SCORE JEE (Advanced)
HOME ASSIGNMENT # 05

SOLUTION — MATHEMATICS

1. Ans. (B)

Case-I :

Two rows contain 2 letters each and one row has 1 letter.

Possible ways $= 4 \times {}^3c_2 + {}^4c_2 \times {}^3c_1 + {}^4c_2 \times {}^3c_2 \times 2$
$= 12 + 18 + 36 = 66$

Case-II :

One row has 3 letters and two others have 1 letter each.

Possible ways $= {}^4c_3 \times {}^3c_1 \times {}^2c_1 + {}^4c_1 \times {}^2c_1$
$= 4 \times 3 \times 2 + 4 \times 2 = 24 + 8 = 32$

Hence total arrangements $= \dfrac{(66+32)5!}{2!} = 5880$

2. Ans. (D)

Suppose we consider {Ace, 2, 3, 4, 5} for each card we have 4 possible suits. So, total ways of such a straight $= (4)^5 = 2^{10}$.

But in this we have counted those cases when all are of same suit = 4.

So total ways of such a straight $= 2^{10} - 4$

But we have 10 such straights - {Ace, 2, 3, 4, 5}, {2, 3, 4, 5, 6} .... {10, J, Q, K, Ace}

So, total ways $= 10 \,.\, (2^{10} - 4) = 10200$.

3. Ans. (B)

Case-I :

a, b, c having exactly one 5 with :

(i) 2 even digits (different or same) :

$${}^4C_2.3! + {}^4C_1.\frac{3!}{2!} = 48$$

(ii) one digit divisible by 4 and one odd :
${}^2C_1.{}^4C_1.\,3! = 48$

Case-II :

a, b, c having exactly two 5 with :

(i) one digit divisible by 4 : $2\,.\,\dfrac{3!}{2!} = 6$

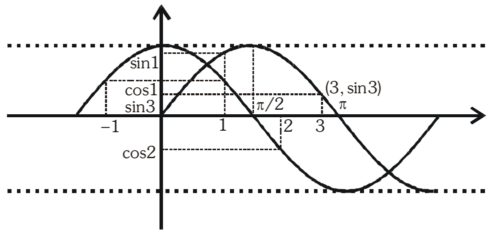
Hence number of ways = 102

4. Ans. (A)

By the graph it is clear that

$\cos 2 < \sin 3 < \cos 1 < \sin 1$

$\therefore$ sin1 is greatest



5. Ans. (A)

$\tan\theta = k$

$$\frac{3\tan\frac{\theta}{3} - \tan^3\frac{\theta}{3}}{1 - 3\tan^2\frac{\theta}{3}} = k$$

$$\tan^3\frac{\theta}{3} - 3k\tan^2\frac{\theta}{3} - 3\tan\frac{\theta}{3} + k = 0$$

$$\sum \tan\frac{\alpha}{3}\tan\frac{\beta}{3} = -3.$$

6. Ans. (B)

$$R = \log_2 \sin\left(\frac{\pi}{5}\right) + \log_2 \sin\left(\frac{2\pi}{5}\right) + \log_2 \sin\left(\frac{3\pi}{5}\right) + \log_2 \sin\left(\frac{4\pi}{5}\right) - \log_2 5$$

$$R = \log_2\left(\sin\frac{\pi}{5}.\sin\frac{2\pi}{5}.\sin\frac{3\pi}{5}.\sin\frac{4\pi}{5}\right) - \log_2 5$$

$$= \log_2\left(\sin^2\frac{\pi}{5}.\sin^2\frac{2\pi}{5}\right) - \log_2 5$$

$$= \log_2 \frac{1}{4}\left(2\sin^2\frac{\pi}{5}.2\sin^2\frac{2\pi}{5}\right) - \log_2 5$$

$$= \log_2 \frac{1}{4}\{(1-\cos 72°)(1-\cos 144°)\} - \log_2 5$$

$$= \log_2 \frac{1}{4}\{(1-\sin 18°)(1+\cos 36°)\} - \log_2 5$$

$$= \log_2 \frac{1}{4}\left\{\left(1-\frac{\sqrt{5}-1}{4}\right)\left(1+\frac{\sqrt{5}+1}{4}\right)\right\} - \log_2 5$$

$$= \log_2 \frac{1}{4}\left(\frac{5-\sqrt{5}}{4}\right)\left(\frac{5+\sqrt{5}}{4}\right) - \log_2 5$$

$$= \log_2 \frac{1}{4}\left(\frac{20}{4\times 4}\right) - \log_2 5 = \log_2\left(\frac{5}{16}\right) - \log_2 5$$

$$= \log_2 5 - \log_2 16 - \log_2 5 = -4.$$

7. Ans. (D)

$6\cos^5\theta - 6\cos^4\theta - 5\cos^3\theta + 5\cos^2\theta + \cos\theta - 1 = 0$

$\Rightarrow\ 6\cos^4\theta(\cos\theta - 1) - 5\cos^2\theta(\cos\theta - 1) + (\cos\theta - 1) = 0$

$\Rightarrow\ (\cos\theta - 1)\{6\cos^4\theta - 5\cos^2\theta + 1\} = 0$

$\Rightarrow\ (\cos\theta - 1)(3\cos^2\theta - 1)(2\cos^2\theta - 1) = 0$

$\cos\theta = 1;\ \cos\theta = \pm\frac{1}{\sqrt{3}};\ \cos\theta = \pm\frac{1}{\sqrt{2}}$

$\downarrow \quad \downarrow \quad \downarrow$

$0 \quad 4 \quad 4$

$\Rightarrow$ 8 solutions

8. Ans. (D)

$3\sin^2x - \sin x + \ell n(\text{sgn}(\cot^{-1}x)) = 0$

$\because\ \cot^{-1}x \in (0, \pi)$

$\therefore\ \text{sgn}(\cot^{-1}x) = 1 \Rightarrow \ell n(\text{sgn}(\cot^{-1}x)) = 0$

Now, $3\sin^2x - \sin x = 0$

$\sin x = 0$ or $\frac{1}{3}$

$\sin x = 0$ at $x = -\pi, 0, \pi$

$\sin x = \frac{1}{3}$ at $x = \alpha, \beta;\ \alpha \in (0, \pi/2)$

& $\beta \in (\pi/2, \pi)$

$\therefore$ Number of solutions in $[-\pi, \pi]$ is 5

9. Ans. (A)

Let the boxes be B_1, B_2, B_3, B_4. Let us assume that two specific balls have been put in box B_i $(i = 1,2,3,4)$.

It means in box B_i we have to put 3 balls from the remaining 18 balls.

Thus the probability that the two specific balls have been put in the particular box

$$P(B_i) = \frac{{}^{18}C_3}{{}^{20}C_5} = \frac{5\times4}{20\times19} = \frac{1}{19}$$

10. Ans. (A)

$\sin A = \sin B$ and $\cos A = \cos B$

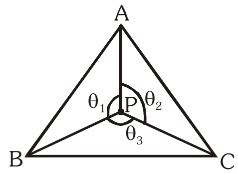
$\Rightarrow\ 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right) = 0$

& $\ 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right) = 0$

$\Rightarrow\ \sin\left(\frac{A-B}{2}\right) = 0$

as $\sin\frac{A+B}{2}$ & $\cos\frac{A+B}{2}$ cannot be zero simultaneously.

11. Ans. (B)



$\theta_1 + \theta_2 + \theta_3 = 2\pi$

$\frac{\theta_1}{2} + \frac{\theta_2}{2} + \frac{\theta_3}{2} = \pi$

$\tan\frac{\theta_1}{2} + \tan\frac{\theta_2}{2} + \tan\frac{\theta_3}{2} = \tan\frac{\theta_1}{2}\tan\frac{\theta_2}{2}\tan\frac{\theta_3}{2}$

$r_x = s\tan\frac{\theta_1}{2};\ r_y = \frac{s}{2}\tan\frac{\theta_2}{2};\ r_z = \frac{s}{3}\tan\frac{\theta_3}{2}$

$\frac{r_x}{s} + \frac{2r_y}{s} + \frac{3r_z}{s} = \frac{6r_xr_yr_z}{s^3}$

$\frac{r_x}{6} + \frac{r_y}{3} + \frac{r_z}{2} = \frac{r_xr_yr_z}{s^2}$

12. Ans. (B)

$\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

$\Rightarrow\ 2\sin 2x\cos x - 3\sin 2x = 2\cos 2x\cos x - 3\cos 2x$

$\Rightarrow\ \sin 2x(2\cos x - 3) = \cos 2x(2\cos x - 3)$

$\Rightarrow\ (2\cos x - 3)(\sin 2x - \cos 2x) = 0$

$\therefore\ \cos x = \frac{3}{2}$ or $\tan 2x = 1$ with $\cos 2x \neq 0$

$\Rightarrow\ 2x = n\pi + \frac{\pi}{4}$ with $2x \neq (2k+1)\frac{\pi}{2}$

$\therefore\ x = \frac{n\pi}{2} + \frac{\pi}{8}$

13. Ans. (B)

$y + \frac{1}{y} \geq 2 \Rightarrow \sqrt{y + \frac{1}{y}} \geq \sqrt{2}$

$\sin x + \cos x = \sqrt{2}$ is only possible case. When $y = 1$

$\Rightarrow\ \cos x\frac{1}{\sqrt{2}} + \sin x\frac{1}{\sqrt{2}} = 1$

$\Rightarrow\ \cos\left(x - \frac{\pi}{4}\right) = \cos 0$

$\Rightarrow\ x - \frac{\pi}{4} = 2n\pi \quad \Rightarrow\ x = 2n\pi + \frac{\pi}{4}$

$\therefore$ at $n = 0,\ x = \frac{\pi}{4},\ y = 1$

14. Ans. (A)

$AH^2 + BC^2 = 4R^2\cos^2A + 4R\cos^2A = 4R^2$

$\Rightarrow \frac{1}{64}(AH^2 + BC^2)(BH^2 + AC^2)(CH^2 + AB^2)$

$= \frac{64R^6}{64} = R^6 = 64.$

15. Ans. (D)

tan A + tanB + tanC = (tanA tanB)tanC

tanA + tanB + tanC = $\frac{3}{4}$ tanC

given that

tanA tanB = $\frac{3}{4}$ $\Rightarrow$ tan B = $\frac{3}{4\tan A}$

tanA + $\frac{3}{4\tan A}$ + tanC = $\frac{3}{4}$tanC

Let tanA = t

t + $\frac{3}{4t}$ + $\frac{1}{4}$tanC =0

$4t^2$ + (tanC)t + 3 = 0

t is real $\Rightarrow$ D $\geq$ 0

$\tan^2C - 4\,(3)(4) \geq 0$

$\tan^2C \geq 48$

Possible only for D option because

$\tan^2 75^0 = (2+\sqrt{3})^2 = 7+2\sqrt{3}$

16. Ans. (C)

$\lambda = \tan x - \cot y$

$\lambda^2 = \tan^2 x + \cot^2 y - 2\tan x \cot y$

$2\tan x \cot y = \tan^2 x + \cot^2 y - \lambda^2$

$\tan^4 x + \cot^4 y + 8 = 4\tan^2 x + 4\cot^2 y - 4\lambda^2$

$(\tan^2 x - 2)^2 + (\cot^2 y - 2)^2 + 4\lambda^2 = 0$

possible if $\tan x = \sqrt{2}$; $\cot y = \sqrt{2}$; $\lambda = 0$

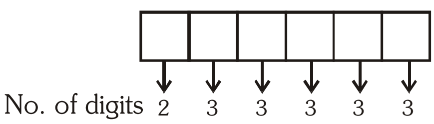
17. Ans. (B)

