Physics

TARGET : JEE 2013

SCORE

JEE (Advanced)

Home Assignment # 05

ALLEN CAREER INSTITUTE
KOTA (RAJASTHAN)
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MECHANICAL WAVE

EXERCISE # (O)

1. A stiff wire is bent into a circular loop of diameter D. It is clamped rigidly by knife edges at two points A and B diametrically opposite each other. A transverse wave is sent around the loop by means of a small vibrator. The resonant frequencies of the loop in term of wave speed v and diameter D (n is a positive integer) will be given by :-

(A) $f_n = n\left[\frac{2v}{3D}\right]$ (B) $f_n = n\left[\frac{2v}{\pi D}\right]$ (C) $f_n = n\left[\frac{v}{\pi D}\right]$ (D) $f_n = n\left[\frac{v}{2D}\right]$

2. A wave propagates in a string in the positive x-direction with velocity v. The shape of the string at $t = t_0$ is given by $f(x, t_0) = A \sin\left(\frac{x^2}{a^2}\right)$: then, the wave equation at any instant t is given by

(A) $g(x,t) = A \sin \frac{\left[x - v(t - t_0)\right]^2}{a^2}$ (B) $g(x,t) = A\sin\frac{\left[x + v(t - t_0)\right]^2}{a^2}$

(C) $g(x,t) = A\sin\frac{\left[x - v(t + t_0)\right]^2}{a^2}$ (D) $g(x,t) = A\sin\frac{\left[x + v(t + t_0)\right]^2}{a^2}$

3. A source of sound of single frequency ν_0 flies along a straight line which is at a finite distance from the observer. Then the observer hears

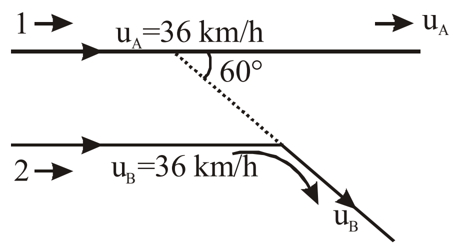
(A) a frequency ν_0 at the instant when the source is nearest to him

(B) a frequency greater than ν_0 the instant when the source is nearest to him

(C) a frequency ν_0 at an instant later than the instant of nearest position of the source

(D) the variation of frequency with time is linear.

4. Two trains move in the same direction on two close tracks. The train A sounds a horn of single frequency of 1 kHz. At a certain instant of time, when both trains move at the same speed of 36 km/hr, the two straight tracks deviate at an angle of 60° with each other. Velocity of sound in air is 300 m/s. Which of the following statement(s) is/are true ?



(A) The apparent frequency heard by a passenger on the second train B just after he passes the bend will be 1.05 kHz.

(B) The apparent frequency heard by a passenger on the first train A, just after the second train B passes the bend will be 1 kHz.

(C) The apparent frequency heard by a passenger on the second train B just after he passes the bend will be 983 Hz.

(D) If the first train A, that produces the sound, were to pass the bend while the other train B goes on the straight track then the apparent frequency heard by a passenger on the latter train B will be less than 1 kHz.

5. A source is moving across a circle given by the equation $x^2 + y^2 = R^2$, with constant speed $\frac{330\pi}{6\sqrt{3}}$ m/s, in anti-clockwise sense. A detector is at rest at point (2R, 0) w.r.t. the centre of the circle. If the frequency emitted by the source is f and the speed of sound, c = 330 m/s. Then

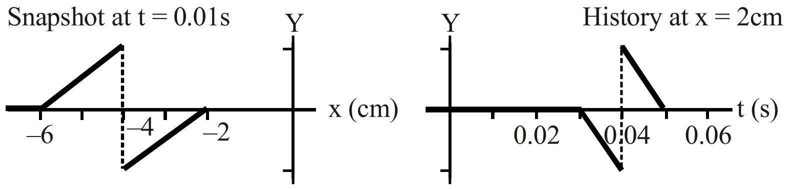
(A) the position of the source when the detector records the maximum frequency is $\left(+\frac{\sqrt{3}}{2}R, -\frac{R}{2}\right)$

(B) the coordinate of the source when the detector records minimum frequency is (0, R)

(C) the minimum frequency recorded by the detector is $\frac{6\sqrt{3}}{\pi+6\sqrt{3}}f$

(D) the maximum frequency recorded by the detector is $\frac{6\sqrt{3}}{6\sqrt{3}-\pi}f$

6. Figure shows a snapshot graph and a history graph for a wave pulse on a stretched string. They describe the same wave from two perspectives.



(A) the wave is travelling in positive x-direction
(B) the wave is travelling in negative x-direction
(C) the speed of the wave is 2 m/s.
(D) the peak is located at x = –6 cm at t = 0.

7. A pulse is started at a time t = 0 along the +x direction on a long, taut string. The shape of the pulse at t = 0 is given by function f(x) with

$$f(x) = \begin{cases} \frac{x}{4}+1 & for \quad -4 < x \le 0 \\ -x+1 & for \quad 0 < x < 1 \\ 0 & otherwise \end{cases}$$

here f and x are in centimeters. The linear mass density of the string is 50 g/m and it is under a tension of 5N.

(A) The shape of the string is drawn at t = 0 then the area of the pulse enclosed by the string and the x-axis is 2.5 cm^2.
(B) The shape of the string is drawn at t = 0 then the area of the pulse enclosed by the string and the x-axis is 5 cm^2.
(C) The transverse velocity of the particle at x = 13 cm and t = 0.015 s will be – 250 cm/s
(D) The transverse velocity of the particle at x = 13 cm and t = 0.015 s will be 250 cm/s

8. A standing wave of time period T is set up in a string clamped between two rigid supports. At t = 0 antinode is at its maximum displacement 2A.

(A) The energy density of a node is equal to energy density of an antinode for the first time at t = T/4.

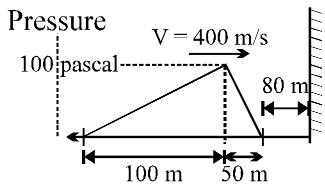
(B) The energy density of node and antinode becomes equal after T/2 second.

(C) The displacement of the particle at antinode at $t = \frac{T}{8}$ is $\sqrt{2}A$

(D) The displacement of the particle at node is zero

Paragraph for Question Nos. 9 to 12

A plane pressure pulse triangular in shape approaches a rigid wall along normal at a speed of 400 m/s. At time t = 0, situation is shown in the figure. The peak pressure is 100 P. By the wall pulse gets reflected and pressure near the wall gets doubled. Height of the wall is 2m and width is also 2m. A detector on the wall records a minimum excess pressure of 16 pascal.



9. When for the first time detector will record the pressure pulse

(A) 22×10^{-2} s (B) 21×10^{-2} s (C) 31.5×10^{-2} s (D) None of these

10. For how much time the detector will record the pulse

(A) 37.5×10^{-2} s (B) 34.5×10^{-2} s (C) 31.5×10^{-2} s (D) None of these

11. What is the maximum force applied by the pulse on the wall

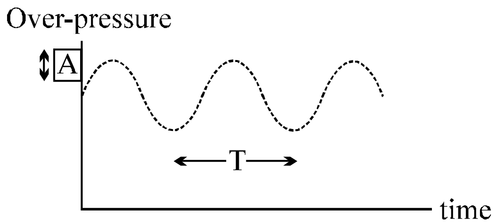
(A) 400 N (B) 800 N (C) 200 N (D) None of these

12. Total impulse imparted by the pulse on the wall will be

(A) 300 Ns (B) 150 Ns (C) 750 Ns (D) None of these

Paragraph for Question Nos. 13 to 15

The next three questions refer to this graph which shows pressure versus time at the position of a listener's ear as a sinusoidal sound wave passes by. The wave has period T and amplitude A.



13. If the amplitude A of the sound wave is decreased, the listener reports that .....

(A) The loudness (intensity) decreases (B) The loudness (intensity) increases

(C) The pitch (frequency) decreases (D) There is no change in the sound

14. If the period T of the sinusoidal sound wave is increased keeping pressure amplitude constant, the listener reports that

(A) The loudness (intensity) decreases (B) The loudness (intensity) increases

(C) The pitch (frequency) decreases (D) The pitch (frequency) increases

(E) There is no change in the sound

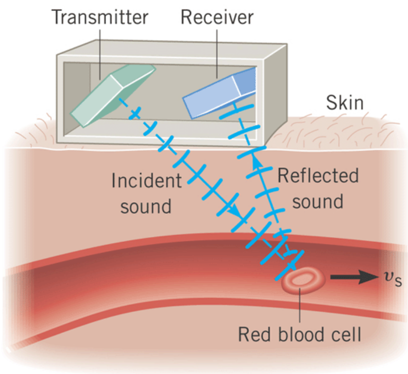
15. If the period T of the sinusoidal sound wave is increased, what happens to the speed of the sound?

(A) Speed increases (B) Speed decreases

(C) Speed remains the same (D) None of these

Paragraph for Question Nos. 16 to 18

The Doppler flow meter is a particularly interesting medical application of the Doppler effect. This device measures the speed of blood flow, using transmitting and receiving elements that are placed directly on the skin, as in Figure. The transmitter emits a continuous sound whose frequency is typically about 5 MHz. When the sound is reflected from the red blood cells, its frequency is changed in a kind of Doppler effect because the cells are moving with the same velocity as the blood. The receiving element detects the reflected sound, and an electronic counter measures its frequency, which is Doppler-shifted relative to the transmitter frequency. From the change in frequency the speed of the blood flow can be determined. Typically, the change in frequency is around 600 Hz for flow speeds of about 0.1 m/s.



16. Assume that the red blood cell is directly moving away from the source and the receiver. What is the (approx) speed of the sound wave in the blood?

(A) 1700 m/s (B) 330 m/s (C) 5000 m/s (D) 3000 m/s

17. An abnormal segment of the artery is narrowed down by an arteriosclerotic plaque to one-fourth the normal cross-sectional area. What will be the change in frequency due to reflection from the red blood cell in that region?

(A) 150 Hz (B) 300 Hz (C) 600 Hz (D) 2400 Hz

18. At what extra rate does the heart have to work due to this narrowing down of the artery? Assume the density to be 1.5 gm/cc and the area of the normal artery to be 0.1 cm^2.

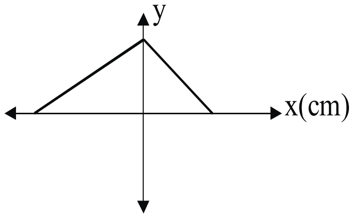
(A) 1.125×10^{-4} W (B) 2.5×10^{-4} W (C) 6.25×10^{-5} W (D) 5.625×10^{5} W

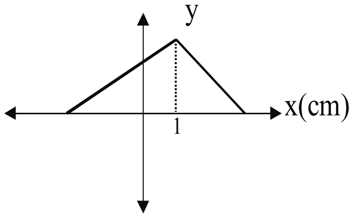
Paragraph for Question Nos. 19 to 21

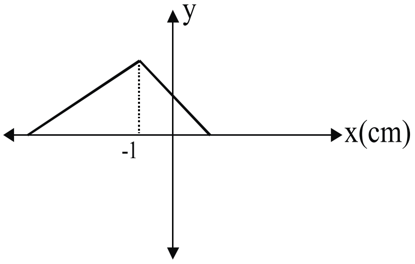
At a time t =0 a pulse moving in the +ve x-direction with velocity v in the shape given by the function

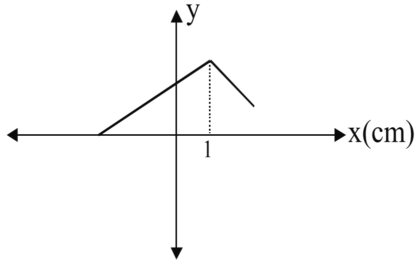
between with $y = \begin{cases} \dfrac{x}{4}+1 & -4 < x < 0 \\ -x+1 & 0 < x < 1 \\ 0 & \textit{otherwise} \end{cases}$, where x and y are in cm.

19. If velocity is 10 m/s, which of the graphs shows the shape and position of the string at a time t = 1 millisec

(A)


(B)


(C)


(D)


20. The wave equation that explicitly gives the displacement of any point at any time is

(A) $y = \begin{cases} \frac{1}{4}(x - Vt) + 1 & -4 < (x - Vt) < 0 \\ -(x - Vt) + 1 & 0 < (x - Vt) < 1 \\ 0 & otherwise \end{cases}$

(B) $y = \begin{cases} \frac{1}{4}(x + Vt) + 1 & -4 < (x + Vt) < 0 \\ -(x + Vt) + 1 & 0 < (x + Vt) < 1 \\ 0 & otherwise \end{cases}$

(C) $y = \begin{cases} -\frac{1}{4}(x + Vt) + 1 & -4 < (x + Vt) < 0 \\ (x + Vt) + 1 & 0 < (x + Vt) < 1 \\ 0 & otherwise \end{cases}$

(D) $y = \begin{cases} -\frac{1}{4}(x - Vt) + 1 & -4 < (x - Vt) < 0 \\ (x - Vt) + 1 & 0 < (x - Vt) < 1 \\ 0 & otherwise \end{cases}$

21. Velocity of a point at x = 20 m as a function of time is given as

(A) $v = \begin{cases} \frac{1}{4}\text{ m/s} & 1.999 < t < 2\text{ s} \\ -1\text{ m/s} & 2 < t < 2.004\text{ s} \\ 0 & otherwise \end{cases}$

(B) $v = \begin{cases} 1\text{m/s} & 1.999 < t < 2\text{ s} \\ -\frac{1}{4}\text{ m/s} & 2 < t < 2.004\text{ s} \\ 0 & otherwise \end{cases}$

(C) $v = \begin{cases} 10\text{m/s} & 1.999 < t < 2\text{ s} \\ -\frac{5}{2}\text{ m/s} & 2 < t < 2.004\text{ s} \\ 0 & otherwise \end{cases}$

(D) $v = \begin{cases} \frac{5}{2}\text{ m/s} & 1.999 < t < 2\text{ s} \\ -10\text{ m/s} & 2 < t < 2.004\text{ s} \\ 0 & otherwise \end{cases}$

Paragraph for Question Nos. 22 to 24

Sinusoidal waves travel on five identical strings. Use the mathematical forms of the waves given below. In the expressions given below x is in m and y is in millimeters and t is in seconds.

