

After second collision : $v_X = -20\hat{i}$; $v_y = +15\hat{j}$ Range = 60 m Net : 60 m + 80 m = 140 m

4. Ans. 5

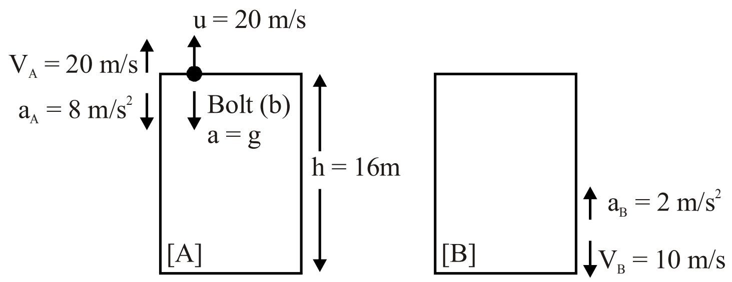
For collision $u=\sqrt{u_B^2+u_C^2}=\sqrt{\left(\frac{72}{5}\times\frac{5}{18}\right)^2+\left(\frac{54}{5}\times\frac{5}{18}\right)^2}=\sqrt{4^2+3^2}=5\text{ m/s}$

5. Ans. 7

As $vdv=\vec{a}.\overrightarrow{dr}=adx=5dx \Rightarrow \int_0^v vdv=5\int_0^{4.9}dx \Rightarrow \frac{v^2}{2}=5(4.9) \Rightarrow v^2=49 \Rightarrow v=7\text{ m/s}$

6. Ans: 27R/28

7. Ans. 51 m



In the ref. frame of A :

$V_{b/A}=0$ $\qquad$ $a_{b/A}=2\text{m/s}^2$ –

$S_{b/A}=16\text{ m}$ –

$T=\sqrt{\left(\frac{2\times16}{2}\right)}=4\text{s}$ $\qquad$ **for Bolt (b) :** $\qquad$ $0=30-12(t_1)$

$t_1=\frac{30}{12}=\frac{10}{4}=2.5\text{s} \Rightarrow$ $\qquad$ $h_1=\frac{(30)^2}{2(12)}=\frac{900}{24}$

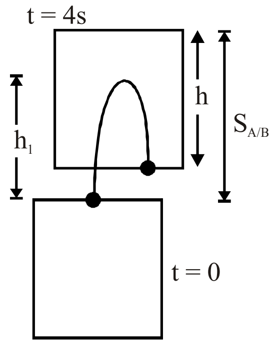
$h_1=\frac{75}{2}$

In the ref. frame of B :

$V_{b/B}=30\text{ m/s}$ - $\qquad$ $a_{b/B}=12\text{m/s}^2$ – $\qquad$ $T=4\text{s}$

$V_{A/B}=30\text{ m/s}$ - $\qquad$ $a_{A/B}=10\text{m/s}^2$ –

for A :



$S_{A/B}=30(4)-\frac{1}{2}(10)(4)^2=120-80=40$

distance $=2h_1+h-S_{A/B}=75+16-40=51\text{ m}$

8. Ans. 8

$a=-3$

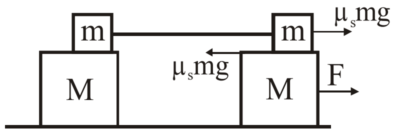
$u=\sqrt{\left(25\times\frac{4}{5}-8\right)^2+\left(25\times\frac{3}{5}\right)^2}=\sqrt{(12)^2+(15)^2}=3\sqrt{41}$

at t = $\frac{3\sqrt{41}}{3}$ so it will stop relative motion before 10 sec. So it will have same velocity as plank.

9. Ans. 5

Direction of friction will be opposite to the direction of relative slipping.

10. Ans. F = 2$\mu_s$mg $\left(\frac{m+M}{2m+M}\right)$



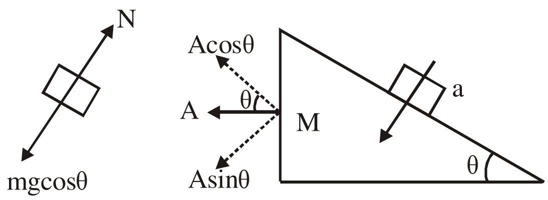
$\mu_s$mg = (M+2m)a $\Rightarrow a = \frac{\mu_s mg}{M+2m}$

Also $F - \mu_s mg = Ma \Rightarrow 2\mu_s mg\frac{(m+M)}{(M+2m)}$

11. Ans. 2

mgcosθ – N = mAsinθ...(i)

Nsinθ = MA...(ii)



$A = \frac{mg\sin\theta\cos\theta}{M+m\sin^2\theta}$, $a = \frac{mg\sin^2\theta\cos\theta}{M+m\sin^2\theta}$