Total – (Dearrangement)

4! – (Dearrangement of 4 objects)

24 – 9 = 15

18. Ans. (B)



Single digit number = 2

Two digit number = 2 × 3

Three digit number = 2 × 3 × 3

Four digit number = 2 × 3 × 3 × 3

Five digit number = 2 × 3 × 3 × 3 × 3

Six digit number = 2 × 3 × 3 × 3 × 3 × 3

$\therefore$ Total numbers

= 2 + 6 + 18 + 54 + 162 + 486 = 728

19. Ans. (D)

P(atleast two digit same) = 1 – P(All digits different)

$=1-\frac{9.9.8}{900}=\frac{7}{25}$

20. Ans. (D)

$\sqrt{2}(2\cos 2x - 1) + \sqrt{3 - 4\cos 2x + (2\cos^2 2x - 1)} = \sqrt{2}$

or $\sqrt{2}(2\cos 2x - 1) + \sqrt{2(1-\cos 2x)^2} = \sqrt{2}$

$\Rightarrow$ $2\cos 2x - 1 + 1 - \cos 2x = 1$ or cos2x = 1

$\therefore$ 2x = 2nπ, n $\in$ I orx = nπ

21. Ans. (A)

A A D H I K R

words starting from A = $6! = 720$

words starting from D, H, I, K=$\left(\frac{6!}{2!}\right).4$

$= (360)4 = 1440$

words starting from RAA = 4! = 24

RADA = 3! = 6

RADHA = 2! = 2

RADHIAK = 1

$\therefore$ Number of words before RADHIKA = 2193

22. Ans. (B)

Each of the N persons from a pair with (N – 3) person (i.e. excluding the person himself and the adjacent two)

So total number of pairs that can be formed

$= \frac{N(N-3)}{2}$

Total time they sing = $\frac{N(N-3)}{2} \times 2 = 28$

$\Rightarrow$ on solving N = 7 or – 4

$\Rightarrow$ N = 7 (as N > 0)

23. Ans. (D)

$x = n\pi - \tan^{-1}3$ $\Rightarrow$ tanx = –3

Now, tan2x = $\frac{2\tan x}{1-\tan^2 x} = \frac{3}{4}$ and

cosx = $\pm\frac{1}{\sqrt{1+\tan^2 x}} = \pm\frac{1}{\sqrt{10}}$

on substituting these values in the given equation, we find only cosx = $-\frac{1}{\sqrt{10}}$ satisfies the equation, so equation holds true for tanx $= -3$ and $\cos x = -\frac{1}{\sqrt{10}}$

which is possible if x lies in II quadrant.

So, n must be odd integer.

24. Ans. (C)

$x + y \le 5 \qquad x, y \ge 1$

$x + y \le 3 \qquad x, y \ge 0$

$x + y + z = 3 \qquad x, y, z \ge 0$

number of non-negative integral solutions of the above equation is -

${}^{3+3-1}C_{3-1} = {}^5C_2 = 10$

number of integral points which lies inside or on the square is 8.

i.e (1,1) (1,2) (1,3) (2,1) (2,2) (2,3) (3,1) (3,2)

$\therefore$ Required probability is $= \frac{2}{10} = \frac{1}{5}$

25. Ans. (A)

$f(\theta) = 3 + 2\sin^2\theta - 3\sin 2\theta$

$= 3 + 1 - \cos 2\theta - 3\sin 2\theta$

$= 4 - (\cos 2\theta + 3\sin 2\theta)$

$-\sqrt{10} \le \cos 2\theta + 3\sin 2\theta \le \sqrt{10}$

$-(\cos 2\theta + 3\sin 2\theta) \le \sqrt{10}$

$4 - (\cos 2\theta + 3\sin 2\theta) \le 4 + \sqrt{10}$

$\log_{(4+\sqrt{10})} f(\theta) \le 1$

26. Ans. (A)

$$\frac{\tan 500° + 2\tan 470°}{1 + \tan 500° \tan 490°} = \frac{\tan 140° + 2\tan 110°}{1 + \tan 140° \tan 130°}$$

$$= \frac{-\tan 40° - 2\cot 20°}{1 + (-\tan 40°)(-\cot 40°)}$$

$$= -\frac{1}{2}\left(\frac{2\tan 20°}{1-\tan^2 20°} + \frac{2}{\tan 20°}\right) = -\frac{1}{2}\left(\frac{2t}{1-t^2} + \frac{2}{t}\right)$$

$$= \frac{1}{t(t^2-1)}$$

27. Ans. (C)

$A > B$

$3\sin x - 4\sin^3 x = k \quad \Rightarrow \sin 3x = k$

A & B are roots of $\sin 3x = k$

$\Rightarrow \quad 3B = \sin^{-1} k, \ \ 3A = \pi - \sin^{-1} k$

Now $C = \pi - (A + B)$

$$= \pi - \left(\frac{\pi}{3} - \frac{1}{3}\sin^{-1} k + \frac{1}{3}\sin^{-1} k\right) = \frac{2\pi}{3}$$

28. Ans. (D)

$9! = 2^7 \times 3^4 \times 5 \times 7$

odd factors of the form 3m + 2 are neither multiple of 2 nor multiple of 3. So the factors may be 1, 5, 7, 35 of which 5 and 35 are of the form 3m + 2, their sum is 40.

29. Ans. (A)

$-\pi \le \pi \sin x \le \pi$

$\Rightarrow \quad -1 \le \cos(\pi \sin x) \le 1$

$\Rightarrow \quad -1 \le \sin\left(\frac{\pi}{2}(\cos \pi \sin x)\right) \le 1$

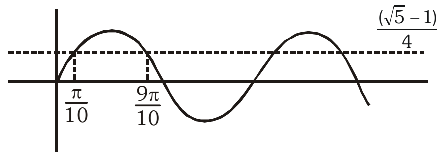
$\Rightarrow \quad -\pi \le \pi \sin\left(\frac{\pi}{2}\cos(\pi \sin x)\right) \le \pi$

$\Rightarrow \quad -1 \le y \le 1$

30. Ans. (C)

$\sin x \ge \frac{1}{\sqrt{5}+1}$

$\Rightarrow \sin x \ge \frac{\sqrt{5}-1}{4} \ \Rightarrow \sin x \ge \sin\frac{\pi}{10}$



$x \in \left[\frac{\pi}{10}, \frac{9\pi}{10}\right]$ general solution $\left[2k\pi + \frac{\pi}{10}, 2k\pi + \frac{9\pi}{10}\right]$

31. Ans. (A)

$3^{\sin 2x + 2\cos^2 x} + 3^{1 - \sin 2x + 2\sin^2 x} = 28$

$1 - \sin 2x + 2\sin^2 x$

$= 1 - (\sin 2x - 2\sin^2 x)$

$= 3 - (\sin 2x + 2\cos^2 x)$

$\therefore \quad 3^{\sin 2x + 2\cos^2 x} + 3^3 . 3^{-(\sin 2x + 2\cos^2 x)} = 28$

Put $3^{\sin 2x + 2\cos^2 x} = t$

$t + \frac{27}{t} = 28$

$t^2 - 28t + 27 = 0$

$t = 1, 27$

$\therefore \quad \sin 2x + 2\cos^2 x = 0, 3$

$\Rightarrow \quad 2\sin x \cos x + 2\cos^2 x = 0, 3$

Case-I : $2\cos x(\sin x + \cos x) = 0$

$\Rightarrow \quad \cos x = 0, \tan x = -1$

Case-II : $2\cos x(\sin x + \cos x) = 3$

$\Rightarrow \quad \cos 2x + \sin 2x = 2$ which is not possible from case-I general solution is

$x = (2K+1)\frac{\pi}{2}, K\pi - \frac{\pi}{4}, K \in I$

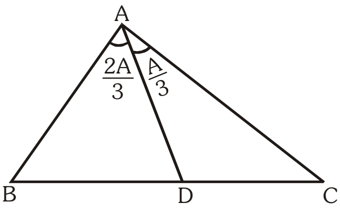
32. Ans. (C)

$2079000 = 2^3 \times 3^3 \times 5^3 \times 7 \times 11$

For the divisors to be even and divisible by 15; 2, 3 and 5 must occur atleast once.

Therefore the total number of required divisors are $3 \times 3 \times 3 \times 2 \times 2 = 108$

33. Ans. (D)



Using sine rule in ΔABD

$$\frac{\sin\frac{2A}{3}}{BD} = \frac{\sin B}{AD} \quad \text{.......... (i)}$$

Using sine rule in ΔADC

$$\frac{\sin C}{AD} = \frac{\sin\frac{A}{3}}{CD} \quad \text{.......... (ii)}$$

from (i) & (ii)

$$\frac{BD\sin B}{\sin\frac{2A}{3}} = \frac{CD\sin C}{\sin\frac{A}{3}}$$

$$\Rightarrow \frac{\sin B}{\sin C} = \frac{CD}{BD}\frac{\sin\frac{2A}{3}}{\sin\frac{A}{3}} = 2\cos\frac{A}{3}$$

34. Ans. (C)

Given set of numbers is {1, 2, ... 9} is which 4 are even and 5 are odd, so in the given product it is not possible to arrange to substract only even number from odd number. There must be atleast one factor involving substraction of an odd number from another odd number. So atleast one of the factor is even. Hence product is always even.

∴ Required probability = 1.

35. Ans. (D)

$\cos^6 x + \sin^4 x \geq 1 + 2x^4$

$f(x) = (1-\sin^2 x)^3 + (\sin^2 x)^2$

Let $\sin^2 x = t$

$g(t) = (1-t)^3 + t^2$, where $0 \leq t \leq 1$

$g'(t) = 2t - 3(1-t)^2 = -\{3t^2 - 8t + 3\}$

$= -\left(t - \frac{4-\sqrt{7}}{3}\right)\left(t - \frac{4+\sqrt{7}}{3}\right)$

$\Rightarrow g(t) \leq g(0) \quad \Rightarrow g(t) \leq 1$

$\Rightarrow f(x) \leq 1 \;\&\; 1 + 2x^4 \geq 1$

⇒ Inequality holds only for x = 0.