(a) y = (2mm) sin(2x - 4t) (b) y = (3mm) sin(4x - 10t)

(c) y = (2.5mm) sin(6x - 12t) (d) y = (1mm) sin(8x - 16t)

(e) y = (4mm) sin(10x - 20t)

22. Four of the strings have the same tension, but the fifth has a different tension. Identify it.

(A) a (B) b (C) d (D) e

23. Two of the strings have the same energy in one wavelength, Identify the pair.

(A) b and c (B) a and e (C) a and d (D) b and d

24. Which string has maximum slope at x = 8m and at t = 4 sec.

(A) a (B) c (C) d (D) e

Paragraph for Question Nos. 25 to 27

A metallic rod of length 1m has one end free and other end rigidly clamped. Longitudinal stationary waves are set up in the rod in such a way that there are total six antinodes present along the rod. The amplitude of an antinode is 4×10^{-6} m. Young's modulus and density of the rod are 6.4×10^{10} N/m^2 and 4×10^3 Kg/m^3 respectively. Consider the free end to be at origin and at t=0 particles at free end are at positive extreme.

25. The equation describing displacements of particles about their mean positions is

(A) $s = 4\times10^{-6}\cos\left(\frac{11\pi}{2}x\right)\cos\left(22\pi\times10^3 t\right)$ (B) $s = 4\times10^{-6}\cos\left(\frac{11\pi}{2}x\right)\sin\left(22\pi\times10^3 t\right)$

(C) $s = 4\times10^{-6}\cos(5\pi x)\cos\left(20\pi\times10^3 t\right)$ (D) $s = 4\times10^{-6}\cos(5\pi x)\sin\left(20\pi\times10^3 t\right)$

26. The equation describing stress developed in the rod is

(A) $140.8\pi\times10^4\cos\left(\frac{11}{2}\pi x+\pi\right)\cos\left(22\pi\times10^3 t\right)$

(B) $140.8\pi\times10^4\sin\left(\frac{11}{2}\pi x+\pi\right)\cos\left(22\pi\times10^3 t\right)$

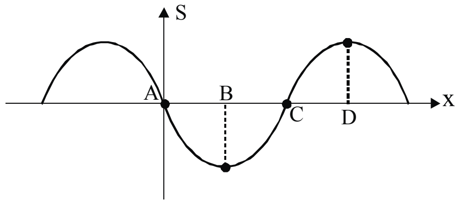
(C) $128\pi\times10^4\cos(5\pi x+\pi)\cos\left(20\pi\times10^3 t\right)$

(D) $128\pi\times10^4\sin(5\pi x+\pi)\cos\left(20\pi\times10^3 t\right)$

27. The magnitude of strain at midpoint of the rod at t= 1 sec is

(A) $11\sqrt{3}\pi\times10^{-6}$ (B) $11\sqrt{2}\pi\times10^{-6}$ (C) $10\sqrt{3}\pi\times10^{-6}$ (D) $10\sqrt{2}\pi\times10^{-6}$

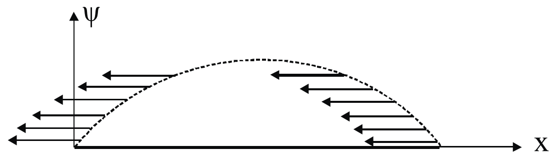
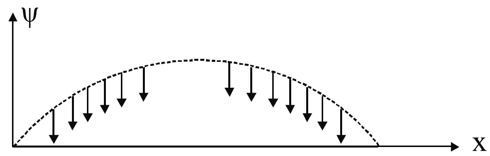
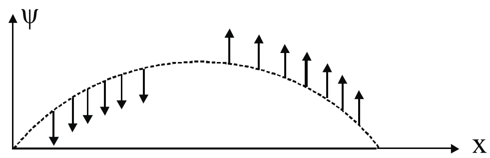
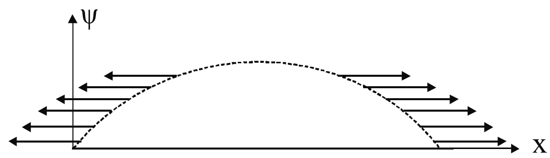
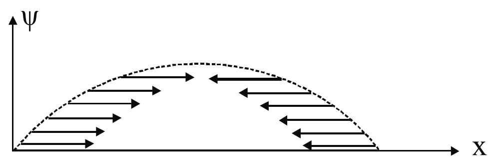
28. Figure shows a graph of particle displacement function of x at t =0 for a longitudinal wave travelling in positive x-direction in a gas. A,B,C,D denote position of particles in space.



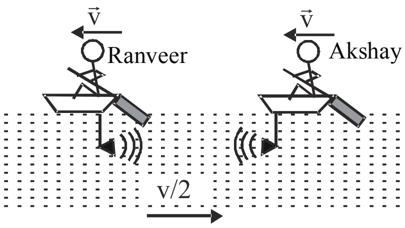
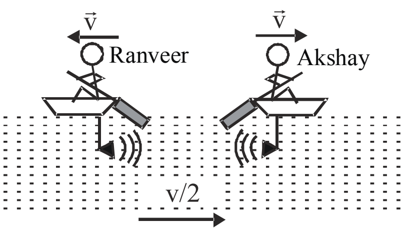
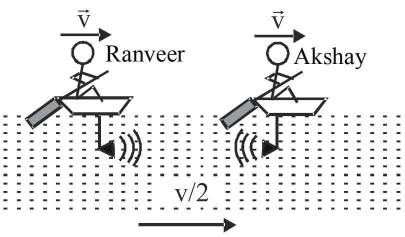
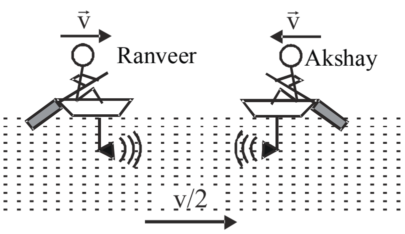
	Column-I		**Column II**
(A)	point A	(P)	Particle velocity is in direction of wave propagation
(B)	point B	(Q)	Maximum magnitude of strain
(C)	point C	(R)	Excess pressure is zero
(D)	point D	(S)	Maximum density
		(T)	Maximum magnitude of excess pressure

E

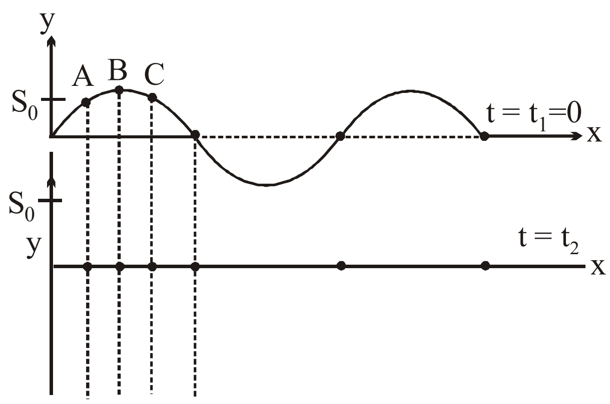
29. The 5 graphs shown in column-II are plots of displacement from mean position at a given instant. The arrows represent the direction of velocity of particles at that moment. Match with description in column-I.

Column I		**Column II**
(A) Standing wave	(P)	 All velocities are along negative x.
(B) Wave travelling along x > 0 (positive x)	(Q)	 All velocities are along negative y.
(C) Wave travelling along x < 0 (negative x)	(R)	 Half of the velocities are along negative y and other half are along positive y.
(D) Longitudinal wave	(S)	 Half of the velocities are along negative x and other half are along positive x.
	(T)	 Half of the velocities are along negative x and other half are along positive x.

30. Ranvir and Akshay are rowing two boats in a river with the same speed v relative to the river. Ranvir is upstream relative to Akshay. The river is flowing with speed v/2. Each boat sends, through the water, a signal to the other. The frequencies f_0 of the generated continuous sonic signals are the same. Each observer observes the wave sent by the other person. In each of the situation in column-I, Match the appropriate entries.

Column I	Column II
(A) 	(P) The times it takes for the signals to travel from one boat to the other will be the same.
(B) 	(Q) The wavelength observed by Akshay is greater than the wavelength observed by Ranvir.
(C) 	(R) The frequency observed by Akshay is greater than the frequency observed by Ranvir.
(D) 	(S) The wavelength observed by Akshay is lesser than the wavelength observed by Ranvir.
	(T) The frequency observed by Akshay is equal to the frequency observed by Ranvir.

31. The graphs show the standing wave on a string at two successive instants of time t_1, t_2. A, B, C are points on the string (so is the maximum displacement amplitude of the standing wave) table II gives observations about net mechanical energy for the time interval between t_1 & t_2.



Column-I	Column-II
(A) A	(P) Kinetic energy at this element is increasing.
(B) B	(Q) Energy is flowing towards right through this point at t_1.
(C) C	(R) Energy is flowing left through this point at t_1.
	(S) No net energy ever crosses this point

EXERCISE # (S)

1. A train approaching a hill at a speed of 40 km/hr sounds a whistle of frequency 580 Hz. When it is at a distance of 1 km from a hill, a wind with a speed of 40 km/hr is blowing in the direction of motion of the train find

(a) the frequency of the whistle as heard by an observer on the hill.

(b) the distance from the hill at which the echo from the hill is heard by the driver and its frequency. (Velocity of sound in air = 1200 km/hr.)

2. The equation of a plane wave is given by $\Psi = (10^{-3}) \sin \pi (10t - \sqrt{3}x - y)$ where x, y and Ψ are expressed in metre and t in second. Find (a) the direction of propagation of the wave (b) wavelength and (c) phase difference between two points A ($\sqrt{3}$ m, 1m) and B($2\sqrt{3}$ m, 2m).

3. Sinusoidal waves 5 mm in amplitude are to be transmitted along a string having a linear mass density equal to 4×10^{-2} kg/m. If the source can deliver an average power of 90 W and the string is under a tension of 100 N, then find the frequency (in Hz) at which the source can operate. (take $\pi^2 = 10$) :

4. Two identical wires, one made of iron and the other of aluminum are stretched along - side on a sonometer board by equal stretching forces. [Density of iron = 7.5 gm/cc, density aluminum = 2.7 g/cc] . The frequency of lowest harmonic for which both wires vibrate in unison, given that the length of the wires is 1 m, their diameters 1 mm and tension 75π is given as β kHz. Find the value of 12 β.

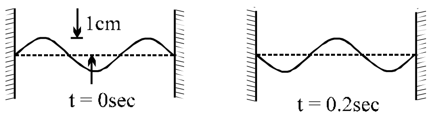
5. A string will break apart if it is placed under too much tensile stress. One type of steel has density. $\rho_{steel} = 10^4$ kg/m^3 and breaking stress $\sigma = 9 \times 10^8$ N/m^2. We make a guitar string from (4π) gram of this type of steel. It should be able to withstand $(900\,\pi)$N without breaking. What is highest possible fundamental frequency (in Hz) of standing waves on the string if the entire length of the string vibrates.

6. A string of linear mass density 5.0×10^{-3} kg/m is stretched under a tension of 65 N between two rigid supports 60 cm apart

(a) If the string is vibrating in its second overtone so that the amplitude at one of its antinodes is 0.25 cm, what are the maximum transverse speed and acceleration of the string at antinodes?

(b) What are these quantities at a distance 5.0 cm from an node?

7. Stationary waves are formed in a string of length 0.3 m and its photograph at interval of 0.2 sec is shown below:



It is also noted that the string is at rest at these times. Find

(a) The possible velocities of constant travelling wave in the string

(b) Draw V – t graph of particle at x = 0.15 m if wave is travelling with minimum possible velocity.