$\frac{A}{a} = \frac{1}{\sin 30°} = 2$

12. Ans. 9

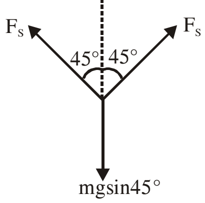
$a = \frac{(5-4)g}{9} = \frac{g}{9}; V_1 = \sqrt{2a\times H} = \sqrt{2\times\frac{g}{9}\times 2} = \frac{2}{3}\sqrt{g}$; $V_1 = \frac{D}{t} = 2 = \frac{2}{3}\sqrt{g} \Rightarrow$ g = 9

13. Ans. 4

Slipping occurs : $a_{m_1} = 10m/s^2, a_{m_2} = 2m/s^2$

$a_{m_1/m_2} = 8m/s^2$, S = 16 m ⇒ t = 2sec

14. Ans. 5



$\frac{2F_S}{\sqrt{2}} = \frac{mg}{\sqrt{2}}$; $F_S = \frac{mg}{2}$ ∴ acceleration = $\frac{F_S}{m} = \frac{g}{2} = 5$ m/s²

15. Ans 5

The girl will catch the ball when their horizontal displacement becomes equal

$$\therefore 12t = \frac{1}{2}\left(g\sin 37^\circ \cos 37^\circ\right)t^2 \Rightarrow t = 5\,\text{sec}$$

16. Ans. 200 N

$$\cos\theta = \frac{1.5}{2} = \frac{3}{4}$$

$45g = 3T\cos\theta$

$$\frac{150}{3/4} = T$$

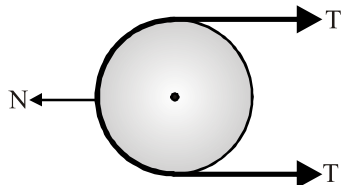
$200 = T$

17. Ans. 12 N

Sol. $2T - N = m\omega^2 R_{cm} = \mu \times \pi R \times \left(\frac{V}{R}\right)^2 \times \frac{2R}{\pi}$

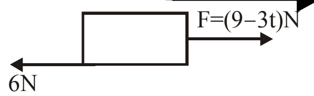
$2T - N = 2\mu v^2$

$N = 2T - 2\mu v^2 = 2 \times 10 - 2 \times 1 \times 4 = 12\ N$



18. Ans. 0

$f_k = 6$ N which will act till velocity is not zero.



Let velocity becomes zero at t, then $0 - 4.5 = \int_0^t (3 - 3t)dt \Rightarrow t = 3$ so at t = 3, v = 0

19. Ans. 5

Location of COM when they all fall simultaneously = 1100 m east.

For Y-axis, $\frac{m \times 120 - m \times 60 + 2m \times y}{4m} = 0$

For X-axis $\frac{m \times 550 + m \times 550 + 2m \times x}{4m} = 1100$

$\therefore$ x = 1650, y = 30

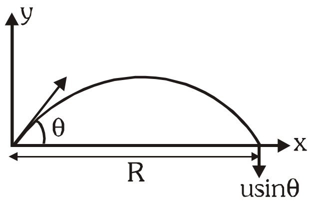
20. Ans. 50

$v_1 = v_2 = v_3$

$h = v(t) + \frac{1}{2}gt^2$

$h = -\frac{v}{2}t + \frac{1}{2}gt^2$

21. Ans. 2



u = 18 km/hr = 5 m/s

$$\omega = \frac{v_1}{r} = \frac{u\sin\theta}{R} = \frac{u\sin\theta}{\frac{2(u\cos\theta)(u\sin\theta)}{g}} = \frac{g}{2u\cos\theta} = \frac{10}{2\times 5\times \frac{1}{2}} = 2\text{rad}/\text{s}$$

22. **Ans. 729**

$$\frac{dv}{dt} = \frac{v^2}{R} \Rightarrow \int_{v_0}^{v} \frac{dv}{v^2} = \frac{1}{R}\int_0^t dt \Rightarrow \left(-\frac{1}{v}\right)_{v_0}^{v} = \frac{t}{R} \Rightarrow v = \frac{v_0}{1-\frac{v_0}{R}t} \Rightarrow \frac{ds}{dt} = \frac{v_0}{1-\frac{v_0}{R}t} \Rightarrow \int_0^{2\pi R} ds = \int_0^t \frac{v_0}{\left(1-\frac{v_0}{R}t\right)}dt$$

$$\Rightarrow 2\pi R = -R\left[\ell n\left(1-\frac{v_0}{R}t\right)\right]_0^t \Rightarrow 2\pi = -\ell n\left(1-\frac{v_0}{R}t\right) \Rightarrow 1-\frac{v_0}{R}t = e^{-2\pi} \Rightarrow t = \frac{R}{v_0}\left(1-e^{-2\pi}\right)$$

$$\Rightarrow a = 1, \beta = 2 \Rightarrow (\alpha+\beta)^6 = (1+2)^6 = 729$$

23. **Ans. 9**

$$\frac{dx}{dt} = v_0, \ \frac{dy}{dt} = a\omega\cos\omega t \Rightarrow x = v_0 t, \ y = a\sin\omega t$$

$$\Rightarrow y = a\sin\left(\frac{\omega x}{v_0}\right) \Rightarrow m=1, n=1 \Rightarrow (m+2n)^2 = (1+2)^2 = 9$$