36. Ans. (A)

We have $\tan(A+B)\tan(A-B) = 1$

$\Rightarrow 2A = \frac{\pi}{2}$ or $A = \frac{\pi}{4}$

Also $\tan\left(\frac{\pi}{4}+B\right) + \tan\left(\frac{\pi}{4}-B\right) = 4$

$\Rightarrow \frac{1+\tan B}{1-\tan B} + \frac{1-\tan B}{1+\tan B} = 4$

$\Rightarrow \frac{1-\tan^2 B}{1+\tan^2 B} = \frac{1}{2}$ or $\cos 2B = \frac{1}{2} \quad \therefore B = \frac{\pi}{6}$

37. Ans. (C)

$\frac{\sqrt{abc(a+b+c)}}{\Delta} = \frac{1}{\Delta}\sqrt{4R\Delta.2s}$

$= \sqrt{\frac{8Rs}{\Delta}} = \sqrt{\frac{8R}{r}} \geq \sqrt{8.2} = 4 \quad (\because R \geq 2r)$

38. Ans. (D)

Number must be divisible by 3 & 5 as the sum is 48 so every number will be divisible by 3. For divisibility of 5, unit digit must be '5'.

(i) 99988[5] → $\frac{5!}{2!\,3!} = 10$

(ii) 99997[5] → $\frac{5!}{4!} = 5$

15

39. Ans. (D)

	Case		Number of ways
(i)	$x_1x_2x_3x_4x_5$	${}^6C_5$	= 6
(ii)	$x_1x_1x_2x_3x_4$	${}^3C_1.{}^5C_3$	= 30
(iii)	$x_1x_1x_1x_2x_3$	${}^2C_1.{}^5C_2$	= 20
(iv)	$x_1x_1x_2x_2x_3$	${}^3C_2.{}^4C_1$	= 12
(v)	$x_1x_1x_1x_2x_2$	${}^2C_1.{}^2C_1$	= 4

Total 72

40. Ans. (C)

$\cos^2(\pi x) - \sin^2(\pi y) = \frac{1}{2}$

$\Rightarrow \cos\pi(x + y)\cos\pi(x - y) = \frac{1}{2}$

$\Rightarrow \cos\pi(x + y)\cos\frac{\pi}{3} = \frac{1}{2} \quad (\because\ x - y = \frac{1}{3})$

$\Rightarrow \cos\pi(x + y) = 1 \quad \Rightarrow x + y = 2n,\ n \in I$

$\therefore \quad x + y = 2n\ \&\ x - y = \frac{1}{3}$

$\Rightarrow x = n + \frac{1}{6},\quad y = n - \frac{1}{6},\ n \in I$

for $n = -1$, $(x, y) = \left(-\frac{5}{6}, -\frac{7}{6}\right)$

41. Ans. (D)

First of all select $1 + 2 + 3 + 4 = 10$ pens out of 25 identical pens and distribute them as desired. It can happen only in one way. Now let x_1, x_2, x_3 and x_4 pens are given to them respectively (here $x_1, x_2, x_3, x_4 \geq 0$).

As now any one can get any number of pens.

$\therefore$ non-negative integral solution of $x_1 + x_2 + x_3 + x_4 = 15$ will be the number of ways so ${}^{15+4-1}C_{4-1} = {}^{18}C_3$.

42. Ans. (B)

The prime digits are 2, 3, 5, 7.

If we fix 2 at first place, then other eleven places are filled by all four digits.

So, total number of ways $= 4^{11}$.

Now sum of 2 consecutive digits is prime when consecutive digits are (2,3) or (2, 5), then 2 will be fixed at all alternate places.

2		2		2		2		2		2	

So, favourable cases $= 2^6$

$\therefore$ Required probability $= \frac{2^6}{2^{22}} = \frac{1}{2^{16}}$

43. Ans. (C)

$\vec{a}.\vec{b} = 0$

$\Rightarrow x_1 - 2x_2 + x_3 = 0 \Rightarrow x_1 + x_3 = 2x_2$

x_1, x_2, x_3 are selected from the set $\{1, 2, 3, ..., 10\}$,

Case-I : 1, 3, 5, 7, 9 = 5 numbers

2, 4, 6, 8, 10 = 5 numbers

3 numbers in A.P. can be selected by ${}^5C_2 + {}^5C_2 = 20$ ways.

In each way x_1 & x_3 can be interchanged

$\therefore$ Number of required triplets are $2 \times 20 = 40$

Case-II : When numbers are equal $x_1 = x_2 = x_3$ then number of triplets = 10

Hence number of vectors $\vec{a}$ are $40 + 10 = 50$

44. Ans. (C)

Without any loss of generality assume the wts to be 1, 2, 3, 4, 5, 6

It is obvious that 1 should be at the top of pyramid.

If 2, 3 make second row then

1

2 3-------2!

4 5 6-------3!

$\Rightarrow$ 12 ways

If 2, 4 make second row

1

2 4-------2

3 5 6-------2

$\Rightarrow$ 4 ways

Total number of ways = 16

45. Ans. (B)

$x, y, z, w = k\frac{\pi}{2} \in [0, 10]$

$\because \quad 2^{\sin^2 x}\, 3^{\cos^2 y}\, 4^{\sin^2 z}\, 5^{\cos^2 w} \geq 120$

$\Rightarrow 2^{\sin^2 x}\, 3^{\cos^2 y}\, 4^{\sin^2 z}\, 5^{\cos^2 w} \geq 2.\ 3.\ 4.\ 5$

Taking logarithm both sides we have

$\Rightarrow \sin^2 x \log 2 + \cos^2 y \log 3 + \sin^2 z \log 4 + \cos^2 w \log 5 \geq \log 2 + \log 3 + \log 4 + \log 5$

$\Rightarrow \cos^2 x \log 2 + \sin^2 y \log 3 + \cos^2 z \log 4 + \sin^2 w \log 5 \leq 0$

which is possible only when

$\cos^2 x = 0 \Rightarrow x = m\pi + \frac{\pi}{2}, m \in I$

$\sin^2 y = 0 \Rightarrow y = n\pi, n \in I$

$\cos^2 z = 0 \Rightarrow z = r\pi + \frac{\pi}{2},\ r \in I$

$\sin^2 w = 0 \Rightarrow w = p\pi\ , p \in I$

$\because \quad x, y, z, w \in [0, 10]$

$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ (three solutions)

$\Rightarrow y = 0, \pi, 2\pi, 3\pi$ (four solutions)

$\Rightarrow \quad z = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ (three solutions)

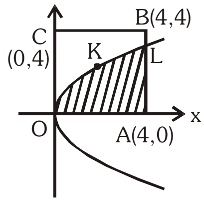
$\Rightarrow \quad w = 0, \pi, 2\pi, 3\pi$ (four solutions)

Hence the number of ordered 4–tuple (x, y, z, w) is 3 . 4 . 3 . 4 = 144

And probability $= \frac{144}{7^4}$

46. Ans. (D)

Probability $= \frac{\text{Area OALKO}}{\text{Area OABC}}$



$= \frac{\int_0^4 \sqrt{x}dx}{4 \times 4}$

$= \frac{\frac{2}{3}\left[x^{3/2}\right]}{16} = \frac{\frac{16}{3}}{16} = \frac{1}{3}$

47. Ans. (B)

1 3 2 4 2 1 4 3 3 1 4 2 4 1 3 2

1 4 3 2 2 4 1 3 3 2 1 4 4 1 1 3

2 4 3 1 3 2 4 1 4 3 2 1

Total 11 numbers

48. Ans. (C)

(i) For two digit number = 9 → (11, 22,....99)

(ii) For three digit number all same = 9 (111, 222, ...)

(iii) For 3 digit when 2 digit same and 1 different.

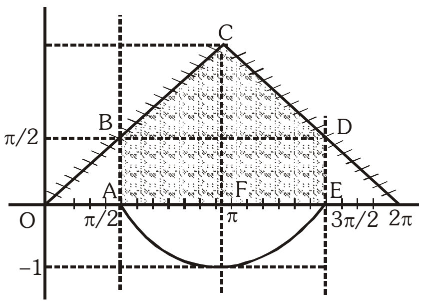
(a) | × | × | 0 | or | × | 0 | × | = 9 + 9 = 18

(b) without zero $^3C_1 . 9 . 8 = 216$

Total 216 + 18 + 9 + 9 = 252

$p = \frac{252}{999} = \frac{28}{111}$

49. Ans. (B)

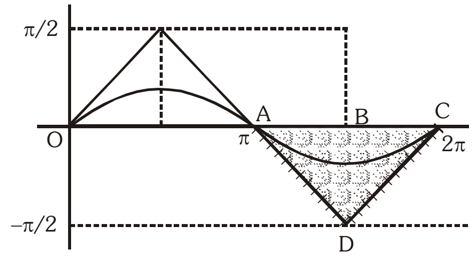


$f(x) = \max(\cos x, \cos^{-1}(\cos x))$

A_1 = Area of ABCDEFA

= 2 . area of trapezium ABCFA

$= 2 . \frac{1}{2} . (AB + CF).AF = \frac{3\pi^2}{4}$



$g(x) = \min(\sin x, \sin^{-1}(\sin x))$

A_2 = Area of triangle ABCDA

$= \frac{1}{2} . AC . BD = \frac{1}{2} . \pi . \frac{\pi}{2} = \frac{\pi^2}{4} \Rightarrow \frac{A_2}{A_1} = \frac{1}{3}$

50. Ans. (A,B,C)

$E^c = F$; $E \cap E^c = Q$; P(E) + P(not E) = 1; E and E^c can always be equally likely

51. Ans. (A,B)

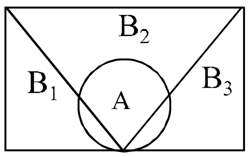
B_1 = Journey by car

B_2 = Journey by motor cycle

B_3 = Journey on foot

$P(B_1) = \frac{1}{2}$;

$P(B_2) = \frac{1}{6}$;



$P(B_3) = \frac{1}{3}$

Let A = accident occurs

$P(A/B_1) = \frac{1}{5}$; $P(A/B_2) = \frac{2}{5}$; $P(A/B_3) = \frac{1}{10}$

$P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3)$

$= P(B_1) \cdot P(A/B_1) + P(B_2) \cdot P(A/B_2)$

$+ P(B_3) \cdot P(A/B_3)$

$= \frac{1}{2} \cdot \frac{1}{5} + \frac{1}{6} \cdot \frac{2}{5} + \frac{1}{3} \cdot \frac{1}{10}$

$= \frac{3}{30} + \frac{2}{30} + \frac{1}{30} = \frac{6}{30} = \frac{1}{5}$

$\Rightarrow$ **(A) is correct**

$P(B_1/A) = \frac{\frac{1}{2} \cdot \frac{1}{5}}{\frac{1}{5}} = \frac{1}{10} \cdot \frac{5}{1} = \frac{1}{2}$

$\Rightarrow$ **(B) is correct**

$$P(B_2/A) = \frac{\frac{1}{6}\cdot\frac{2}{5}}{\frac{1}{5}} = \frac{2}{30}\cdot\frac{5}{1} = \frac{1}{3}$$

$\Rightarrow$ **(C) is incorrect**

$$P(B_3/A) = \frac{\frac{1}{3}\cdot\frac{1}{10}}{\frac{1}{5}} = \frac{1}{30}\cdot\frac{5}{1} = \frac{1}{6}$$