ANSWER KEY

EXERCISE # (O)

1. Ans. (C) **2. Ans. (A)** **3. Ans. (B, C)** **4. Ans. (B, D)**

5. Ans. (A, B, C, D) **6. Ans. (A, C, D)** **7. Ans. (A, C)** **8. Ans. (C, D)**

9. Ans. (B) **10. Ans. (B)** **11. Ans. (B)** **12. Ans. (B)**

13. Ans. (A) **14. Ans. (C)** **15. Ans. (C)** **16. Ans. (A)**

17. Ans. (D) **18. Ans. (A)** **19. Ans. (B)** **20. Ans. (A)**

21. Ans. (C) **22. Ans. (B)** **23. Ans. (C)** **24. Ans. (D)**

25. Ans. (A) **26. Ans. (B)** **27. Ans. (B)**

28. Ans. (A) → (P,Q,S,T) ; (B) → (R) ; (C) → (Q,T) ; (D) → (R)

29. Ans. (A) → (P, Q) ; (B) → (R,S) ; (C) → (T) ; (D) → (P,S,T)

30. Ans. (A) → (Q, T) ; (B) → (Q, R) ; (C) → (S,T) ; (D) → (Q)

31. Ans. (A) → (P, Q) ; (B) → (P,S) ; (C) → (P, R)

EXERCISE # (S)

1. (a) ≈ 599 Hz. (b) 0.935 km.

2. Ans. (a) Along a straight in xy plane through origin in at 30° with x-axis, (b) 1m, (c) 4π

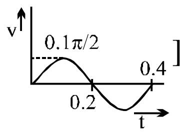
3. Ans. 300

4. Ans. 6

5. Ans. 375

6. Ans. (a) max speed = 4.48 m/s, max acceleration = 8.0×10^3 m/s^2 ;

(b) max speed = 3.14 m/s, max acceleration = 5.6×10^3 m/s^2]

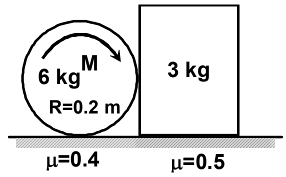
7. Ans. (a) $v = \frac{w}{k} = \frac{1}{2}$ m/s, $\frac{3}{2}$ m/s...... (b) 

]

RIGID BODY DYNAMICS

EXERCISE # (O)

1. A clockwise torque of 6N-m is applied to the circular cylinder as shown in the figure. There is no friction between the cylinder and the block.



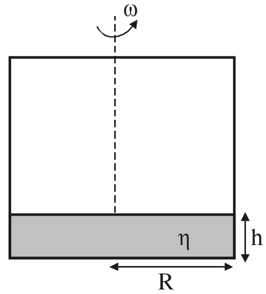
(A) The cylinder will be slipping but the system does not move forward

(B) The system cannot move forward for any torque applied to the cylinder

(C) The acceleration of the system will be $1 m/s^2$ forward

(D) The angular acceleration of the cylinder is $10 rad/s^2$.

2. A hollow cylinder of radius R and height h (<<R) is completely filled with a liquid having coefficient of viscosity η. Now a cylinder of radius R is placed over it and is rotated with constant angular velocity ω by an external agent as shown.



(A) Net viscous force acting on the cylinder is zero.

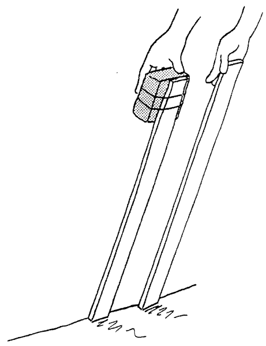
(B) external torque required to rotate the cylinder with constant angular velocity ω is $\frac{\pi\omega R^4\eta}{h}$.

(C) external torque required to rotate the cylinder with constant angular velocity ω is $\frac{\pi\omega R^4\eta}{2h}$.

(D) external power required to rotate the cylinder with constant angular velocity ω is $\frac{\pi\omega^2 R^4\eta}{h}$.

3. **Statement-1 :** A pair of upright metersticks, with their lower ends against a wall, are allowed to fall to the floor. One is bare, and the other has a heavy weight attached to its upper end. The stick to hit the floor first is the weighted stick.

Statement-2 : The torque acting on weighted stick is more than the bare stick.



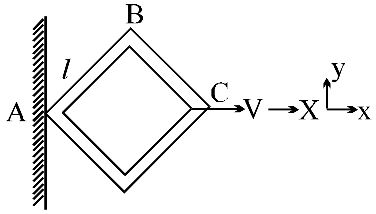
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

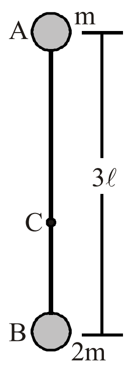
(D) Statement-1 is false, statement-2 is true

4. Four rods side length l have been hinged to form a rhombus. Vertex A is fixed to rigid support, vertex C is being moved along the X-axis with a constant velocity V as shown in figure. The rate at which vertex B is nearing the X-axis at the moment the rhombus is in the form of a square is



(A) V/4 (B) $\frac{V}{\sqrt{2}}$ (C) V/2 (D) V^2/g

5. A light rigid rod AB of length 3ℓ has a point mass m at end A and a point mass 2m at end B is kept on a smooth horizontal surface. Point C is the center of mass of the system. Initially the system is at rest. The mass 2m is suddenly given a velocity v_0 towards right. Take Z axis to be perpendicular to the plane of the paper.



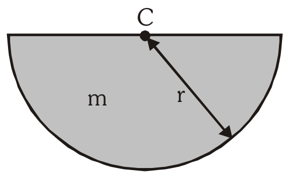
(A) The minimum moment of inertia (about Z-axis), I_{zz} of the system is $5m\ell^2$.

(B) The magnitude of tension in the rod in subsequent motion is $\frac{2mv_0^2}{9\ell}$

(C) The ratio of moment of inertia about Z-axis at points A and B, $\frac{I_{zz}^A}{I_{zz}^B} = 2$

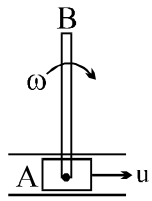
(D) Point C remains stationary during subsequent motion.

6. A semicircular lamina of mass m has radius r and centre C. Its moment of inertia about an axis through its centre of mass and perpendicular to its plane is–



(A) $\frac{1}{2}mr^2$ (B) $\left(\frac{1}{2}-\frac{4}{\pi^2}\right)mr^2$ (C) $\left(\frac{1}{2}-\frac{8}{9\pi^2}\right)mr^2$ (D) $\left(\frac{1}{2}-\frac{16}{9\pi^2}\right)mr^2$

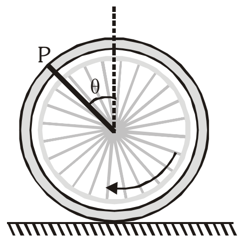
7. A mechanism consists of a part which is translated with a velocity u and a rod AB of length L and mass M hinged at A. The rod rotates about axis A with angular velocity ω. The kinetic energy of rod when it is vertical as shown is



(A) $\frac{1}{2}Mu^2 + \frac{1}{6}ML^2\omega^2$ (B) $\frac{1}{2}Mu^2 + \frac{1}{6}ML\omega u$

(C) $\frac{1}{2}Mu^2 + \frac{1}{6}ML^2\omega^2 + \frac{1}{2}ML\omega u$ (D) None of these

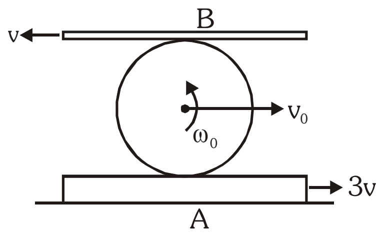
8. A wheel is rolling straight on ground without slipping. If the centre of mass of the wheel has speed v, the instantaneous velocity of a point P on the rim, defined by angle θ, relative to the ground will be –



(A) $v\cos\left(\frac{1}{2}\theta\right)$ (B) $2v\cos\left(\frac{1}{2}\theta\right)$

(C) $v\sin\left(\frac{\theta}{2}\right)$ (D) v (sin θ)

9. The disc of radius r is confined to roll without slipping at A and B. If the plates have the velocities shown, then



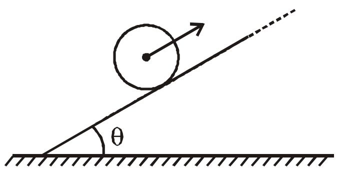
(A) linear velocity $v_0 = v$

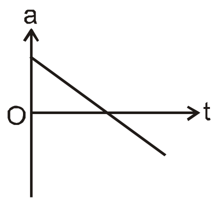
(B) angular velocity of disc is $\frac{3v}{2r}$

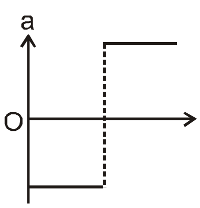
(C) angular velocity of disc is $\frac{2v}{r}$

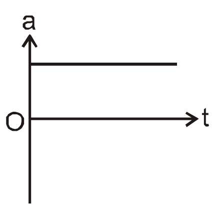
(D) None of these

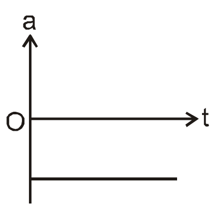
10. A uniform solid sphere rolls up (without slipping) the rough fixed inclined plane, and then back down. Which is the correct graph of acceleration 'a' of centre of mass of solid sphere as function of time t (for the duration sphere is on the incline) ? Assume that the sphere rolling up has a positive velocity.



(A) 

(B) 

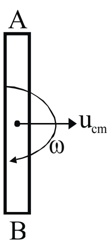
(C) 

(D) 

11. A ring of radius R is rolling purely on the outer surface of a pipe of radius 4R. At some instant, the center of the ring has a constant speed = v. Then , the acceleration of the point on the ring which is in contact with the surface of the pipe is

(A) $4v^2/5R$ (B) $3v^2/5R$ (C) $v^2/4R$ (D) zero

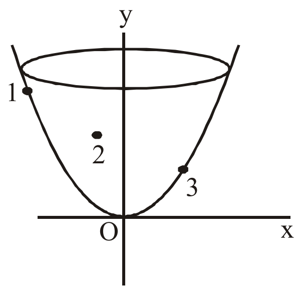
12. A rod of length ℓ is rotating with angular velocity ω and translating such that $u_{cm} = \omega\ell/2$. The distance covered by the point B when the rod completes one full rotation is



(A) $\pi\ell$ (B) 8 ℓ

(C) 4 ℓ (D) 2 $\pi\ell$

13. Three particles, each of mass m are placed at the points (x_1,y_1,z_1), (x_2,y_2,z_2) and (x_3,y_3,z_3) on the inner surface of a paraboloid of revolution obtained by rotating the parabola, $x^2 = 4ay$ about the y-axis. Neglect the mass of the paraboloid. (y-axis is along the vertical)



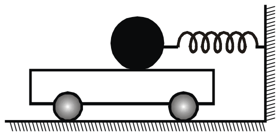
(A) The moment of inertia of the system about the axis of the paraboloid is $I = 4ma\,(y_1 + y_2 + y_3)$.

(B) If potential energy at O is taken to be zero, the potential energy of the system is $mg\,(y_1 + y_2 + y_3)$.

(C) If the particle at (x_1,y_1,z_1) slides down the smooth surface, its speed at O is $\sqrt{2gy_1}$

(D) If the paraboloid spins about OY with an angular speed ω, the kinetic energy of the system will be $2ma\,(y_1 + y_2 + y_3)\omega^2$.

14. In the figure shown, the mass of the disc as well as that of the trolley is M. The spring is ideal and has stiffness k. The trolley can move horizontally on smooth floor and the disc can roll on the trolley surface without slipping. The spring is compressed and the system released so that oscillations begin. The



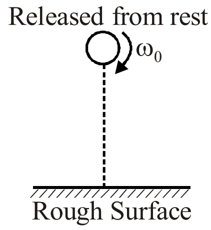
(A) acceleration of centre of disc = twice of that of trolley

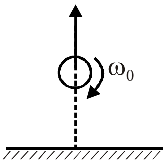
(B) acceleration of centre of disc = thrice of that of trolley

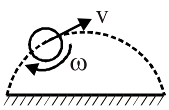
(C) acceleration of centre of disc = half of that of trolley

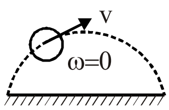
(D) acceleration of centre of disc = that of trolley

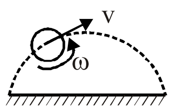
15. A disc is given an angular speed ω_0 and released from a certain height (as shown in figure). Motion of disc is observed after collision with the rough surface. Velocity of centre of mass of ball and direction of ω is shown in figure after the collision. Mark possible path, disc **CAN** follow after the collision.



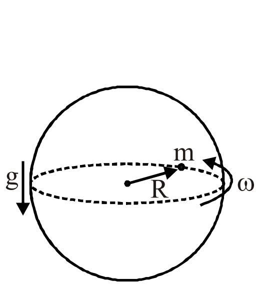
(A) 

(B) 

(C) 

(D) 

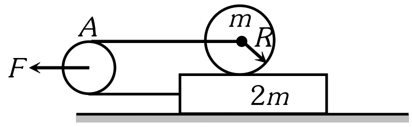
16. A particle constrained to move inside a smooth fixed spherical surface of radius R is projected horizontally (and tangent to the spherical surface at that point) from a point at the level of the center so that its angular velocity relative to the vertical axis is ω. Find approximately the maximum depth z below the level of the center that the ball goes. Take $\omega^2 R >> g$.