24. **Ans. 6**

$$\alpha = \frac{\omega d\omega}{d\theta}$$

$$\int \omega d\omega = \int \alpha d\theta$$

$$\frac{\omega^2}{2} = \text{area under } \alpha \text{ vs } \theta \text{ graph} = \frac{1}{2}(9\times 4)$$

$$\omega = \sqrt{36} = 6 \text{ rad/s}$$

25. **Ans. 3**

26. Ans. 3

(a) $m_1 g \sin\theta = (m_1+m_1)a_1$

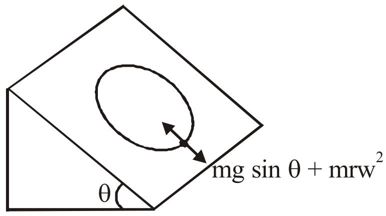
(b) $(m_2-m_1)\, g \sin\theta/2 = (m_1 + m_2)\, a_2$

27. Ans. 6

Sol. $f \geq mg \sin\theta + mrw^2$

$\mu\, mg \cos\theta \geq mg \sin\theta + mrw^2$

or $\mu \geq \tan\theta + \dfrac{w^2 r}{g\cos\theta}$

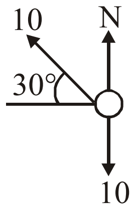


28. Ans. 5

$T\cos\alpha = m\omega^2(0.1)\cos\alpha$

$T = 1\times 100\times 0.1 = 10$

$10\sin 30° + N - 10 = 0 \Rightarrow N = 5$



29. Ans. 1

When the block has fallen by 60°, $\frac{1}{2}mv^2 = mgR(1-\cos\theta) \Rightarrow v = \sqrt{2gR\times\frac{1}{2}} = \sqrt{gR}$

Applying NLM along the radial direction $mg\cos\theta + kx = \dfrac{mv^2}{R} \Rightarrow x = \dfrac{1}{2}m$

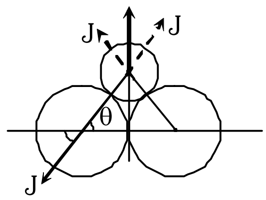
Extension in spring = 0.5 m ⇒Natural length = (R–x) = 1m

30. Ans. 1

For smaller ball

$2\int J dt\cos\theta = M\times v_0$ ...(i)

For larger ball $2\int J dt\sin\theta = Mv$ ...(ii)



$\sin\theta = \dfrac{R}{(3/2)R} = \dfrac{2}{3}$

$v = \dfrac{v_0}{2}\tan\theta = \dfrac{v_0}{2}\dfrac{2}{\sqrt{5}} = \dfrac{v_0}{\sqrt{5}}$

Therefore speed is 1m/s

31. Ans. 2

$$T = 2\left[\frac{1}{2}\sqrt{\frac{2R}{F/m}} + \frac{1}{4}\sqrt{\frac{2R}{(F/m)}} + \frac{1}{8}\sqrt{\frac{2R}{(F/m)}}.....\infty\right] = \sqrt{\frac{2R}{(F/m)}}\left[+1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+.........+.....\infty\right]$$

32. Ans. $T = 2\sqrt{\frac{2h}{g}}$, $x = \nu\sqrt{\frac{h}{2g}}$

33. Ans. straight line, $\frac{2\pi r}{v_0}$, $\frac{\sqrt{3}v_0}{2}$

34. Ans. 10

$50(L-x) - 150x = 0 \Rightarrow 50L - 200x = 0 \Rightarrow x = \frac{L}{4}$

Similarly in y direction : $y = \frac{L}{4}$

$$S = \sqrt{x^2 + y^2} \Rightarrow S = \frac{L}{2\sqrt{2}} = \frac{20\sqrt{2}}{2\sqrt{2}} = 10$$

35. Ans. 8

$F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$ when $\theta = \tan^{-1}(\mu) \Rightarrow W = (F_{min})\, S\cos\theta = \frac{\mu mgS}{(1+\mu^2)} = 8$ joule

36. Ans. 60

$$\mu = \frac{1}{10}\left(\frac{x}{2}+1\right)$$

$$W = \int \mu mg dx = \frac{mg}{10}\left[\frac{x^2}{4} + x\right]_0^2$$

$$W = \frac{mg}{10}[3-0] = \frac{20\times 10\times 3}{10} = 60J$$

37. Ans. 100

$W = \Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$ Here $W = \vec{F}.\vec{d} = (30\hat{i} + 40\hat{j}).(100\hat{i} + 100\hat{j}) = 3000 + 4000 = 7000$

$$\Rightarrow 7000 = \frac{1}{2}\times 2\times v_2^2 - \frac{1}{2}\times 2\times 3000 \Rightarrow v_2^2 = 10000 \Rightarrow v_2 = 100$$