$\Rightarrow$ **(D) is incorrect**

52. Ans. (A,B,C,D)

$P(A) + P(B) - P(A \cap B) = 0.8$

$\therefore \quad P(A \cap B) = 0.5 + 0.4 - 0.8 = 0.1$

$P(\overline{A} \cap B) = P(B) - P(A \cap B)$

$= 0.5 - 0.1 = 0.4 \quad \Rightarrow$ **(A)** is correct

$$P(B/\overline{A}) = \frac{P(B \cap \overline{A})}{P(\overline{A})} = \frac{0.4}{1-P(A)} = \frac{0.4}{0.6} = \frac{2}{3}$$

$\Rightarrow$ **(B)** is correct

$P(A) \cdot P(B) = 0.2$

$\Rightarrow \quad P(A \cap B) < P(A) \cdot P(B)$

$\Rightarrow$ **(C)** is correct

P(A or B but not both) $= 0.9 - 2 \times 0.1 = 0.7$

53. Ans. (A,B,C)

$\cos^2x - \sin^2x = (\cos x - \sin x)(1 + \sin x \cos x)$

$\cos x - \sin x = 0$ or $\cos x + \sin x = 1 + \sin x \cos x$

$\cos x = \sin x$

or $(\cos x + \sin x) = 1 + \frac{(\cos x + \sin x)^2 - 1}{2}$

$\tan x = 1$ Let $\cos x + \sin x = t$

$x = n\pi + \frac{\pi}{4}, n \in I$ $\quad t = 1 + \frac{t^2-1}{2}$

$t^2 - 2t + 1 = 0$

$t = 1$

$\sin x + \cos x = 1$

$\cos x \cdot \frac{1}{\sqrt{2}} + \sin x \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$

$x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

$x = 2n\pi + \frac{\pi}{2}$

$x = 2n\pi$

54. Ans. (C,D)

$2\sin x - 2\cos 2x - \sin 2x - 2\sin x \sin 2x + 2\cos x = 0$

$2\sin x - 2\cos x - \sin 2x - \cos x + \cos 3x + 2\cos x = 0$

$2\sin x(1 - \cos x) + 4\cos^3x - 3\cos x$

$- 2(2\cos^2x - 1) + \cos x = 0$

$2\sin x(1 - \cos x) + 4\cos^3x - 4\cos^2x - 2\cos x + 2 = 0$

$2\sin x(1 - \cos x) + 4\cos^2x(\cos x - 1) - 2(\cos x - 1) = 0$

$(1 - \cos x)\{2\sin x + 2 - 4(1 - \sin^2x)\} = 0$

$(1 - \cos x)\{4\sin^2x + 2\sin x - 2\} = 0$

$(1 - \cos x)(\sin x + 1)(2\sin x - 1) = 0$

$x = 0,\ 2\pi, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

55. Ans. (A,B,C,D)

Total number of permutations $= \frac{(2n)!}{(n!)^2} = {}^{2n}c_n$

Also ${}^{2n}c_n = \frac{2^n\, n!\,[1 \cdot 3 \cdot ...... (2n-1)]}{(n!)^2}$

$= \frac{2^n\,[1 \cdot 3 \cdot ...... (2n-1)]}{n!}$

Similarly (B), (D) follows .

Hence (A), (B), (C), (D).

56. Ans. (A,D)

(A) $\frac{{}^{6+4}C_{4-1}}{{}^{10+4-1}C_{4-1}} = \frac{{}^9C_3}{{}^{13}C_3} = \frac{42}{143}$

(D) $\frac{P(A \cap B)}{P(B)} = \frac{{}^9C_3}{{}^{13}C_3}$

57. Ans. (A,B,C,D)

$P(A) = \frac{9}{10};\ P(B) = P(A \cap B) = \frac{9}{10} \times \frac{9}{10} = \frac{81}{100}$

$$P\left(\frac{A}{\overline{B}}\right) = \frac{P(A \cap \overline{B})}{P(\overline{B})} = \frac{\frac{9}{10} \times \frac{1}{10}}{1 - \frac{81}{100}}$$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{9}{10}$

58. Ans. (A,D)

(A) Digits at unit places of 4^m are 4 or 6

Digits at unit places of 3^n are 3, 9, 7, 1

For divisibility by 5 ; select (4, 1) or (6, 9) at unit places

$\Rightarrow \quad {}^{10}C_1 \times {}^{5}C_1 + {}^{10}C_1 \times {}^{5}C_1 = 100$ ways

(D) $10^2 = 100$ ways.

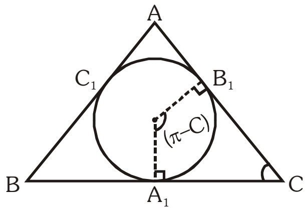
59. Ans. (A,B,C,D)

$r(\pi - C) = 6$ ..... (i)

$r(\pi - B) = 8$ ..... (ii)

$r(\pi - A) = 10$ ..... (iii)

(i) + (ii) + (iii)



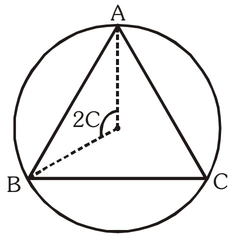
$\Rightarrow \quad r(3\pi - (A + B + C)) = 24 \Rightarrow r = \frac{12}{\pi}$

$C = \frac{\pi}{2};\ B = \frac{\pi}{3};\ A = \frac{\pi}{6}$

Angles are in A.P.

$r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$R = \frac{6\sqrt{2}}{\pi\sin(15°)}$



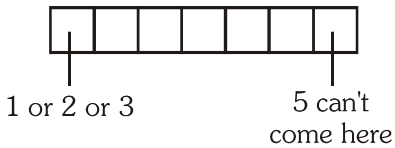
arc length AB = R(2C)

arc length AB = R(2A)

arc length CA = R(2B)

60. Ans. (B,D)

Total numbers which are not divisible by 5 are $= 6! \times 6 = 4320$



Now when 1 or 2 or 3 occupies the 7^{th} place, then the number of numbers

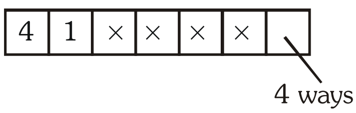
$= 3 \times 5! \times 5 = 1800$

↓

(last can be filled only in 5 ways)

$\therefore$ 1800^{th} number in the list is 3765421.

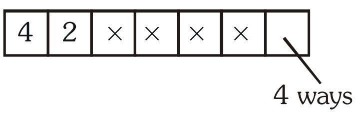
when 1^{st} two places are 41 .... then



number of numbers $= 4! \times 4 = 96$

$\therefore$ 1897^{th} number in the list is 4213567.

with 42 ....

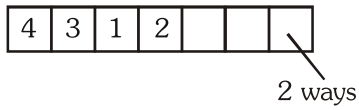


number of numbers $= 4! \times 4 = 96$

Total so far $= 1800 + 192 = 1992$.

$\therefore$ 1994^{th} number in the list is 4312576.

1^{st} three places are filled as 4 3 1 2 ....



number of numbers $= 2! \times 2 = 4$

[Total = 1992 + 4 = 1996]

Now, when first 4 places are , 4 3 1 5 × × ×

then the remaining 3 places in each case be filled in $3! = 6$ ways

which makes total numbers = 2002 and the $(2002)^{th}$ number is 4315762

Hence $(2001)^{st}$ number is just before it

= 4315726

61. Ans. (A,D)

$2^{(2\sin^2 x - 3\sin x + 1)} + 2^{3-(2\sin^2 x - 3\sin x + 1)} = 9$

$2^{2\sin^2 x - 3\sin x + 1} = t$

$t + \frac{8}{t} = 9 \Rightarrow \quad t^2 - 9t + 8 = 0$

$\Rightarrow \quad (t - 8)(t - 1) = 0$

$\Rightarrow \quad 2\sin^2 x - 3\sin x + 1 = 3$

or $\quad 2\sin^2 x - 3\sin x + 1 = 0$

$\sin x = -\frac{1}{2}, \quad \sin x = \frac{1}{2}, \quad \sin x = 1$

62. Ans. (A,B,C,D)

Consider $\tan 20° - 2\tan 10°$

$= \frac{\sin 20°\cos 10° - 2\sin 10°\cos 20°}{\cos 10°\cos 20°}$

$= \frac{(\sin 30° + \sin 10°) - 2(\sin 30° - \sin 10°)}{2\cos 10°\cos 20°}$

$= \frac{3\sin 10° - \sin 30°}{2\cos 10°\cos 20°} = \frac{4\sin^3 10°}{2\cos 10°\cos 20°} > 0$

$\Rightarrow \quad \tan 20° > 2\tan 10°$

$\therefore \quad y > x \ \& \ z > w$ ...... (i)

Now $2w - 2y$

$= 2\tan 10° + \tan 70° - \tan 20° - \tan 50°$

$= 2\tan 10° + \tan 20° \tan 50° \tan 70°$

$= 2\tan 10° + \tan 50°$

$= 2x$ which is positive

$\therefore \quad w = x + y \ \& \ w > y$ ..... (ii)

Combining (i) & (ii) $z > w > y > x$

Also $2y - z$

$$= \tan 20° + \tan 50° - \frac{1}{2}\tan 20° - \frac{1}{2}\tan 70°$$

$$= \frac{1}{2}\tan 20° + \tan 50° - \frac{1}{2}\tan 70°$$

$$= \frac{1}{2}(\tan 20° + 2\tan 50° - \tan 70°)$$

$$= \frac{1}{2}\left(\frac{\sin 70°}{\cos 20° \cos 50°} - \frac{\sin 20°}{\cos 50° \cos 70°}\right)$$

$$= \frac{\sin 140° - \sin 40°}{4\cos 20° \cos 50° \cos 70°} = 0$$

63. Ans. (A,B)

M must have exactly 10 divisors

So $M = p_1^9$ or $p_1^4 p_2$

$p_1^9 <$ $2^9 = 512$, $3^9 > 1000$

$2^4\ 61 = 976$

$3^4.11 = 891 < 976$

$5^4.\ 2 > 1000$

$\Rightarrow$ maximum value of M = $2^4.61$ which is divisible by 2 & 61

64. Ans. (B,C)

$(t - [|\sin x|])! = 3!\ 5!\ 7!$

If $x = n\pi + \frac{\pi}{2}$, $(n \in I)$

then $(t - 1)! = 3!\ 5!\ 7!$

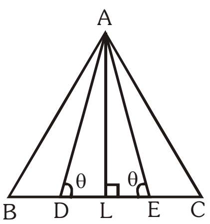
$\Rightarrow (t - 1)! = 10!$

$\Rightarrow t - 1 = 10 \Rightarrow t = 11$

If $x \neq n\pi + \frac{\pi}{2}$, $(n \in I)$

then $(t - 0)! = 10! \Rightarrow t = 10$

65. Ans. (A,B,C,D)



ΔADE is isosceles

as BD = DE = EC

$\Rightarrow \Delta ABC$ is isosceles

$\Rightarrow \angle B = \angle C$

$$\tan\theta = \frac{AL}{DL} = \frac{\frac{9}{2}\tan B}{\frac{9}{6}} = 3\tan B = 3\tan C$$

Also $\tan A = \tan(180 - 2B) = -\tan 2B$

$$= \frac{2\tan B}{\tan^2 B - 1} = \frac{6\tan\theta}{\tan^2\theta - 9}$$

Also $\cot\frac{A}{2} = \tan B \Rightarrow 9\cot^2\frac{A}{2} = \tan^2\theta$

66. Ans. (B,C)

(A) $\frac{a}{\cos A} = \frac{2abc}{b^2 + c^2 - a^2}$, $\frac{b}{\cos B}$

$$= \frac{2abc}{c^2 + a^2 - b^2}$$

& $\frac{c}{\cos c} = \frac{2abc}{a^2 + c^2 - a^2}$

$\because$ $b^2 + c^2 - a^2 \neq c^2 + a^2 - b^2$

$\neq a^2 + b^2 - c^2$ for every ΔABC.