(A) $\dfrac{g}{2\omega^2}$ (B) $\dfrac{g}{\omega^2}$ (C) $\dfrac{2g}{\omega^2}$ (D) $\dfrac{4g}{\omega^2}$

Paragraph for Question Nos. 17 to 19

A solid cylinder of mass m and radius R is kept at rest on a plank of mass 2m lying on a smooth horizontal surface. Massless and inextensible string connecting cylinder to the plank is passing over a massless pulley. The friction between the cylinder and the plank is sufficient to prevent slipping. Pulley A is pulled with a constant horizontal force F.



17. Acceleration of cylinder with respect to earth is

(A) $\frac{5F}{21m}$ (B) $\frac{F}{7m}$ (C) $\frac{3F}{7m}$ (D) $\frac{2F}{7m}$

18. Acceleration of plank with respect to earth is

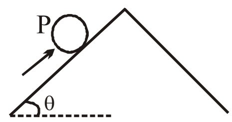
(A) $\frac{5F}{21m}$ (B) $\frac{F}{7m}$ (C) $\frac{3F}{7m}$ (D) $\frac{2F}{7m}$

19. Magnitude of friction force acting on the plank is

(A) $\frac{F}{7}$ (B) $\frac{F}{14}$ (C) $\frac{F}{21}$ (D) $\frac{2F}{7}$

Paragraph for Question Nos. 20 to 22

Greek lores have a fable of Sisyphus, a celebrated philosopher. He was punished in Hades for rebelling against the king. The punishment was to roll a spherical stone up a hill from where it would invariably roll down to the other side. He would run after the stone, to catch it back and roll it back uphill. The top of the hill was always a position of unstable equilibrium. The sphere and Sisyphus have same mass.



Being a great philosopher, Sisyphus compared this act with our attempts to succeed in life. No matter how high we go, the stone of success always rolls down back.

Philosophy apart, answer the following questions.

20. At what height from the centre should sisyphus push in a direction parallel to hill to roll the sphere slowly with minimum force.

(A) 0 (B) R/2 (C) 2/5 R (D) R

21. If the sphere is rolling down, and Sisyphus wants to run along with the same acceleration without exerting any force on it.

(A) friction on Sisyphus would be backwards and equal to that on sphere

(B) friction on Sisyphus would be forwards and greater than on sphere

(C) friction on Sisyphus would be backwards and less than on sphere

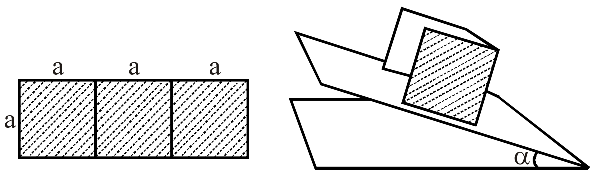
(D) friction on Sisyphus would be forwards and equal to that on sphere

22. What is the minimum friction coefficient on the hill for Sisyphus to just push the sphere up (assume it to be same for Sisyphus and Rock.) He pushes in a direction parallel to the hill.

(A) $\frac{3}{2}\tan\theta$ (B) $\tan\frac{\theta}{2}$ (C) $\frac{2}{7}\tan\theta$ (D) $\frac{5}{7}\tan\theta$

Paragraph for Question nos. 23 to 25

A cardboard strip, bent in the shape of the letter C, is put on a rough inclined plane, as shown in the figure.



23. At what angle of inclination to the horizontal plane will it topple? (assume that it does not slide).

(A) $\tan^{-1}\left(\frac{2}{3}\right)$ (B) $\tan^{-1}\left(\frac{3}{2}\right)$ (C) 45° (D) $\tan^{-1}\left(\frac{1}{3}\right)$

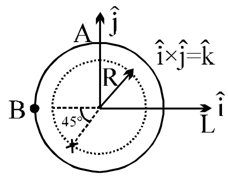
24. What should be the coefficient of friction so that it does not slide before toppling.

(A) 0.66 (B) 0.75 (C) 1.0 (D) 0.33

25. What should be the time taken to travel a distance a if the inclined plane is made completely smooth? [Take angle of inclined plane as α]

(A) $\sqrt{\frac{a}{g\sin\alpha}}$ (B) $\sqrt{\frac{2a}{g\sin\alpha}}$ (C) $\sqrt{\frac{2a}{3g\sin\alpha}}$ (D) None of these

Paragraph for Question Nos. 26 & 27 (2 questions)



A uniform spherical ball initially rotates about the $\hat{i}$ – axis, such that the point A moves out of the page. A bullet moving straight out of the page (parallel to the $\hat{k}$ – axis) is shot through the ball at a perpendicular distance R from the center of the ball. As a result of the momentum $\Delta p\hat{k}$ that the bullet transfers to the ball, the ball no longer rotates about the $\hat{i}$ – axis but instead rotates about the $\hat{j}$ – axis, with the point B moving out of the paper, and the magnitude of the angular velocity the same as it was initially. The bullet goes through the ball so quickly that the net effect of the bullet is to apply an impulse $\Delta p\hat{k}$ to the ball, a perpendicular distance R from the center of the ball. The balls moment of inertia about any axis passing through its centre is I and magnitude of its angular velocity is ω.

26. The magnitude of linear momentum Δp lost by the bullet in traversing the ball is

(A) $I\omega/R$ (B) $I\omega/(2R)$ (C) $I\omega/(\sqrt{2}R)$ (D) $I\omega\sqrt{2}/R$

27. The work done by the impulsive force is

(A) $I\omega^2$ (B) $2I\omega^2$ (C) $I\omega^2/2$ (D) zero

Paragraph for Question Nos. 28 & 29 (2 questions)

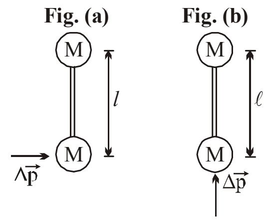


Figure shows a dumbbell that consists of a massless rod and two particle size spheres. In figure (a) impulse is imparted perpendicular to the rod and in figure (b) impulse is imparted parallel to the rod. Answer following questions.

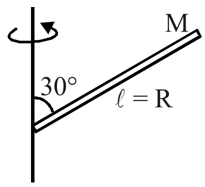
28. Velocity of C.M. and angular velocity of system in figure (a) are respectively

(A) $\frac{\Delta\vec{p}}{M}$; $\omega = \frac{\Delta p}{2Ml}$ (B) $\frac{\Delta\vec{p}}{2M}$; $\omega = \frac{\Delta p}{Ml}$ (C) $\frac{2\Delta\vec{p}}{M}$; $\omega = \frac{\Delta p}{4Ml}$ (D) $\frac{\Delta\vec{p}}{2M}$; $\omega = \frac{\Delta p}{2Ml}$

29. Energies imparted to dumbbell in figure (a) and figure (b) are respectively

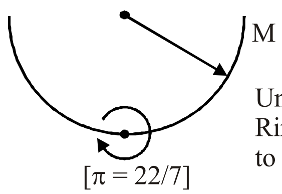
(A) $\frac{(\Delta p)^2}{2M}; \frac{(\Delta p)^2}{4M}$ (B) $\frac{(\Delta p)^2}{4M}; \frac{(\Delta p)^2}{2M}$ (C) $\frac{(\Delta p)^2}{2M}; \frac{(\Delta p)^2}{2M}$ (D) $\frac{(\Delta p)^2}{4M}; \frac{(\Delta p)^2}{4M}$

30. **Column-I** **Column-II**

(A) 

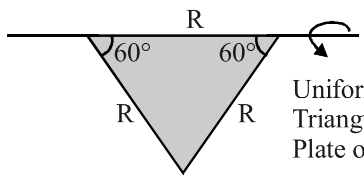
Uniform Rod

(P) $\frac{8MR^2}{11}$

(B) 

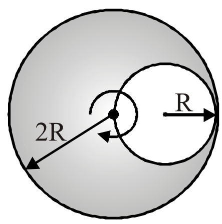
Uniform Semicircular Ring. Axis is perpendicular to plane of ring.

(Q) $\frac{MR^2}{12}$

(C) 

Uniform Triangular Plate of mass M

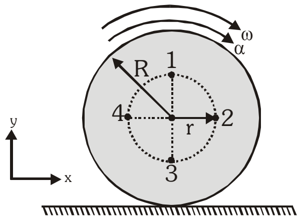
(R) $\frac{13MR^2}{8}$

(D) 

Uniform disk of initial mass M from which circular portion of radius R is then removed. M.I. of remaining mass about axis which is perpendicular to plane of plate

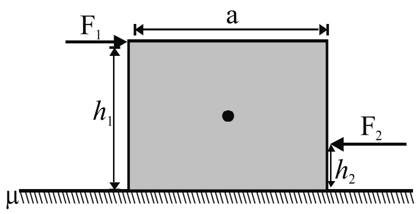
(S) $\frac{MR^2}{8}$

31. A disc of radius R is rolling without slipping with an angular acceleration α, on a horizontal plane. Four points are marked at the end of horizontal and vertical diameter of a circle of radius r (<R) on the disc. If horizontal and vertical direction are chosen as x and y axis as shown in the figure, then acceleration of points 1, 2, 3 and 4 are $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$ and $\vec{a}_4$ respectively, at the moment when angular velocity of the disc is ω. Match the following



Column-I	**Column-II**
(A) $\vec{a}_1$	(P) $\left(R\alpha - r\alpha\right)\hat{i} + \left(r\omega^2\right)\hat{j}$
(B) $\vec{a}_2$	(Q) $\left(R\alpha + r\alpha\right)\hat{i} - \left(r\omega^2\right)\hat{j}$
(C) $\vec{a}_3$	(R) $\left(R\alpha - r\omega^2\right)\hat{i} - \left(r\alpha\right)\hat{j}$
(D) $\vec{a}_4$	(S) $\left(R\alpha + r\omega^2\right)\hat{i} + \left(r\alpha\right)\hat{j}$
	(T) None of these

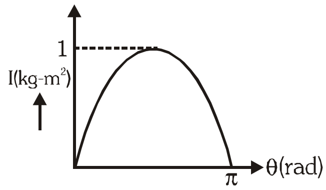
32. Consider a square plate of mass 'm' kept on a rough horizontal ground with friction coefficient μ=1. Two anti parallel forces F_1 & F_2 is applied at height h_1 & h_2 respectively as shown in figure. Column I gives various values of forces & height of point of application of that force while column II gives various effects.



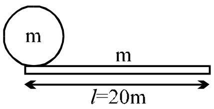
Column I	**Column II**
(A) $F_1=F_2=\frac{mg}{2}$; $h_1 = a$; $h_2 = \frac{a}{8}$	(P) Normal reaction shifts towards right
(B) $F_1=\frac{mg}{2}$; $F_2 = mg$; $h_1 = a$; $h_2 = \frac{a}{8}$	(Q) Torque due to friction is into the plane of plate
(C) $F_1 = mg$; $F_2 = 0$; $h_1 = a$	(R) Torque due to friction is out of the plane of plate
(D) $F_1=\frac{mg}{2}$; $F_2=mg$; $h_1 = h_2=\frac{a}{2}$	(S) Toppling of plate occurs
	(T) Friction force is zero.

EXERCISE # (S)

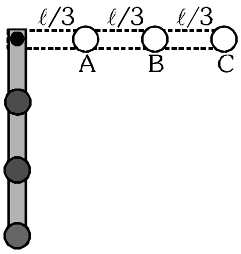
1. Figure shows the variation of the moment of inertia of a uniform rod, about an axis passing through its centre and inclined at an angle θ to the length. The moment of inertia (in kg-m^2) of the rod about an axis passing through one of its ends and making an angle $\theta = \frac{\pi}{3}$ will be



2. A plank of length 20 m and mass 1 kg is kept on a horizontal smooth surface. A cylinder of mass 1 kg is kept near one end of the plank. The coefficient of friction between the two surfaces is 0.5. Plank is suddenly given a velocity 20 m/s towards left. Find the time which plank and cylinder separate.



3. A light rod carries three equal masses A, B and C as shown in figure. What will be velocity of B in vertical position of rod, if it is released from horizontal position as shown in figure ?



4. Water flowing along an open channel drives an undershot waterwheel of radius 2 m (figure). The water approaches the wheel with a speed of 5.0 m/s and leaves with a speed of 2.5 m/s; the amount of water passing by is 200 kg per second. At what rate does the water deliver angular momentum to the wheel (in kJ) ?

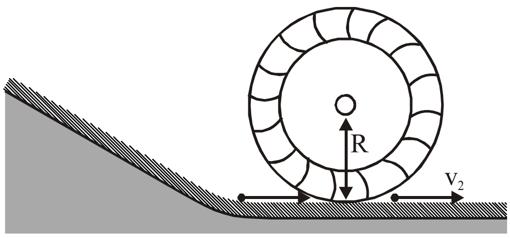
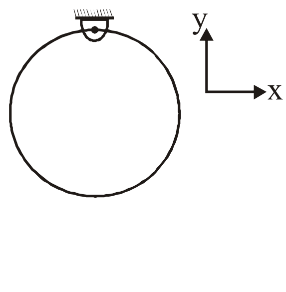
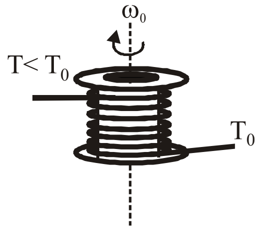


fig. : Undershot water wheel

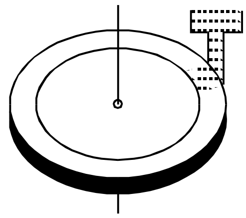
5. A bullet of mass m moving with velocity $v_0(-\hat{k})$ strikes the bottom of a stationary vertical uniform ring of same mass m and radius R = 1 m. The ring lies in XY plane with its topmost point hinged on the ceiling. The ring can rotate about X-axis. There is no friction between the hinge and the ring. The bullet gets embedded in the ring immidiately after collision. Find the angular velocity of the system (in radian /s) just after collision. [Take v_0 = 11 m/s]



6. A capstan is a rotating drum (cylinder) over which a rope or cord slides in order to increase the tension due to friction. If the difference in tension between the two ends of the rope is 500 N and the capstan has a diameter of 10 cm and rotates with angular velocity 10 rad/s. Capstan is made of iron and has mass 5 kg, specific heat 1000 J/kg K. At what rate does temperature rise ? Assume that the temperature in the capstan is uniform and all the thermal energy generated flows into it. Express your answer as x × 10^{-4} °C/s. Fill up value of x.



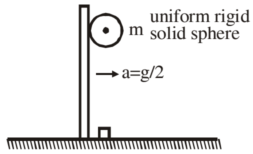
7. A uniform circular stone of mass M and radius R rotates around a vertical axle through its centre at an angular velocity ω_0 at time t = 0 as shown in the figure. The stone has a deep groove along its circumference. Sand is dumped into this groove at a constant rate of q. Sand does not spill out or slip relative to the stone once it falls into the groove.



(i) Assuming there is no external torque, find the angular velocity of the stone as a function of time t.

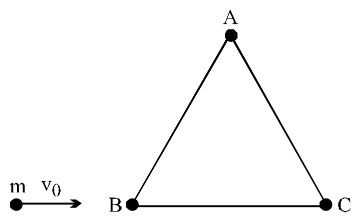
(ii) How much external torque would be needed to maintain the stone at a constant angular velocity?

8. A uniform solid sphere rolls down on a vertical rigid surface without slipping. The vertical surface moves with an acceleration a = g/2 in horizontal direction as shown. The minimum coefficient of friction between the sphere and vertical surface so as to prevent slipping is μ. Write 14 μ as answer.



9. A solid cylinder rolls without slipping along a horizontal plane with velocity 66 m/s and reaches a plane inclined at 60° with the horizontal. If there is sufficient friction to prevent slipping then the body starts rolling up the inclined plane with the velocity v. Find the value of v (in m/s).

10. Three particles A, B, C of mass m each are joined to each other by massless rigid rods to form an equilateral triangle of side a. Another particle of mass m hits B with a velocity v_0 directed along BC as shown. The colliding particle stops immediately after impact .

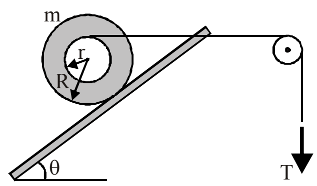


(i) Calculate the time required by the triangle ABC to complete half-revolution in its subsequent motion.

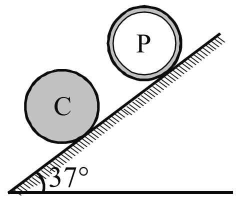
(ii) What is the net displacement of point B during this interval ?

11. A cyclist turns her bicycle upside down to tinker with it. After she gets it upside down, notices the front wheel executing a small-amplitude, back-and-forth rotational motion with a period of 12s. Considering the wheel to be a thin ring of total mass 800g and radius 20cm, whose only irregularity is the presence of the small tyre valve stem, determine the mass of the valve stem (in gm).
[Take : π^2 = g and appropriate approximation]

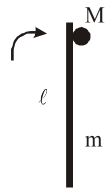
12. A spool, initially at rest, is kept on a frictionless incline making an angle $\theta = 37°$ with the horizontal. The mass of the spool is 2kg and it is pulled by a string as shown with a force of T = 10N. The string connecting the spool and the pulley is initially horizontal. Find the initial acceleration of the spool on the incline. Express your answer in m/s^2.



13. A solid cylinder C and a hollow pipe P of same diameter are in contact when they are released from rest as shown in the figure on a long incline plane. Cylinder C and pipe P roll without slipping. Determine the clear gap (in m) between them after 4 seconds.



14. A uniform stick of mass 18 kg and length 3 m spins around on a frictionless horizontal plane, with its Centre of Mass stationary. A mass M is placed on the plane, and the stick collides elastically with it, as shown (with the contact point being the end of the stick). What should M (in kg) be so that after the collision the stick has translational motion, but no rotational motion?



ANSWER KEY

EXERCISE # (O)

1. Ans. (C, D)	**2. Ans. (AC)**	**3. Ans. (D)**	**4. Ans. (C)**
5. Ans. (BC)	**6. Ans. (D)**	**7. Ans. (C)**	**8. Ans. (B)**
9. Ans. (A,C)	**10. Ans. (D)**	**11. Ans. (A)**	**12. Ans. (C)**
13. Ans. (A,B,C,D)	**14. Ans. (B)**	**15. Ans. (B)**	**16. Ans. (C)**
17. Ans. (C)	**18. Ans. (D)**	**19. Ans. (B)**	**20. Ans. (D)**
21. Ans. (A)	**22. Ans. (A)**	**23. Ans. (A)**	**24. Ans. (A)**
25. Ans. (B)	**26. Ans. (D)**	**27. Ans. (D)**	**28. Ans. (B)**
29. Ans. (A)			

30. Ans. (A) $\rightarrow$ (Q) ; (B) $\rightarrow$ (P) ; (C) $\rightarrow$ (S) ; (D) $\rightarrow$ (R)

31. Ans. (A) $\rightarrow$ (Q) ; (B) $\rightarrow$ (R) ; (C) $\rightarrow$ (P) ; (D) $\rightarrow$ (S)

32. Ans. (A) $\rightarrow$ (P,T) ; (B) $\rightarrow$ (P, R) ; (C) $\rightarrow$ (P,Q,S) ; (D) $\rightarrow$ (R)

EXERCISE # (S)

1. Ans. 3 **2. Ans.** t = 1.5 sec

3. Ans. $\sqrt{\frac{8g\ell}{7}}$

4. Ans. 1 **5. Ans.** 4 **6. Ans.** 0500

7. Ans. (i) $\frac{M\omega_0}{M+2qt}$, (ii) $q\omega_0R^2$ **8. Ans.** 8 **9. Ans.** 44

10. Ans. (i) $t = \frac{6a\pi}{\sqrt{3}\,v_0}$; (ii) $s = \frac{a}{\sqrt{3}}\sqrt{1+\left(2\pi+\sqrt{3}\right)^2}$

11. Ans. 5 **12. Ans.** 2 **13. Ans.** 8 **14. Ans.** 9

SIMPLE HARMONIC MOTION

EXERCISE # (O)

1. The position vector of a particle that is moving in three dimensions is given by $\vec{r} = (1 + 2\cos 2\omega t)\hat{i} + (3\sin^2 \omega t)\hat{j} + (3)\hat{k}$ in the ground frame. All units are in SI. Choose the correct statement (s) :

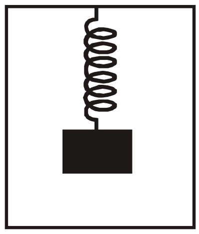
(A) The particle executes SHM in the ground frame about the mean position $\left(1, \frac{3}{2}, 3\right)$

(B) The particle executes SHM in a frame moving along the z-axis with a velocity of 3 m/s.

(C) The amplitude of the SHM of the particle is $\frac{5}{2}$ m.

(D) The direction of the SHM of the particle is given by the vector $\left(\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}\right)$

2. A spring mass system is hanging from the ceiling of an elevator in equilibrium. The elevator suddenly starts accelerating upwards with acceleration a, consider all the options in the reference frame of elevator.



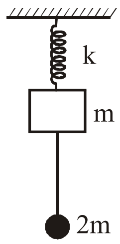
(A) the frequency of oscillation is $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(B) the amplitude of the resulting SHM is $\frac{ma}{k}$

(C) amplitude of resulting SHM is $\frac{m(g+a)}{k}$

(D) maximum speed of block during oscillation is $\left(\sqrt{\frac{m}{k}}\right)a$

3. A bob of mass 2m hangs by a string attached to the block of mass m of a spring block system. The whole arrangement is in a state of equilibrium. The bob of mass 2m is pulled down slowly by a distance x_0 and released.



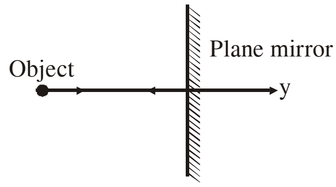
(A) For $x_0 = \frac{3mg}{k}$ maximum tension in string is 4mg

(B) For $x_0 > \frac{3mg}{k}$, minimum tension in string is mg

(C) Frequency of oscillation of system is $\frac{1}{2\pi}\sqrt{\frac{k}{3m}}$, for all non-zero values of x_0

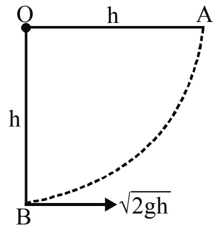
(D) The motion will remain simple harmonic for $x_0 \leq \frac{3mg}{k}$

4. In the diagram shown, the object is performing SHM according to the equation y = 2A sin(ωt) and the plane mirror is performing SHM according to the equation $Y = -A\sin\left(\omega t - \frac{\pi}{3}\right)$. The diagram shows the state of the object and the mirror at time t = 0 sec. The minimum time from t = 0 sec after which the velocity of the image becomes equal to zero ?



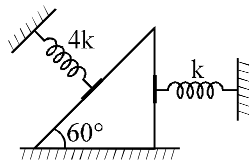
(A) $\frac{\pi}{3\omega}$ (B) $\frac{3\pi}{\omega}$ (C) $\frac{\pi}{6\omega}$ (D) $\frac{2\pi}{3\omega}$

5. Two pendulums A and B having equal lengths and equal bob masses are attached to a common point O and were initially in the position, as shown in the figure. The bob of pendulum A is released from rest and simultaneously that to B was given a horizontal velocity $\sqrt{2gh}$ as shown in the figure, when the two bobs collide, what will be the ratio of their kinetic energy just before collision ?



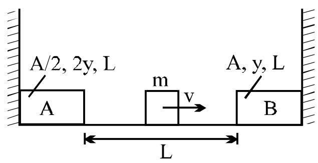
(A) 1 : √2 (B) √2 : 1 (C) 1 : 1 (D) 1 :2

6. A smooth wedge of mass m and angle of inclination 60^0 rests unattached between two springs of spring constant k and 4k, on a smooth horizontal plane, both springs in the unextended position. The time period of small oscillations of the wedge (assuming that the springs are constrained to get compressed along their length) equals



(A) $\pi\left(1+\frac{1}{2}\right)\sqrt{\frac{m}{k}}$ (B) $\pi\left(1+\frac{1}{\sqrt{3}}\right)\sqrt{\frac{m}{k}}$ (C) $\pi\left(1+\frac{2}{\sqrt{3}}\right)\sqrt{\frac{m}{k}}$ (D) none of the above

7. In the given figure, two elastic rods A & B are rigidly joined to end supports. A small mass 'm' is moving with velocity v between the rods. All collisions are assumed to be elastic & the surface is given to be smooth. The time period of small mass 'm' will be :[A=area of cross section, y=young's modulus, L=length of each rod]



(A) $\frac{2L}{v}+2\pi\sqrt{\frac{mL}{Ay}}$ (B) $\frac{2L}{v}+2\pi\sqrt{\frac{2mL}{Ay}}$ (C) $\frac{2L}{v}+\pi\sqrt{\frac{mL}{Ay}}$ (D) $\frac{2L}{v}$

8. The maximum acceleration of a particle in SHM is made two times keeping the maximum speed to be constant. It is possible when

(A) amplitude of oscillation is doubled while frequency remains constant

(B) amplitude is doubled while frequency is halved

(C) frequency is doubled while amplitude is halved

(D) frequency is doubled while amplitude remains constant

9. The energy of a particle executing simple harmonic motion is given by $E = Ax^2 + Bv^2$ where x is the displacement from mean position x =0 and v is the velocity of the particle at x then choose the **CORRECT** statement(s).

(A) Amplitude of SHM is $\sqrt{\dfrac{2E}{A}}$

(B) Maximum velocity of the particle during SHM is $\sqrt{\dfrac{E}{B}}$

(C) Time period of motion is $2\pi\sqrt{\dfrac{B}{A}}$

(D) Displacement of the particle is directly proportional to the velocity of the particle.

10. Consider a travelling simple harmonic wave on a string of mass per unit length μ and tension T. Kinetic energy per unit length is given by $u_k = \dfrac{1}{2}\mu\left(\dfrac{\partial y}{\partial t}\right)^2$ and potential energy per unit length is given by $u_p = \dfrac{1}{2}T\left(\dfrac{\partial y}{\partial x}\right)^2$. Mark the **CORRECT** option(s).