$\therefore$ (A) is not correct

(D) $\frac{\sin 2A}{a^2} = \frac{2\sin A\cos A}{2R\sin A\, a} = \frac{\cos A}{aR}$

$\therefore$ clearly (D) is not correct.

(B) $\frac{a\cos A + b\cos B + \cos C}{a + b + c}$

$$= \frac{R(\sin 2A + \sin 2B + \sin 2C)}{2s}$$

$$= \frac{R \times 4\sin A\sin B\sin C}{2s}$$

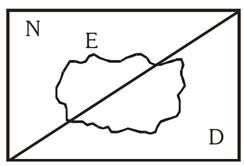
$$= \frac{4R \times \frac{a}{2R} \times \frac{b}{2R} \times \frac{c}{2R}}{2s} = \frac{abc}{4R^2/s} = \frac{r}{R}$$

(C) $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$

$$= \frac{(b^2 + c^2 - a^2) + (c^2 + a^2 - b^2) + (a^2 + b^2 - c^2)}{2abc}$$

$$= \frac{a^2 + b^2 + c^2}{2abc}$$

67. Ans. (A,B,D)



N = Normal coin

D = Double headed coin

E = Head on the 2nd toss

E_1 = Head on the 1st toss

E_2 = Tail on the 1st toss

(A) $E = (E \cap N) + (E \cap D)$

$= P(N) . P(E/N) + P(D) . P(E/D)$

$$= \frac{1}{2}\left[\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot 1\right] + \frac{1}{2}\left[1\cdot\frac{1}{2}\right]$$

T H H H

$$= \frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$

(B) $P(E_1/E) = \frac{P(E_1 \cap E)}{P(E)} = \frac{\frac{1}{4}+\frac{1}{4}}{\frac{5}{8}} = \frac{4}{5}$

(C) $P(E_2/E) = \frac{P(E_2 \cap E)}{P(E)} = \frac{\frac{1}{2}\left(\frac{1}{2}\times\frac{1}{2}\right)}{\frac{5}{8}} = \frac{1}{5}$

(D) $\frac{1}{2}\left(\frac{1}{2}\times\frac{1}{2}\right) = \frac{1}{8}$

T T

68. Ans. (A,B,D)

(A) Last two digits should be 08, 16, 36, 60, 68, 80

$\therefore$ Number of number ending with 08, 60 and 80 = (3 × 2 × 1) × 3 = 18

Number ending with 16, 36, 68

= (2 × 2 × 1) × 3 = 12

$\therefore$ Total number divisible by 4 = 18 + 12 = 30

(B) Numbers divisible by 11 is possible only when

Case-I : 0, 1, 8, occurs alternately and 3, 6, occurs alternately

i.e. odd places are occupied by 0, 1, 8 and even places are occupied by 3, 6.

But the number has to be greater than 30,000

$\therefore$ such numbers are 2! × 2! = 4

Case-II : odd places are occupied by 0, 3, 6 and even places are occupied by 1, 8.

$\therefore$ such numbers are 2 × 2! × 2! = 8

Total required numbers = 8 + 4 = 12

(C)

			8	

2 × 3 × 2 × 1 × 1 = 12

(D) Sum of digits = 18 (divisible by 3)

$\therefore$ All even numbers formed using the digits 0, 1, 3, 6, 8 will be divisible by 6.

$\therefore$ Required numbers = 3! × 3 = 18

69. Ans. (A,C)

$a^2 + b^2 + c^2 = ca + ab\sqrt{3}$

$\Rightarrow \underbrace{a^2 - ab\sqrt{3} + \frac{3}{4}b^2} + \underbrace{\frac{b^2}{4} - ca + c^2} = 0$

$\Rightarrow \left(a - \frac{\sqrt{3}}{2}b\right)^2 + \left(\frac{b}{2} - c\right)^2 = 0$

$\therefore \frac{a}{b} = \frac{\sqrt{3}}{2}$ & $\frac{c}{b} = \frac{1}{2}$

$\Rightarrow a = b\,.\,\frac{\sqrt{3}}{2}$ & $c = \frac{b}{2}$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{\frac{3}{4}b^2 + \frac{b^2}{4} - b^2}{2ac} = 0$$

$\therefore \angle B = 90°$

Similarly $\angle A = 60°$ & $\angle C = 30°$

$\therefore \Delta ABC$ is right angled but not isosceles

$$[\tan A + \tan C] = \left[\sqrt{3} + \frac{1}{\sqrt{3}}\right] = \left[\frac{4}{\sqrt{3}}\right] = 2$$

$$\left\{\frac{r_2}{r_1}\right\} = \left\{\frac{r\tan B/2}{r\tan A/2}\right\} = \left\{\frac{\tan 45°}{\tan 30°}\right\} = \{\sqrt{3}\} = \sqrt{3} - 1$$

70. Ans. (B,D)

$$(a+2)\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} + (2a-1)\frac{\left(1-\tan^2\frac{\alpha}{2}\right)}{\left(1+\tan^2\frac{\alpha}{2}\right)}$$

= 2a + 1

Let $\tan\frac{\alpha}{2} = t$

$(a + 2)\,2t + (2a - 1)(1 - t^2) = (2a + 1)(1 + t^2)$

$\Rightarrow 4a\,.\,t^2 - 2t(a + 2) + 2 = 0$

$\Rightarrow t^2 - t\left(\frac{1}{2} + \frac{1}{a}\right) + \frac{1}{2a} = 0$

$\Rightarrow \left(t - \frac{1}{2}\right)\left(t - \frac{1}{a}\right) = 0$

$\therefore \tan\frac{\alpha}{2} = \frac{1}{2}$ or $\tan\frac{\alpha}{2} = \frac{1}{a}$

If $\tan\frac{\alpha}{2} = \frac{1}{2}$ or If $\tan\frac{\alpha}{2} = \frac{1}{a}$

$$\therefore \quad \tan\alpha = \frac{2\times\frac{1}{2}}{1-\frac{1}{4}} = \frac{4}{3}$$

$$\Rightarrow \quad \tan\alpha = \frac{2\times\frac{1}{a}}{1-\frac{1}{a^2}} = \frac{2a}{a^2-1}.$$

71. Ans. (A,B,C)

$$P(C_4/H) = \frac{P(HC_4)}{P(H)} = \frac{P(HC_4)}{P(HC_1 \cup HC_2 \cup ... \cup HC_7)}$$

$$= \frac{P(C_4)P(H/C_4)}{\sum P(C_i)P(H/C_i)}$$

also $P(H/C_i) = \frac{i+1}{7}$

so $P(H/C_1) = \frac{2}{8}, P(H/C_2) = \frac{3}{8} ..... P(H/C_7) = 1$

$$P(C_4/H) = \frac{\frac{1}{7}\times\frac{5}{8}}{\frac{1}{7}\left[\frac{2}{8}+\frac{3}{8}+.......\frac{8}{8}\right]} = \frac{1}{7}$$

Also P(H) = $\frac{1}{7}\times\frac{35}{8} = \frac{5}{8}$

$$P(C_1/H) = \frac{\frac{1}{7}\times\frac{2}{8}}{\frac{5}{8}} = \frac{2}{35}$$

72. Ans. (A,C)

Bag < R R R ; G G G

withdraw < 3 R ; 3 G ; R G G ⇒ B B B R R G ; G R R ⇒ B B B R G G

In the top two cases the probability of the required event is zero. Hence,

$$P(E) = \frac{2\,.\,{}^3C_1\,.\,{}^3C_2}{{}^6C_3}\,.\left[\frac{{}^3C_1\,.\,{}^2C_1\,.\,{}^1C_1\,.\,2}{{}^6C_3}\right]$$

$$= \frac{18}{20}\cdot\frac{12}{20} = \frac{54}{100} = \frac{27}{50}$$

Also $x^2 - 33x + 90 \le 0 \Rightarrow (x-3)(x-30) \le 0$

$\Rightarrow x \in [3, 30] \quad \Rightarrow$ Probability $= \frac{27}{100}$

73. Ans. (B,C)

$a^2 - b^2 = c \quad \Rightarrow \quad (a-b)(a+b) = c$

$\Rightarrow a - b = 1 \Rightarrow a = 3, b = 2, c = 5$ & $d = 7$

and sum of all possible numbers

= 3!(1111)(2 + 3 + 5 + 7) = 113322

74. Ans. (A,C,D)

(A) $\left[\frac{2009}{11}\right]+\left[\frac{2009}{11^2}\right]+\left[\frac{2009}{11^3}\right]$

= 182 + 16 + 1 = 199

(B) ${}^9C_4 - 4 = 122$

(C) $\frac{{}^{12}C_1 \times {}^{12-4-1}C_{4-1}}{4} = 105$

(D) (2 + 1)(1 + 1)(1 + 1)(2 + 1) – 1 = 35

Paragraph for Question 75 to 77

75. Ans. (A)

$$\left[\frac{300}{7}\right]+\left[\frac{300}{7^2}\right]+\left[\frac{300}{7^3}\right] = 42+6+0 = 48$$

76. Ans. (D)

$$\left[\frac{3250}{5}\right]+\left[\frac{3250}{25}\right]+\left[\frac{3250}{125}\right]+\left[\frac{3250}{625}\right]+\left[\frac{3250}{3125}\right]$$

= 650 + 130 + 26 + 5 + 1 = 812

77. Ans. (B)

$${}^{200}C_{100} = \frac{200!}{(100!)^2}$$

Exponent of 2 in 200!

$$= \left[\frac{200}{2}\right]+\left[\frac{200}{2^2}\right]+\left[\frac{200}{2^3}\right]+\left[\frac{200}{2^4}\right]$$
$$+\left[\frac{200}{2^5}\right]+\left[\frac{200}{2^6}\right]+\left[\frac{200}{2^7}\right]+\left[\frac{200}{2^8}\right]$$

= 100 + 50 + 25 + 12 + 6 + 3 + 1 + 0 = 197

Exponent of 2 in 100!

$$= \left[\frac{100}{2}\right]+\left[\frac{100}{2^2}\right]..........+\left[\frac{100}{2^7}\right]$$