(A) Power transmitted by wave equals $(u_k + u_p)\sqrt{\dfrac{T}{\mu}}$.

(B) u_k and u_p simultaneously attain their maximum and minimum values.

(C) Total energy per unit length of a string is constant when a harmonic wave travels on it.

(D) A small part of string has maximum potential energy when it is at its equilibrium position.

11. A body is performing S.H.M., then its

(A) average total energy per cycle is equal to its maximum kinetic energy.

(B) average kinetic energy per cycle is equal to half of its maximum kinetic energy.

(C) mean velocity over a complete cycle is equal to $2/\pi$ times of its maximum velocity.

(D) root mean square velocity is $1/\sqrt{2}$ times of its maximum velocity

12. Consider a spring that exerts the following restoring force

$F = -kx$ for $x > 0$

$F = -4kx$ for $x < 0$

A mass m on a frictionless surface is attached to the spring displaced to x = A by stretching the spring and released :

(A) The period of motion will be $T = \dfrac{3}{2}\pi\sqrt{\dfrac{m}{k}}$

(B) the most negative value the mass m can reach will be $x = -\dfrac{A}{2}$

(C) The time taken to move from x = A to $x = -\dfrac{A}{\sqrt{2}}$, straight away will be equal to $\dfrac{5\pi}{8}\sqrt{\dfrac{m}{k}}$

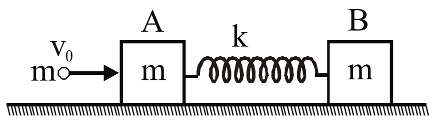
(D) The total energy of oscillations will be $\dfrac{5}{2}kA^2$

13. Two blocks A and B, each of mass m, are connected by an ideal spring of stiffness k and placed on a smooth horizontal surface. A ball of mass m moving with a velocity v_0 strikes the block A and gets embedded to it. Then,

(A) velocity of block A just after collision is $v_0/2$

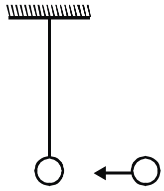
(B) velocity of block B just after collision is zero

(C) the maximum compression produced in the spring is $v_0\sqrt{\dfrac{m}{6k}}$



(D) the kinetic energy lost during collision is $\dfrac{1}{4}mv_0^2$

14. A ball is swinging on a swing like a simple pendulum. Its time period of oscillation is T and amplitude is A. When it is at the bottom of the swing, another ball of equal mass strikes and sticks to it while both of them are travelling in same direction. Choose the correct option(s).



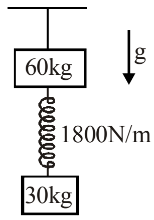
(A) The time period of oscillation remains same.

(B) The amplitude increases.

(C) The time period of oscillation increases.

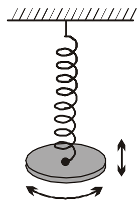
(D) The time period of oscillation decreases.

15. The system shown is hanging in equilibrium well above the ground. When the string is cut at t = 0, the time at which the spring is at its natural length will be (Take $\pi^2 = 10$)



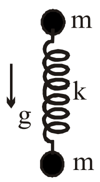
(A) $\dfrac{1}{6}$ s (B) $\dfrac{1}{2}$ s (C) $\dfrac{1}{3}$ s (D) $\dfrac{2}{3}$ s

16. A solid disk of radius R is suspended from a spring of linear spring constant k and torsional constant c, as shown in figure. In terms of k and c, what value of R will give the same period for the vertical and torsional oscillations of this system?



(A) $\sqrt{\dfrac{2c}{k}}$ (B) $\sqrt{\dfrac{c}{2k}}$ (C) $2\sqrt{\dfrac{c}{k}}$ (D) $\dfrac{1}{2}\sqrt{\dfrac{c}{k}}$

17. A mass m is hung on an ideal massless spring. Another equal mass is connected to the other end of the spring. The whole system is at rest. At t = 0, m is released and the system falls freely under gravity. Assume that natural length of the spring is L_0, its initial stretched length is L and the acceleration due to gravity is g. What is distance between masses as function of time.



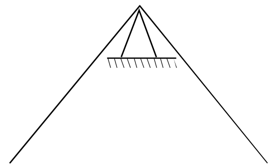
(A) $L_0 + (L - L_0)\cos\sqrt{\dfrac{2k}{m}}\,t$

(B) $L_0\cos\sqrt{\dfrac{2k}{m}}\,t$

(C) $L_0\sin\sqrt{\dfrac{2k}{m}}\,t$

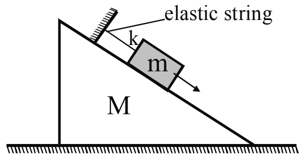
(D) $L_0 + (L - L_0)\sin\sqrt{\dfrac{2k}{m}}\,t$

18. Two identical rods of length l and mass m are welded together at right angles and then suspended from a knife-edge as shown. Angular frequency of small oscillations of the system in its own plane about the point of suspension is



(A) $\sqrt{\frac{3g}{4\sqrt{2}\,l}}$ (B) $\sqrt{\frac{3g}{2\sqrt{2}\,l}}$ (C) $\sqrt{\frac{3g}{\sqrt{2}\,l}}$ (D) None of these

19. An object of mass m is tied to an elastic string of length ℓ and force constant k. The other end of the string is tied to a wedge of mass M placed on a frictionless surface. The system is gently released along the incline from rest. Assuming frictionless incline.

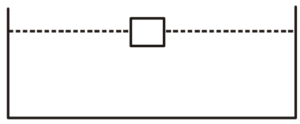


(A) motion of M and m are periodic but not oscillatory
(B) motion of M and m are oscillatory.
(C) only M oscillates while motion of m is periodic but non-oscillatory.
(D) only m oscillates while motion of M is periodic but non-oscillatory.

Paragraph for Question Nos. 20 to 22

A cube made of wood having specific gravity 0.4 and side length a is floated in a large tank full of water.

20. Which action would change the depth to which block is submerged?



(A) more water is added in the tank
(B) atmospheric pressure increases
(C) the tank is accelerated upwards
(D) None of these

21. If the cube is depressed slightly, it executes SHM from it's position. What is it's time period?

(A) $2\pi\sqrt{\frac{a}{g}}$ (B) $2\pi\sqrt{\frac{5a}{2g}}$ (C) $2\pi\sqrt{\frac{2a}{5g}}$ (D) $\frac{4\pi}{5}\sqrt{\frac{a}{g}}$

22. What can be the maximum amplitude of it's vertical simple harmonic motion?

(A) $\frac{a}{2}$ (B) 0.4 a (C) 0.6 a (D) 0.2 a

Paragraph for Question Nos. 23 to 25

The time–period of a simple pendulum performing simple harmonic motion, having length ℓ is given $T = 2\pi\sqrt{\frac{\ell}{g}}$

This formula holds good if following conditions are satisfied.

(i) String is massless and extensible.
(ii) Bob is a point mass.
(iii) Angular displacement about mean position is very small, and
(iv) Length of string is very small as compared to the radius of earth (R_e).

23. If the bob is a solid sphere of radius R and length of string is L, then equivalent length of pendulum will be

(A) $\frac{2}{5}\frac{R^2}{(L+R)}+(R+L)$ (B) $\frac{2}{5}\frac{R^2}{L}+L$ (C) R + L (D) L

24. If the bob is a hollow spherical shell of radius R and length string is L. The time period of pendulum is T_1. Now bob is filled with water completes and now its time period become T_2 then T_1/T_2 is

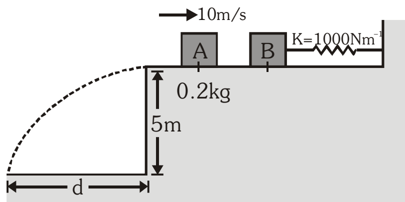
(A) = 1 (B) <1 (C) >1 (D) None of these

25. Now the water of bob is completely freezes into ice and this phenomena does not produce any change in L. The new time period becomes T_3 then T_2/T_3 is

(A) = 1 (B) < 1 (C) 1 (D) None of these

Paragraph for Question Nos. 26 to 28

Figure shows block A of mass 0.2 kg sliding to the right over a frictionless elevated surface at a speed of 10 m/s. The block undergoes a collision with stationary block B, which is connected to a non-deformed spring of spring constant 1000 Nm^{-1}. The coefficient of restitution between the blocks is 0.5. After the collision, block B oscillates in SHM with a period of 0.2s, and block A slides off the opposite end of the elevated surface, landing a distance 'd' from the base of that surface after falling height 5m. (use $\pi^2 = 10$; g=10m/s^2) Assume that the spring does not affect the collision.



26. Mass of the block B is-

(A) 0.4 kg (B) 0.8 kg (C) 1 kg (D) 1.2 kg

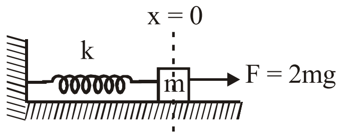
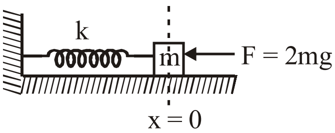
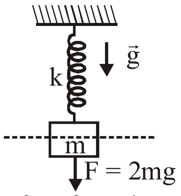
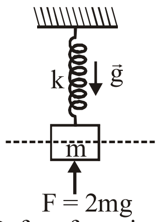
27. Amplitude of the SHM as being executed by block B-spring system is-

(A) $2.5\sqrt{10}$ cm (B) 10 cm (C) $3\sqrt{10}$ cm (D) $5\sqrt{10}$ cm

28. The distance 'd' will be equal to-

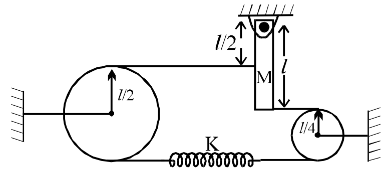
(A) 2m (B) 2.5 m (C) 4m (D) 6.25 m

29. Column I shows spring block system with a constant force permanently acting on block match entries of column I with column II.

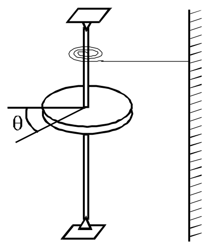
Column I	**Column II**
(A)  spring is initially relaxed when force is applied	(P) Time period of oscillation $T = 2\pi\sqrt{\frac{m}{k}}$
(B)  spring is initially relaxed when force is applied	(Q) Amplitude of oscillation is $A = \frac{2mg}{k}$
(C)  Before force is applied block is in equilibrium position	(R) Maximum velocity attained by block is $2g\left[\sqrt{\frac{m}{k}}\right]$
(D)  Before force is applied block is in equilibrium position.	(S) Maximum magnitude of acceleration of block is 2g.
	(T) Velocity of block when spring is in natural length is zero. (If spring acquire natural length).

EXERCISE # (S)

1. A light inextensible string carrying a spring (as shown) is passed over two smooth fixed pulleys. Find the time period of oscillation, if the rod is slightly displaced (about the hinge) from it's equilibrium position (when the spring is unstreched).



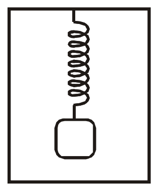
2. A solid disk of mass M and radius R on a vertical shaft. The shaft is attached to a coil spring which exerts a linear restoring torque of magnitude $C\theta$ where θ is the angle measured from the static equilibrium position and C is a constant. Neglect the mass of the shaft and the spring and assume the bearings to be frictionless.



(a) Show that the disks can undergo SHM and find the frequency of motion.

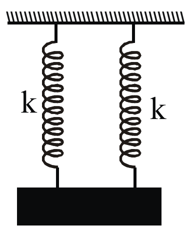
(b) Suppose that the disk is moving according to the $\theta = \theta_0 \sin(\omega t)$ where ω is the frequency found in part (a) At time $t_1 = \pi/\omega$ a ring of stickly putty of mass M and radius R is dropped concentrically on the disk, Find (i) the new frequency of motion (ii) the new amplitude of motion.

3. A mass m is resting at equilibrium suspended from a vertical spring of natural length L and spring constant k inside a box as shown :
The box begins accelerating upward with acceleration a. Find the maximum speed of the mass (in cm/s) relative to the box in subsequent motion. (Given : $a = 1\ m/s^2$, m = 3kg, k = 1200 N/m)



4. A particle of mass 2 kg is executing SHM and it s potential energy at extreme is 10 J. Velocity at mean position is $\sqrt{8}$ m/s. If potential energy at A/2 (where A is amplitude) is U_0 then find the value of $2U_0$ (in J).

5. A small block of mass m = 0.5 kg is attached to two springs each of force constant k = 10 N/m as shown in figure. The block is executive SHM with amplitude $A = \frac{1}{\sqrt{8}}$ m. When the block is at equilibrium position one of the spring breaks without changing momentum of block. New amplitude of oscillation is $\frac{e}{16}$ m. Find the value of e ?



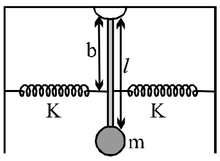
6. A mass m is pushed against a horizontal spring with spring constank k and held in place with a catch. The spring compresses an unknown distance x. When the catch is removed, the mass leaves the spring and slides inside a frictionless circular loop in vertical plane of radius R. When the mass reaches the top of the loop, the force of the loop on the mass (the normal force) is equal to twice the weight of the mass.

(a) Using conservation of energy, find the kinetic energy at the top of the loop. Express your answer as a function of k, m, x, g and R.

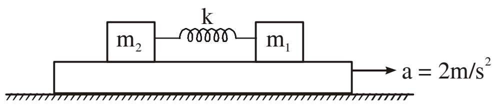
(b) Write the force equation for the mass when it is at the top of the loop.

(c) How far was the spring compressed?

7. Find the time period of small oscillations of the spring loaded pendulum. The equilibrium position is vertical as shown. The mass of the rod is negligible and treat mass as a particle.

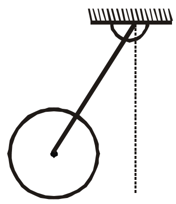


8. Two blocks of masses $m_1 = 1$kg and $m_2 = 2$kg are connected by a spring of constant k = 100 N/m and placed over a plank as shown. The coefficient of friction between m_2 and plank is 0.5 and there is no friction between m_1 and plank.

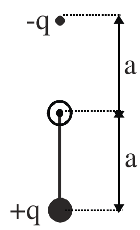


Initially the whole system, is at rest. The plank is now pulled to right with constant acceleration a = 2 m/s^2. Find the amplitude (in cm) of oscillation of block m_1 relative to plank.

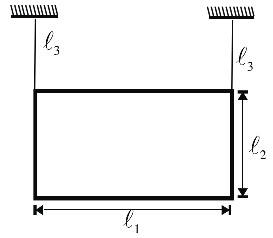
9. A metal rod of length 1.25 m and mass 'm' is pivoted at one end. A solid sphere of same mass and radius 0.25 m is attached at its center to the free end of the rod and the sphere is free to rotate about its center. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Angular frequency of small oscillations is n rad/s. Find the value of n^2.



10. A particle of charge q and mass m is suspended from a point on the wall by a rigid massless rod of length a = 3 m as shown. Above the point of suspension we have fixed a particle of charge – q at a distance a. On slight displacement from the mean position, the suspended particle executes SHM. Find the time period of SHM is second. [Take $\frac{kq^2}{2(a^2)} = mg, g = \pi^2$]



11. A rectangular plate of mass M, sides ℓ_1 and ℓ_2 is suspended from a ceiling by two parallel strings of length ℓ_3 each as shown in figure. The plate is displaced slightly in its plane keeping the strings tight. The angular frequency of its SHM is ω. Find the value of $2\omega^2$ in SI units.



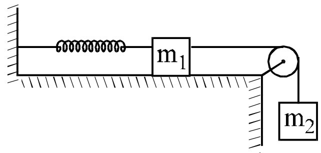
[Take : M = 10 kg, ℓ_1=0.8 m, ℓ_2 = 0.2m, ℓ_3= 0.4 m]

12. A block of mass m_1 = 1 kg is attached to a spring of force constant k = 24 N/cm at one end and attached to a string tensioned by mass m_2 = 5 kg. Deduce the frequency of oscillations of the system. If m_2 is initially supported in hand with spring unstretched and then suddenly released, find

(a) instantaneous tension just after m_2 is released.

(b) the maximum displacement of m_1.

(c) the maximum and minimum tensions in the string during oscillations.



ANSWER KEY

EXERCISE # (O)

1. Ans. (A, C)	**2. Ans. (A, B, D)**	**3. Ans. (A, D)**	**4. Ans. (D)**
5. Ans. (C)	**6. Ans. (B)**	**7. Ans. (A)**	**8. Ans. (C)**
9. Ans. (B, C)	**10. Ans. (A, B, D)**	**11. Ans. (A, B, D)**	**12. Ans. (A, B)**
13. Ans. (A, B, C, D)	**14. Ans. (A, B)**	**15. Ans. (A, B)**	**16. Ans. (A)**
17. Ans. (A)	**18. Ans. (B)**	**19. Ans. (B)**	**20. Ans. (D)**
21. Ans. (C)	**22. Ans. (B)**	**23. Ans. (A)**	**24. Ans. (C)**
25. Ans. (B)	**26. Ans. (C)**	**27. Ans. (A)**	**28. Ans. (B)**

29. Ans. (A) $\rightarrow$(PQRST); (B) $\rightarrow$(PQRST); (C) $\rightarrow$(PQRS); (D) $\rightarrow$(PQRS)

EXERCISE # (S)

1. Ans. $\pi\sqrt{\dfrac{4ML}{3(KL+2Mg)}} + \pi\sqrt{\dfrac{2L}{3g}}$ **2. Ans. (a)** $\dfrac{1}{2\pi}\sqrt{\dfrac{2C}{MR^2}}$, **(b)** $\theta_{new} = \dfrac{\theta_0}{\sqrt{3}}, f' = \dfrac{1}{2\pi}\sqrt{\dfrac{2C}{3MR^2}}$

3. Ans. 5 **4. Ans. 8** **5. Ans. 8**

6. Ans. (a) $\dfrac{1}{2}kx^2 - 2mgR$, **(b)** $3mg = \dfrac{mv_f^2}{R}$, **(c)** $\sqrt{\dfrac{7mgR}{k}}$

7. Ans. $T = 2\pi\sqrt{\dfrac{m\ell^2}{mg\ell + 2Kb^2}}$ **8. Ans. 2** **9. Ans. 9** **10. Ans. 4**

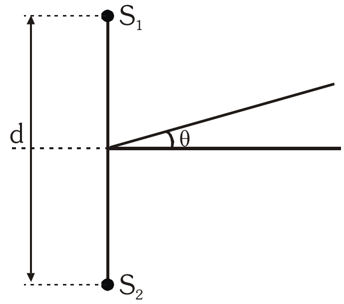
11. Ans. 050

12. Ans: (a) $T = \dfrac{m_1m_2g}{m_1+m_2}$; **(b)** $\dfrac{2m_2g}{k}$; **(c)** $T_{min} = \dfrac{m_1m_2g}{m_1+m_2}; T_{max} = \dfrac{m_2g(m_1+2m_2)}{(m_1+m_2)}$

WAVE OPTICS

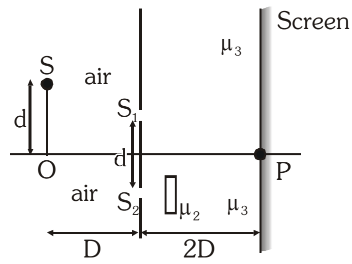
EXERCISE # (O)

1. In an interference arrangement, similar to Young's double–slit experiment, the slits S_1 and S_2 are illuminated with coherent microwave sources, each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by distance d=150 m. The intensity I_θ is measured as a function of θ, where θ is defined as shown in figure. If I_0 is maximum intensity, then $I_{(\theta)}$ for $0 \le \theta \le 90°$ is given by



(A) $I_{(\theta)} = I_0$ for $\theta = 0°$
(B) $I_{(\theta)} = (I_0/2)$ for $\theta = 30°$
(C) $I_{(\theta)} = (I_0/4)$ for $\theta = 90°$
(D) $I_{(\theta)}$ is constant for all values of θ

2. Consider the situation shown in the figure. Two slits S_1 and S_2 are placed symmetrically about the line OP which is perpendicular to screen. The space between screen and slits is filled with a liquid of refractive index μ_3. A plate of thickness t and refractive index μ_2 is placed in front of one of the slit. A source S is placed above OP at a distance d. Choose the correct alternatives.



[(Given that D = 1m, d = 2 mm, t = 6×10^{-6} m, $\mu_2 = 1.2$, $\mu_3 = 1.8$]

(A) Position of central maxima from point P is 2 mm

(B) Position of central maxima from point P is $\frac{2}{9}$ mm

(C) Thickness of glass slab so that central maxima forms at point P is $\frac{20}{3} \times 10^{-6}$m

(D) If glass slab is removed, the central maxima shifts by a distance of 2 mm

3. Mark the **CORRECT** statements(s) :

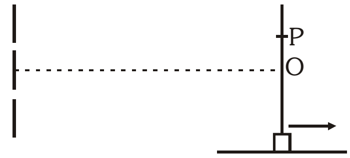
(A) Direction of wave propagation is along the normal to wavefront.

(B) For a point source of light, the shape of wavefront can be considered to be plane at very large distance from the source.

(C) A point source of light is placed at the focus of a thin spherical lens, then the shape of the wavefront for emerged light may be plane.

(D) The shape of the wavefront for the light incident on a thin spherical lens (kept in vacuum) is plane, the shape of the wavefront corresponding to emergent light would be always spherical.

4. In a Young's double–slit experiment, first maxima is observed at a fixed point P on the screen. Now, the screen is continuously moved away from the plane of slits. The ratio of intensity at point P to the intensity at point O (centre of the screen)



(A) remains constant

(B) keeps on decreasing

(C) first decreases and then increases

(D) first decreases and then becomes constant

5. A transparent slab of thickness t and refractive index μ is inserted in front of upper slit of YDSE apparatus. The wavelength of light used is λ. Assume that there is no absorption of light by the slab. Mark the **INCORRECT** statement.

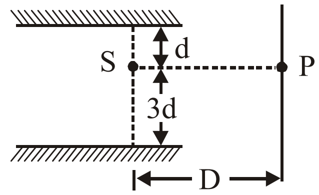
(A) The intensity of dark fringes will be zero, if slits are identical

(B) The change in optical path due to insertion of plate is μt

(C) For marking intensity maximum at the centre of screen, the thickness could be $\dfrac{5\lambda}{(\mu-1)}$

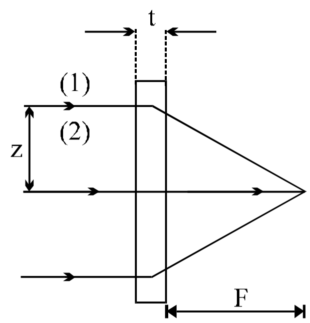
(D) For marking intensity zero at the centre of screen, the thickness could be $\dfrac{5\lambda}{2(\mu-1)}$

6. Consider the optical system shown in the figure that follows. The point source of light S is having wavelength equal to λ. The light is reaching screen only after reflection. For point P to be 2nd maxima, the value of λ would be ($D >> d$ and $d >> \lambda$) :-



(A) $\dfrac{12d^2}{D}$ (B) $\dfrac{6d^2}{D}$ (C) $\dfrac{8d^2}{D}$ (D) $\dfrac{24d^2}{D}$

7. Plane wavefronts are incident on a glass slab which has refractive index as a function of distance Z, according to the relation $\mu = \mu_0(1 - Z^2/Z_0^2)$. This glass slab can acts as lens of focal length F. By using the concept of optical path length calculate the focal length of the slab. Consider t and z to be very small as compared to F.



(A) $Z_0^2/(2\mu_0 t)$ (B) $Z_0^2/(\mu_0 t)$

(C) $\mu_0 Z_0^2/(2t)$ (D) None

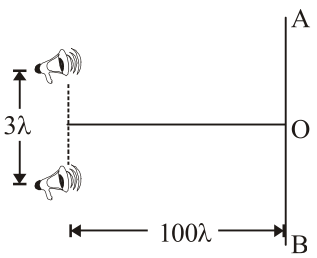
8. One of the slits of a double slit system in young's experiment is wider than the other so that the amplitude of the light reaching the central part of the screen from one slit, acting alone, is **twice** that from the other slit acting alone. Assume that $\beta = \dfrac{\pi d \sin\theta}{\lambda}$, where d is the centre to centre distance between the two slits and λ, the wavelength of light used. Then I_θ, the intensity of the resultant interference wave in the direction θ to central maximum on the screen will be correctly given by :-

(A) $I_\theta = (4\,I_0)\cos^2\beta$ (B) $I_\theta = \dfrac{4I_0}{9}(\cos^2\beta)$

(C) $I_\theta = \dfrac{4I_0}{9}(1+\cos^2\beta)$ (D) $I_\theta = I_0[1+8\cos^2\beta]$

Where I_0 is the intensity of the individual wave due to the narrow slit alone.

9. Two loudspeakers are emitting sound waves of wavelength λ with an initial phase difference of $\dfrac{\pi}{2}$. At what minimum distance from O on line AB will one hear a maxima ?



(A) $25\,\lambda$ (B) $\dfrac{100\lambda}{\sqrt{15}}$

(C) $\dfrac{25\lambda}{3}$ (D) $50\,\lambda$

Paragraph for Question Nos. 10 to 12

In a modified Young's double slit experiment, there are three identical parallel slits S_1, S_2 and S_3. A coherent monochromatic beam of wavelength 700 nm, having plane wavefronts, falls on the slits. The intensity of the central point O on the screen is found to be I_0 W/m². The distance $S_1S_2 = S_2S_3 = 0.7$ mm and screen is 30 cm from slits.