= 50 + 25 + 12 + 6 + 3 + 1 + 0 = 97

so $\quad {}^{200}C_{100} = \frac{2^{197}a}{2^{97}.2^{97}\lambda}$

$\therefore$ exponent of 2 in ${}^{200}C_{100}$ is 3

Paragraph for Question 78 to 80

78. Ans. (A)

$n(S_1)$ is number of positive integral solutions of the equation $x.y.z = 2^2.3.5$

Let $x = 2^{p_1}.3^{q_1}.5^{r_1}$

$y = 2^{p_2}.3^{q_2}.5^{r_2}$

$z = 2^{p_3}.3^{q_3}.5^{r_3}$

$\therefore$ $p_1 + p_2 + p_3 = 2$ .... (i)

$q_1 + q_2 + q_3 = 1$ .... (ii)

$r_1 + r_2 + r_3 = 1$ .... (iii)

Non-negative integral solutions of (i) = ${}^4C_2$

Non-negative integral solutions of (ii) = ${}^3C_1$

Non-negative integral solutions of (iii) = ${}^3C_1$

$\therefore$ $n(S_1) = {}^4C_2 . {}^3C_1 . {}^3C_1 = 54$

79. Ans. (C)

$n(\bar{S}_2 \cap S_3)$ = Number of positive integral solutions of the inequality $15 < x + y + z < 20$

Consider $x + y + z < 20$

$\Rightarrow$ $x + y + z \le 19$ $\{\because x, y, z \in I^+\}$

$\Rightarrow$ $x + y + z + w = 19$

$\therefore$ Number of positive integral solutions = ${}^{19}C_3$

Consider $x + y + z \le 15$

$\Rightarrow$ $x + y + z + w = 15$

$\therefore$ Number of positive integral solutions = ${}^{15}C_3$

$\therefore$ $n(\bar{S}_2 \cap S_3) = {}^{19}C_3 - {}^{15}C_3 = 514$

80. Ans. (A)

$54 = 2 . 3^3$

$\therefore$ Total divisors = 8

Let $a_1, a_2, ... a_8$ be the total divisors

$\therefore$ $\frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_8} = \frac{a_1 + a_2 + ... + a_8}{54}$

$= \frac{(2^0 + 2^1)(3^0 + 3^1 + 3^2 + 3^3)}{54} = \frac{20}{9}$

Paragraph for Questions 81 to 83

81. Ans. (C)

$\Rightarrow$ $a_1 = 2, a_2 = 3, a_3 = 4$

If $P(i) \propto i^2$

$P(1) = \frac{1}{14}$, $P(2) = \frac{4}{14}$, $P(3) = \frac{9}{14}$

$P(R) = \frac{1}{14}\cdot\frac{2}{6} + \frac{4}{14}\cdot\frac{3}{6} + \frac{9}{14}\cdot\frac{4}{6}$

$= \frac{2 + 12 + 36}{84} = \frac{50}{84} = \frac{25}{42}$

$P(G) = \frac{1}{14}\cdot\frac{4}{6} + \frac{4}{14}\cdot\frac{3}{6} + \frac{9}{14}\cdot\frac{2}{6} = \frac{17}{42}$

82. Ans. (B)

P(different color)

$= \frac{1}{3}\left[\frac{{}^2C_1 . {}^4C_1 + {}^3C_1 . {}^3C_1 + {}^2C_1 . {}^4C_1}{{}^6C_2}\right]$

$= \frac{1}{3}\left[\frac{8 + 9 + 8}{15}\right] = \frac{25}{45} = \frac{5}{9}$

83. Ans. (D)

P(all are red) $= \frac{2^{10} + 3^{10} + 4^{10}}{3.6^{10}}$

Paragraph for Question 84 to 86

84. Ans. (D)

EEE, AA, LL, RR, C, N

Cases :

1. 3 alike + 2 alike + 1 different

$${}^1C_1 \times {}^3C_1 \times {}^4C_1 \times \frac{6!}{3!\,2!} = 720$$

2. 3 alike + 3 different

$${}^1C_1 \times {}^5C_3 \times \frac{6!}{3!} = 1200$$

3. 2 alike + 2 alike + 2 alike

$${}^4C_3 \times \frac{6!}{2!\,2!\,2!} = 360$$

4. 2 alike + 2 alike + 2 different

$${}^4C_2 \times {}^4C_2 \times \frac{6!}{2!\,2!} = 6480$$

5. 2 alike + 4 different

$${}^4C_1 \times {}^5C_4 \times \frac{6!}{2!} = 7200$$

6. All different

$${}^6C_6 \times 6! = 720$$

Total words = 16680

85. Ans. (A)

$\frac{5!}{3!\,2!} \times \frac{6!}{2!\,2!} = 10 \times 180 = 1800$

86. Ans. (C)

Arrangement of consonants : $\frac{6!}{2!\,2!}$

Selection of 5 gaps for vowels with arrangement : ${}^7C_5 \times \frac{5!}{3!\,2!}$

Total words $= \frac{6!}{2!\,2!} \times \frac{7!}{5!\,2!} \times \frac{5!}{3!\,2!} = \frac{15.7!}{2}$

Paragraph for Question 87 to 89

87. Ans. (A)

Let a = 2x + 1, b = 2y + 1, c = 2z + 2, where x, y, z $\in$ whole number

$\therefore$ a + b + c = 16

$\Rightarrow$ 2x + 1 + 2y + 1 + 2z + 2 = 16

$\Rightarrow$ x + y + z = 6

$\Rightarrow$ Number of integral solutions

$= {}^{6+3-1}C_{3-1} = {}^8C_2 = 28$

88. Ans. (C)

Possible values of a	1	2	3	....	8
Possible values of b	2 to 9 = 8	3 to 9 = 7	4 to 9 = 6	....	only 9 = 1

C can take any value from 0 to 9

Hence total number $= (8+7+6+.....+2+1)\times 10$

$= \frac{8\times 9}{2}\times 10 = 360$

89. Ans. (B)

The number abc can not be formed, if in the selection of n cards, we get either two of a,b,c (i.e. ab or ac or ca) or only one of a, b, c (i.e. a or b or c only)

$\Rightarrow$ total ways $= 2^n + 2^n + 2^n - 1 - 1 - 1 = 45$

($\because$ only a or b or c are repeated twice)

$\Rightarrow$ $3 \cdot 2^n = 48$

$2^n = 16$

n = 4

Paragraph for Question 90 to 92

90. Ans. (C)

Equation $x^2 + kx + \frac{1}{4}(k+2)$ have real roots if

$D \geq 0 \quad \Rightarrow \quad k^2 - 4\frac{1}{4}.(k+2) \geq 0$

$\Rightarrow$ $(k+1)(k-2) \geq 0$

$\Rightarrow$ $k \in [2, 5]$ hence probability $= \frac{3}{5}$

91. Ans. (A)

Radii of smaller and bigger circles are 1 and 2 respectively

hence probability $= \frac{\text{Area of smaller circle}}{\text{Area of bigger circle}} = \frac{1}{4}$

92. Ans. (B)

The given conditions

$\Rightarrow$ $x > 0, y > 0, 1 - (x+y) > 0$

($\therefore$ $z = 1 - (x+y)$)

$\Rightarrow$ $x > 0, y > 0$ & $x + y < 1$ ...... (i)

Now condition for formation of a triangle

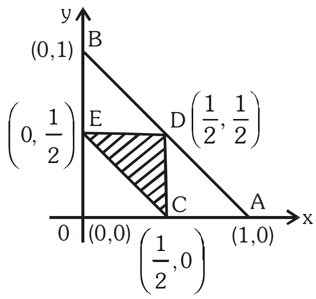
$\Rightarrow$ sum of any two sides is greater than third side

$\Rightarrow$ (i) $x + y > 1 - (x+y) \Rightarrow x + y < \frac{1}{2}$

(ii) $x + 1 - (x+y) > y \Rightarrow y < \frac{1}{2}$

(iii) $y + 1 - (x+y) > x \Rightarrow x < \frac{1}{2}$

plot on x-y plane



$\Rightarrow$ Probability

$= \frac{\text{Area of } \Delta CDE}{\text{Area of } \Delta OAB}$

$= \frac{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times 1 \times 1} = \frac{1}{4}$

Paragraph for Questions 93 to 95

93. Ans. (A)

$P(\overline{w}) = P(E_1 \cap \overline{w}) + P(E_2 \cap \overline{w})$

$= P(E_1).P(\overline{w}/E_1) + P(E_2).P(\overline{w}/E_2)$

$= \frac{1}{3}.\frac{9}{10} + \frac{2}{3}.\frac{8}{10} = \frac{25}{30} = \frac{5}{6}$

94. Ans. (C)

$P(\overline{s}) = P(E_1 \cap \overline{s}) + P(E_2 \cap \overline{s})$

$= P(E_1).P(\overline{s}/E_1) + P(E_2).P(\overline{s}/E_2)$

$= \frac{1}{3}.\frac{8}{10} + \frac{2}{3}.\frac{9}{10} = \frac{26}{30} = \frac{13}{15}$

95. Ans. (D)

$P\left(\frac{E_1}{\overline{w}}\right) = \frac{P(E_1).P(\overline{w}/E_1)}{P(E_1).P(\overline{w}/E_1) + P(E_2).P(\overline{w}/E_2)}$

$= \frac{\frac{1}{3}\cdot\frac{9}{10}}{\frac{1}{3}\cdot\frac{9}{10} + \frac{2}{3}\cdot\frac{8}{10}} = \frac{9}{25}$

96. Ans. (A)→(S), (B)→(P), (C)→(Q), (D)→(R)

(A) Rectangles can be of sizes

$1 \times 1, 1 \times 2, 1 \times 3$ .... $1 \times 8 = 8$ numbers

$2 \times 2 \times 2 \times 3\ 2 \times 8 = 7$ numbers

3×3 .... $3 \times 8 = 6$ numbers

⋮

$8 \times 8 = 1$ number

⇒ Total rectangles

$= (8 + 7 + 6 + ... 1) = 36$

(B) $^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 2^5 - 1 = 31$

(C) $157500 = 2^2\, 3^2\, 5^4\, 7^1$

⇒ Number of divisors divisible by 2, 3, 5 are $(2)(2)(4)(1 + 1) = 32$

(D) 7! onwards are divisible by 35 hence remainder of (1! + 2! + 3! + 4! + 5! + 6!), when divided by 35 is remainder of $\frac{873}{35}$ i.e. = 33.

97. Ans. (A)→(Q), (B)→(P), (C)→(S), (D)→(R)

(A) Odd natural numbers < 100 :

1, 3, 5,........... 99 ⇒ Total 50 numbers

Numbers divisible by 5 are 5, 15, 25, ... 95

⇒ Total 10 numbers.

Now to get product divisible by 5, either select 2 numbers from these 10 numbers or select 1 from these 10 and 1 from remaining 40

⇒ Probability

$$= \frac{^{10}C_2 + {}^{10}C_1 \times {}^{40}C_1}{^{50}C_2} = \frac{45 + 400}{\frac{50 \times 49}{2}} = \frac{89}{245}$$

(B) Using Bay's theorem :

$$\text{Probability} = \frac{\frac{5}{9}}{\frac{5}{9} + \frac{3}{5}} = \frac{25}{52}$$

(C) $\frac{^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8}{2^8}$

$$= \frac{56 + 28 + 8 + 1}{256} = \frac{93}{256}$$

(D) $\frac{50}{52} \times \frac{49}{51} \times \frac{48}{50} \times \frac{2}{49} = \frac{24}{663}$.

98. Ans. (A)→(Q), (B)→(P), (C)→(R), (D)→(Q)

Let E_i denotes the event that the bag contains i black and (14 – i) white balls (i = 0, 1, ..., 14) and A denotes the event that five balls drawn are all black, then

$P(E_i) = \frac{1}{15}$ (i = 0, 1,.....,14)

$P(A/E_i) = 0$ for i = 0, 1, 2, 3, 4.

$P(A/E_i) = \frac{^iC_5}{^{14}C_5}$ for $i \geq 5$

(A) $P(A) = \sum_{i=0}^{14} P(E_i).P(A/E_i)$

$$= \frac{1}{15} \cdot \frac{1}{^{14}C_5}\left({}^5C_5 + {}^6C_5 + ... + {}^{14}C_5\right)$$

$$= \frac{1}{15} \cdot \frac{^{15}C_6}{^{14}C_5} = \frac{1}{6}$$

(B) Clearly $P(A/E_{11}) = \frac{^{11}C_5}{^{14}C_5} = \frac{3}{13}$

(C) By Baye's Theorem,

$$P(A/E_{11}) = \frac{P(E_{11}).P(A/E_{11})}{P(A)} = \frac{\frac{1}{15} \cdot \frac{3}{13}}{\frac{1}{6}} = \frac{6}{65}$$

(D) Let B denotes the probability of 3 black and 2 white balls, then

$P(B/E_i) = 0$ if $i = 0, 1, 2$ or 13, 14

$P(B/E_i) = \frac{^iC_3 . {}^{14-i}C_2}{^{14}C_5}$ for i = 3, 4, ...,12

$\therefore$ $P(B) = \sum_{i=0}^{14} P(E_i).P(B/E_i)$

$$= \frac{1}{15} \cdot \frac{1}{^{14}C_5}\left[{}^3C_3 . {}^{11}C_2 + {}^4C_3 . {}^{10}C_2 + .. + {}^{12}C_3 . {}^2C_2\right]$$

$$= \frac{5005}{15 . {}^{14}C_5} = \frac{1}{6}$$

99. Ans. (A)→(S,T); (B)→(Q); (C)→(R); (D)→(P)

(A) I N D I A T I M E

I		
	I	
		I

I I I N D A T M E

Arrangements of I's in 9 positions

$= {}^3C_1.{}^2C_1.{}^1C_1$

Arrangement of remaining distinct letters $= 6!$

Total required arrangements $= 3\times 2\times 1\times 6! = 6\times 6!$

(B) F O R T U N A T E

T		
	T	

T T F O R U N A E

Arrangements of T's $= {}^3C_2 \times 3 \times 2$

Arrangements of remaining letters (all distinct) $=7!$

Total required arrangements $= 18 \times 7!$

(C) S W E E T N E S S

S	E			S		E
	S	E		E	S	
E		S			E	S

S S S E E E W T N

Arrangements of S's $= 3 \times 2 \times 1$

Arrangements of remaining (all distinct) letters $=3!$

Total required arrangements $= 6\times 2\times 3!=3\times 4!$

(D) C A N D I D A T E

A A D D C N I T E

A		
	A	

Arrangements of A's $= {}^3C_2 \times 3 \times 2$

Arrangements of D's $= \underbrace{3}_{\text{If both the D's are kept in the rows in which A appears}} + \underbrace{(3+3+2)}_{\text{If exactly one D appears in the row in which A doesn't appear}}$

Arrangement of remaining letters (all distinct) $=5!$

Total required arrangement $= 3\times 6 \times 11\times 5! = 33 \times 6!$

100. Ans. (A)→(Q), (B)→(S), (C)→(P), (D)→(R)

(A) Use ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(B) Number of ways $= {}^4C_2 . {}^4C_2 = 36$ ways

(C) Available digits $=1, 2, 2, 3, 3, 4$

Type of selection	Number of selection	Arrangement
2 Alike, 2 others alike	1	4!/2!2!
2 Alike, 2different	${}^2C_1{}^3C_2$	4!/2!
4 different	1	4!

Number of ways $= 6 + 6 \times 12 + 24 = 102$

(D) $(500!) = 2^{k_1}.7^{k_2}.I$

$k_2 < k_1$ & $k_2 = 82$

$\therefore \quad (500!) = (14)^{82} . I$

$\therefore \quad k = 82$

101. Ans. (A)→(S), (B)→(P), (C)→(Q), (D)→(R)

(A) S R G J / 1 1 2 3 $\quad \dfrac{7!}{(1!)^2 2!2!3!}\times 2! = \dfrac{7!}{12}$

(B) S R G J / 2 2 2 1 $\quad \dfrac{7!}{(2!)^3 3!1!}\times 3! = \dfrac{7!}{8}$

(C) S R G J / 1 4 1 1 $\quad \dfrac{7!}{(1!)^3 3!4!}\times 3! = \dfrac{7!}{24}$

(D) Each toy has 4 options.

$\therefore$ number of ways $= 4^7 = \left(\dfrac{4!\,.5!}{45}\right)^2 .4$

102. Ans. (A)→(R), (B)→(S), (C)→(P), (D)→(Q)

(A) Required probability is

$= P(50p, 10p, 1Re) = \dfrac{4}{12}\times\dfrac{5}{11}\times\dfrac{3}{10} = \dfrac{1}{22}$

(B) Required probability is

$= \dfrac{3!}{2!}P(10p, 10p, 1Re)$

$= 3\times\dfrac{5}{12}\times\dfrac{4}{11}\times\dfrac{3}{10} = \dfrac{3}{22}$

(C) Required probability is

$$= \frac{P(50p, 50p, 50p)}{P(10p, 10p, 10p) + P(50p, 50p, 50p) + P(1Re, 1Re, 1Re)}$$

$$= \frac{\frac{4}{12}.\frac{3}{11}.\frac{2}{10}}{\frac{5}{12}.\frac{4}{11}.\frac{3}{10} + \frac{4}{12}.\frac{3}{11}.\frac{2}{10} + \frac{3}{12}.\frac{2}{11}.\frac{1}{10}}$$

$$= \frac{24}{60+24+6} = \frac{4}{15}$$

(D) Required probability is

$$= \frac{\frac{3!}{2!}P(10p, 10p, 1Re)}{1-3!P(10p, 50p, 1Re)}$$

$$\frac{3\times\frac{5}{12}.\frac{4}{11}.\frac{3}{10}}{1-6\times\frac{5}{12}.\frac{4}{11}.\frac{3}{10}} = \frac{3}{16}$$

103. Ans. (A)→(Q); (B)→(S); (C)→(P)

(A) $\Delta = a^2 - (b-c)^2$

$\Delta = a^2 - b^2 - c^2 + 2bc$

$b^2 + c^2 - a^2 = 2bc - \Delta$

$2bc\cos A = 2bc - \frac{1}{2} bc \sin A$

$4\cos A + \sin A = 4$

$4\left(1 - 2\sin^2\frac{A}{2}\right) + 2\sin\frac{A}{2}\cos\frac{A}{2} = 4$

$8\sin^2\frac{A}{2} - 2\sin\frac{A}{2}\cos\frac{A}{2} = 0$

$\Rightarrow 4\tan\frac{A}{2} - 1 = 0$

$\Rightarrow \tan\frac{A}{2} = \frac{1}{4} \quad \Rightarrow \tan A = \frac{8}{15}$

(B) $\tan A = 3k, \tan B = 4k, \tan C = 5k$

In any triangle ABC

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\Rightarrow 12k = 60k^3 \quad \Rightarrow k = \frac{1}{\sqrt{5}}$

$\tan A = \frac{3}{\sqrt{5}} \qquad \tan B = \frac{4}{\sqrt{5}} \qquad \tan C = \sqrt{5}$

$\therefore \sin A \sin B \sin C = \frac{2\sqrt{5}}{7}$

(C) $f(\theta) = \frac{2\cos^2\theta - 2\sin\theta\cos\theta}{2(\cos^2\theta - \sin^2\theta)} = \frac{1}{1+\tan\theta}$

$\alpha + \beta = \frac{5\pi}{4}$

$\tan(\alpha+\beta) = 1$

$\Rightarrow \tan\alpha + \tan\beta = 1 - \tan\alpha\tan\beta$

$f(\alpha) \cdot f(\beta) = \frac{1}{1+\tan\alpha}\cdot\frac{1}{1+\tan\beta}$

$= \frac{1}{1+\tan\alpha+\tan\beta+\tan\alpha\tan\beta}$

$= \frac{1}{1+1-\tan\alpha\tan\beta+\tan\alpha\tan\beta} = \frac{1}{2}$

104. Ans. (A)→(R); (B)→(Q); (C)→(P); (D)→(S)

(A) There are 5 even powers & 6 odd powers

$\therefore$ Number of GP's $= {}^5C_2 + {}^6C_2$

(B) Let the common difference be 'd'.

$\therefore 365 = 5 + (n-1)d$

$\Rightarrow 360 = (n-1)d$

$\therefore$ Total number of AP's = Total number of divisors of 360 $= 4 \times 3 \times 2 = 24$

(C) ${}^nC_2 = {}^nC_3 \Rightarrow n = 5$

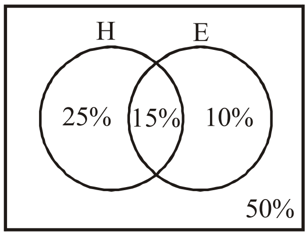
(D) $S = \frac{5!}{2!}(10^0 + 10^1 + 10^2 + 10^3 + 10^4 + 10^5)(1+2+3)$

$= 60\ (1\ 1\ 1\ 1\ 1\ 1)6 = 39999960$

sum of digits = 54

105. Ans. (A)→(Q); (B)→(R); (C)→(S)

Interpret from the venn diagram



106. (A)→(S) ; (B)→(R); (C)→(P)

(A) P(R) = P(T ; HTT ; HTHTT ; HHTTT)

$\Rightarrow$ 11/16

(B) $P(A\cup B) = 0.6$; $P(A\cap B) = 0.2$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\therefore P(A) + P(B) = 0.6 + 0.2 = 0.8$

$P(\overline{A}) + P(\overline{B}) = 2 - 0.8 = 1.2 \Rightarrow$ (R)

(C) $P(X) = \frac{2}{3}$; $P(Y) = \frac{3}{4}$; $P(Z) = p$

E : exactly two bullets hit

$P(E) = P(XY\overline{Z}) + P(YZ\overline{X}) + P(ZX\overline{Y})$

$\frac{11}{24} = \frac{2}{3}\cdot\frac{3}{4}(1-p) + \frac{3}{4}\cdot\frac{1}{3}p + p\frac{2}{3}\cdot\frac{1}{4}$

$\Rightarrow p = \frac{1}{2}$

107. Ans. 168

$12 = 2 \cdot 2 \cdot 3$

The number should be of the form $a^1 b^1 c^2$, where a, b, c are prime numbers.