10. Find the intensity on the screen at O if S_1 and S_3 are covered.

(A) $\dfrac{I_0}{\sqrt{7}}$ (B) $\dfrac{I_0}{7}$ (C) $\dfrac{I_0}{\sqrt{6}}$ (D) $\dfrac{I_0}{6}$

11. Find the intensity on the screen at O if only S_3 is covered.

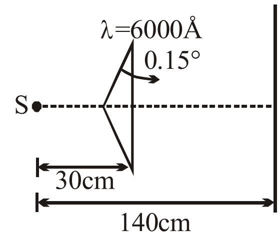
(A) $\dfrac{\sqrt{3}I_o}{7}$ (B) $\dfrac{\sqrt{3}I_0}{\sqrt{7}}$ (C) $\dfrac{3I_0}{7}$ (D) $\dfrac{I_0}{2}$

12. All three slits are now uncovered and a transparent plate of thickness 1.4 μm and refractive index 1.25 is placed in front of S_2. Rresultant intensity at point O is

(A) $3I_0/7$ (C) $4\,I_0/7$ (D) $5\,I_0/7$ (D) $6\,I_0/7$

Paragraph for Question Nos. 13 to 17

Refracting angles of biprism are both 0.15°. Distance of biprism from source is 30 cm. Distance between source and screen is 140 cm. The Fresnel's biprism arrangement shown in the figure uses a source of light of wavelength 6000Å. When a converging lens is moved between the biprism and the screen two images of S are focused on the screen for two positions of the lens. The separations between the two images for the two positions are found to be 0.9 mm and 1.6 mm



13. The focal length of the lens is :-

(A) $\frac{120}{7}$cm (B) $\frac{240}{7}$cm (C) Data not sufficient (D) None

14. Gap between two position of the lens is :-

(A) 80 cm (B) 60 cm (C) 40 cm (D) 20 cm

15. Refractive index of the prism material is approximately :-

(A) 1.46 (B) 1.56 (C) 1.66 (D) 1.76

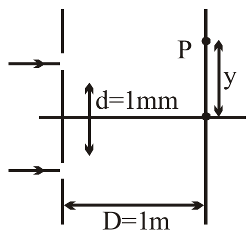
16. The fringe width of the pattern on screen (after the lens has been removed) is :-

(A) 0.7 m (B) 0.35 mm (C) 1.4 mm (D) None

17. The total number of fringes obtained on the screen :-

(A) $\frac{4400}{7}$ (B) $\frac{400}{7}$ (C) $\frac{44}{7}$ (D) None

18. A parallel beam of visible light consisting of wavelengths λ_1 and λ_2 is incident on a standard YDSE apparatus with d = 1 mm, D = 1m. P is a point on the screen at a distance y from center of screen O. At $y = y_1$ is the nearest point above O where the two maxima coincide and at $y = y_2$ is the nearest point above O where the two minima coincide. β_1 & β_2 are fringe width corresponding to wave length λ_1 and λ_2

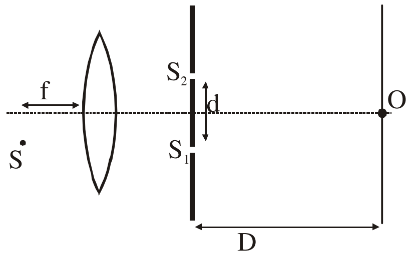


Column-I	**Column-II**
(A) $\beta_1 = 0.3$ mm, $\beta_2 = 0.5$ mm	(P) The 2nd nearest point above O there two maxima coincide is $y = 2y_1$
(B) $\beta_1 = 0.3$ mm, $\beta_2 = 0.4$ mm	(Q) y_2 has no finite value
(C) $\beta_1 = 0.2$ mm, $\beta_2 = 0.4$ mm	(R) The 2nd nearest point above O where the two minima coincide is $y = 3y_2$
	(S) y_1 = LCM of β_1 and β_2

19. Column I shows same modifications in a standard YDSE setup. Column II shows the associated characteristics.

Column I | **Column II**

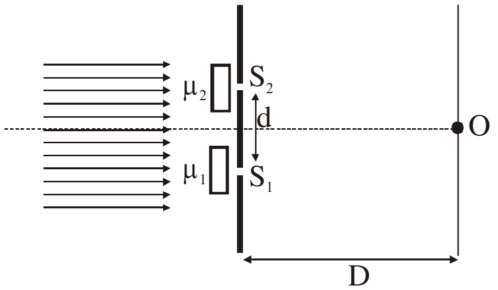
(A)



monochromatic point source
S placed in focal plane.

(P) Zero order maxima lies above O.

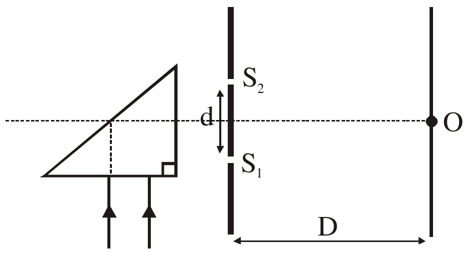
(B)



monochromatic parallel beam incident on S_1S_2 through transparent slabs of same thickness but $\mu_1 > \mu_2$

(Q) If a transparent mica sheet is introduced infront of S_2 central bright fringe can be obtained at O.

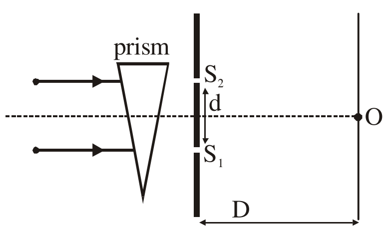
(C)



monochromatic parallel beam incident on a right angled isosceles prism of refractive index 1.50

(R) Fringe width $\beta = \dfrac{\lambda D}{d}$

(D)

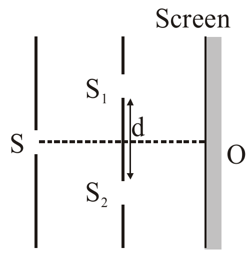
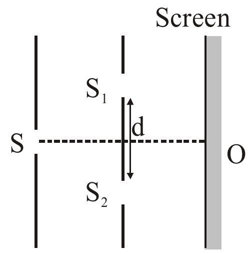
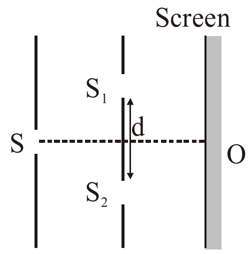
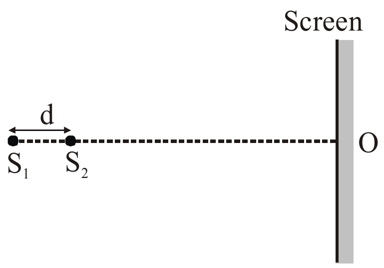


monochromatic parallel beam incident on thin prism

(S) Point O can be a minima

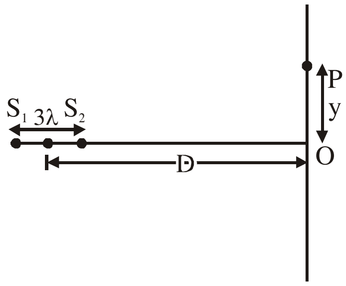
(T) Point O can be a least order minima.

20. Column-I describes various arrangements to obtain interference pattern on screen.

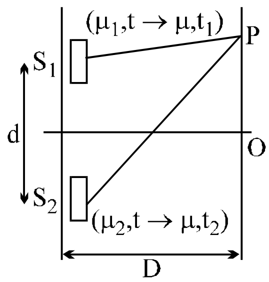
Column–I	**Column–II**
(A)  S_1, S_2 are pin holes. d = 100 λ	(P) Centre of screen 'O' is dark.
(B)  S_1, S_2 are slits. d = 100.5 λ	(Q) First order maxima will not be seen.
(C)  S_1, S_2 are pin holes. d = 0.5 λ	(R) Shape of fringes is hyperbolic
(D)  S_1, S_2 are coherent point sources with zero initial phase difference. d = 2.5 λ	(S) Shape of fringes is circular.
	(T) Shape of fringes is linear.

EXERCISE # (S)

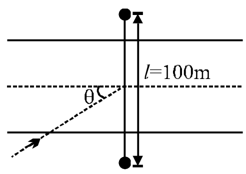
1. Figure shows two coherent microwave source S_1 and S_2 emitting waves of wavelength λ and separated by a distance 3λ. For $\lambda << D$ and $y \neq 0$, the minimum value of y for point P to be an intensity maximum is $\frac{\sqrt{m}\ D}{n}$. Determine the value of m + n, if m and n are coprime numbers.



2. In a YDSE with visible monocromatic light two thin transparent sheets are used in front of the slits S_1 and S_2. $\mu_1 = 1.6$ and $\mu_2 = 1.4$. If both sheets have thickness t, the central maximum is observed at a distance of 5 mm from centre O. Now the sheets are replaced by two sheets of same material of refractive index $\frac{\mu_1 + \mu_2}{2}$ but having thickness t_1 & t_2 such that $t = \frac{t_1 + t_2}{2}$. Now central maximum is observed at distance of 8mm from centre O on the same side as before. Find the thickness t_1(in μm) [Given: d = 1 mm. D = 1m].



3. Two radio transmitter each transmitting a radio of frequency 12 MHz in phase with each other are placed at a distance l = 100 m on either side of the runway on the towers as shown in the figure. An aircraft with a ground speed of V is flying towards the airport such that its velocity makes an angle θ with the runway. The aircraft is heading directly for the mid-point of the towers.

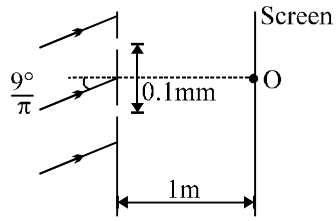


(i) At the aircraft's current position, the intensity of the signal from each tower separately would be I_0. Find the intensity of the combined signal from both towers received by the aircraft for heading of θ = 0 and θ = π/2.

(ii) For what heading would the aircraft receive no signal. Assuming distance of plane much greater than l.

(iii) If the aircraft is heading from large distance directly towards one of the transmitter in the direction perpendicular to 'l' find the position of the aircraft from the nearest tower when the signal dropped to a minimum first time.

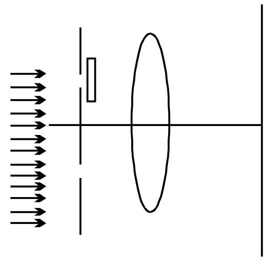
4. In a Young's double slit experiment, a parallel beam of light of wavelength 6000 Å is incident on the slit at an angle of $\frac{9°}{\pi}$ as shown in the figure.



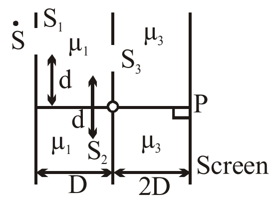
(i) Find fringe width of the interference pattern.

(ii) Find phase difference between the light coming from the two slits at O.

5. The intensity received at the focus of the lens is I when no glass slab has been placed in front of the slit. Both the slits are of the same dimension and the plane wavefront incident perpendicularly on them, has wavelength λ. On placing the glass slab, the intensity reduces to 3I/4 at the focus. Find out the minimum thickness of the glass slab (in Å) if its refractive index is 3/2. Given $\lambda = 6933$Å, $\mu = 1.5$.



6. In the figure shown 'S' is a monochromatic source of light emitting light of wavelength λ (in air). Light falls on slit 'S_1) from 'S' and then reach the slits 'S_2' and 'S_3' through a medium of refractive index 'μ_1'. Light from slits S_2 and S_3 reach the screen through medium of refractive index μ_3. A thin transparent film of refractive index μ_2 and thickness 't' is placed infront of 'S_2' point 'P' is symmetrical w.r.t. 'S_2' and 'S_3' Using the values d = 1 mm, D = 1m, $\mu_1 = 4/3$, $\mu_2 = 3/2$, $\mu_3 = 9/5$ and $t = \frac{4}{9} \times 10^{-5}$ m . Find the



(i) Distance of central maxima from P.

(ii) If the filmin front of S_2 is removed, then by what distance and in which direction will the central maxima shift ?

ANSWER KEY

EXERCISE # (O)

1. Ans. (A,B) **2. Ans. (B, C)** **3. Ans. (A,B,C,D)** **4. Ans. (C)**

5. Ans. (B) **6. Ans. (C)** **7. Ans. (A)** **8. Ans. (D)**

9. Ans. (C) **10. Ans. (B)** **11. Ans. (C)** **12. Ans. (A)**

13. Ans. (B) **14. Ans. (D)** **15. Ans. (D)** **16. Ans. (A)**

17. Ans. (C)

18. Ans. (A) $\rightarrow$ (P,R,S); (B) $\rightarrow$ (P,Q,S); (C) $\rightarrow$ (P, Q, S)

19. Ans. (A) $\rightarrow$ (P,R,S,T); (B) $\rightarrow$ (Q,R,S,T); (C) $\rightarrow$ (R); (D) $\rightarrow$ (P,R,S,T)

20. Ans. (A) $\rightarrow$ (R) ; (B) $\rightarrow$ (T) ; (C) $\rightarrow$ (Q,R) ; (D) $\rightarrow$ (P,S)

EXERCISE # (S)

1. Ans. 7 **2. Ans. 33**

3. Ans. $4I_0$, $\theta = \sin^{-1}\left(\frac{2n+1}{8}\right)$ n = 0, 1, 2, 3, (iii) 393.75 m

4. Ans. (i) 6 mm, (ii) $50\pi/3$ **5. Ans. 2311**

6. Ans. (i) 0, (ii) $\frac{40}{27}\times 10^{-3}\text{m}$ downwards