For number to be least c = 2, b = 3, a = 5

$\therefore$ Least number $= 2^2 \cdot 3^1 \cdot 5^1$

Sum of all divisors

$= (2^0 + 2^1 + 2^2)(3^0 + 3^1)(5^0 + 5^1) = 168$

108. Ans. 510

Required number of ways

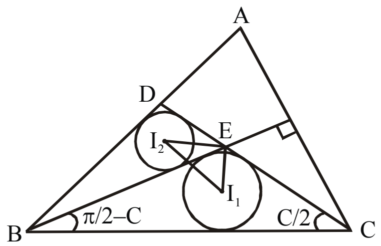
= coefficient of x^6 in $6!\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}\right)^3$

= coefficient of x^6 in

$$6!\left((1+x)^3+3(1+x)^2\left(\frac{x^2}{2!}+\frac{x^3}{3!}\right)+3(1+x)\left(\frac{x^2}{2!}+\frac{x^3}{3!}\right)^2+\left(\frac{x^2}{2!}+\frac{x^3}{3!}\right)^3\right)$$

$$=6!\left(\frac{3}{3!3!}+\frac{3}{3!}+\frac{1}{2!2!2!}\right)=510$$

109. Ans. 1



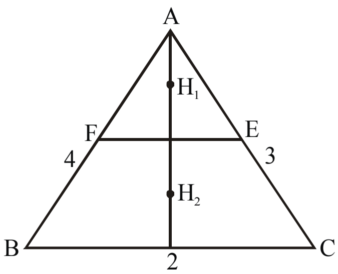
ΔI_1EI_2 is right angle triangle at E

$\therefore$ E is orthocentre

$$\therefore \text{ in } \Delta BEC,\ \frac{BE}{\sin\frac{C}{2}}=\frac{BC}{\sin\left(90+\frac{C}{2}\right)}$$

$$\Rightarrow\ BE=a\tan\frac{C}{2}=1$$

110. Ans. 49



Let H_1 and H_2 are orthocentres of ΔAEF and ΔABC respectively

$AH_2 = 2R\cos A$

$AH_1 = 2R'\cos A$

$\therefore$ distance $d=2\cos A(R-R')$

$$=2.\frac{7}{8}\left(\frac{2}{2.\left(\frac{\sqrt{15}}{8}\right)}-\frac{1}{2\left(\frac{\sqrt{15}}{8}\right)}\right)$$

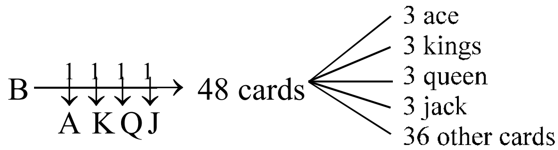
$$=\frac{7}{8}.\frac{8}{\sqrt{15}}=\frac{7}{\sqrt{15}}$$

111. Ans. 35

. Let B_1 : pack A was selected $\Rightarrow P(B_1)=\frac{1}{2}$;

Pack A $\xrightarrow{4\text{ aces}}$ 48 cards in 12 different denominations

B_2 : pack B was selected $\Rightarrow P(B_2)=\frac{1}{2}$; Pack



A : two cards drawn all of same rank.

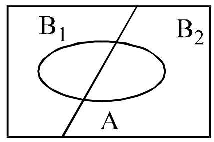
Now $A=(A\cap B_1)+(A\cap B_2)$

$\therefore\ P(A)=P(A\cap B_1)+P(A\cap B_2)$

$=P(B_1)P(A/B_1)+P(B_2)P(A/B_2)$

$$P(A/B_1)=\frac{{}^{12}C_1\cdot{}^4C_2}{{}^{48}C_2}$$

$$P(A/B_2)=\frac{{}^9C_1\cdot{}^4C_2+{}^4C_1\cdot{}^3C_2}{{}^{48}C_2}$$



$$P(B_1/A)=\frac{P(A\cap B_1)}{P(A)}$$

$$=\frac{P(B_1)P(A/B_1)}{P(B_1)P(A/B_1)+P(B_2)P(A/B_2)}$$

$$=\frac{{}^{12}C_1\cdot{}^4C_2}{{}^{12}C_1\cdot{}^4C_2+{}^9C_1\cdot{}^4C_2+{}^4C_1\cdot{}^3C_2}$$

$$=\frac{(12)(6)}{(12)(6)+(9)(6)+(4)(3)}=\frac{12}{23}$$

$\Rightarrow\ m+n=35$ **Ans.**

112. Ans. 3

Area of quadrilateral ADIE is

$$\frac{r^2}{\tan\frac{A}{2}}=5 \qquad \text{......(1)}$$

Area of quadrilateral DIFD is

$$\frac{r^2}{\tan\frac{B}{2}}=10 \qquad \text{......(2)}$$

$$\frac{(2)}{(1)} \qquad \frac{\tan\frac{B}{2}}{\tan\frac{A}{2}}=2$$

$$\frac{\sin\frac{B}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}\cos\frac{B}{2}}=\frac{2}{1}$$

Use C & D

$$\frac{\sin\left(\frac{A}{2}+\frac{B}{2}\right)}{\sin\left(\frac{B}{2}-\frac{A}{2}\right)}=\frac{3}{1}$$

$$\frac{\cos\frac{C}{2}}{\sin\left(\frac{B-A}{2}\right)}=3$$

113. Ans. 10

$$\frac{\tan 8^\circ}{1-3\tan^2 8^\circ}+\frac{3\tan 24^\circ}{1-3\tan^2 24^\circ}+\frac{9\tan 72^\circ}{1-3\tan^2 72^\circ}+\frac{27\tan 216^\circ}{1-3\tan^2 216^\circ}$$

Consider, $\frac{\tan 8^\circ}{1-3\tan^2 8^\circ}$

$$=\frac{1}{8}\left[\frac{8\tan 8^\circ}{1-3\tan^2 8^\circ}+\tan 8^\circ-\tan 8^\circ\right]$$

$$=\frac{1}{8}\left[3\tan 24^\circ-\tan 8^\circ\right]$$

$$=-\frac{1}{8}\tan 8^\circ+\frac{3}{8}\tan 24^\circ$$

Now given expression reduces to -

$$-\frac{1}{8}\tan 8^\circ+\frac{3}{8}\tan 24^\circ-\frac{3}{8}\tan 24^\circ+\frac{9}{8}\tan 72^\circ$$

$$-\frac{9}{8}\tan 72+\frac{27}{8}\tan 216$$

$$-\frac{27}{8}\tan 216^\circ+\frac{27\times 3}{8}\tan(216\times 3)^\circ$$

$$=\frac{81}{8}\tan 108^\circ-\frac{1}{8}\tan 8^\circ$$

$$\Rightarrow\quad x+y=\frac{81-1}{8}=10$$

114. Ans. 3

from given figure $r_1 + r_3 = 2r_2$ ...(1)

and $2\pi r_2^2 = \pi r_1^2 + \pi r_3^2$ ...(2)

from (1) & (2) $r_1 = r_2 = r_3$

115. Ans. 5

$(\sin x - 1)^3 + (\cos x - 1)^3 + (\sin x)^3$

$= (2\sin x + \cos x - 2)^3$

$\Rightarrow (\sin x+\cos x-2)(\cos x+\cos x-1)(2\sin x-1) = 0$

$\Rightarrow \sin x + \cos x = 1$ & $\sin x=\frac{1}{2}$

$\Rightarrow x=0,\frac{\pi}{6},\frac{\pi}{2},\frac{5\pi}{2},2\pi$

116. Ans. 1

$\sin x - 1 < 0$ for all x

$\Rightarrow \sin 6x > 0 \quad \Rightarrow 2n\pi < 6x < 2n\pi + \pi$

or $\frac{n\pi}{3}<x<\frac{n\pi}{3}+\frac{\pi}{6}$

$\therefore$ In $\left(0,\frac{\pi}{2}\right)$, $0<x<\frac{\pi}{6}$, $\frac{\pi}{3}<x<\frac{\pi}{2}$ ....(1)

Also $1 + \tan^2 x - 2\sqrt{2}\tan x \le 0$

$\Rightarrow \sqrt{2}-1\le\tan x\le\sqrt{2}+1$

In $\left(0,\frac{\pi}{2}\right)$, $\frac{\pi}{8}\le x\le\frac{3\pi}{8}$ ...(2)

From (1) & (2), $x\in\left[\frac{\pi}{8},\frac{\pi}{6}\right)\cup\left(\frac{\pi}{3},\frac{3\pi}{8}\right]$

$\Rightarrow a + b + c + d =\frac{\pi}{8}+\frac{\pi}{6}+\frac{\pi}{3}+\frac{3\pi}{8}=\pi$

117. Ans. 9

$2a = 4\sin^2x + \text{cosec}^2x + \sin^2y + 4\text{cosec}^2y$

$= (2\sin x - \text{cosec}x)^2 + 4 + (\sin y - \text{cosec}y)^2 + 3\text{cosec}^2y + 2$

$=6+(2\sin x-\text{cosec}x)^2+(\sin y-\text{cosec}y)^2+3\text{cosec}^2y$

$\Rightarrow$ minimum value of $2a = 6 + 3 = 9$, when $2\sin x = \text{cosec}x$ & $\sin y = \text{cosec}y$

118. Ans. 3

$P=\left(\sqrt{3}+\tan 1^\circ\right)\left(\sqrt{3}+\tan 2^\circ\right)....\left(\sqrt{3}+\tan 29^\circ\right)$

now $\sqrt{3}+\tan 1^\circ=\tan 60^\circ+\tan 1^\circ$

$$=\frac{\sin 60^\circ.\cos 1^\circ.\sin 1^\circ.\cos 60^\circ}{\cos 60^\circ.\cos 1^\circ}=\frac{\sin 61^\circ}{\cos 60^\circ.\cos 1^\circ}$$

hence

$$P=\frac{\sin 61^\circ}{\cos 60^\circ.\cos 1^\circ}.\frac{\sin 62^\circ}{\cos 60^\circ.\cos 2^\circ}.\frac{\sin 63^\circ}{\cos 60^\circ.3^\circ}.......$$

$$\frac{\sin 89^\circ}{\cos 60^\circ.\cos 29^\circ}$$

$$= 2^{29}.\frac{\sin 61°.\sin 62°.\sin 63°......\sin 89°}{\cos 1°.\cos 2°.\cos 3°........\cos 29°} = 2^{29}$$

(as $N^r = D^r$)

119. Ans. 2

We have

$$a = \left(\tan^2\frac{\pi}{24}+\tan^2\frac{11\pi}{24}\right)+\left(\tan^2\frac{2\pi}{24}+\tan^2\frac{10\pi}{24}\right)$$

$$+\left(\tan^2\frac{3\pi}{24}+\tan^2\frac{9\pi}{24}\right)+\left(\tan^2\frac{4\pi}{24}+\tan^2\frac{8\pi}{24}\right)$$

$$+\left(\tan^2\frac{5\pi}{24}+\tan^2\frac{7\pi}{24}\right)+\left(\tan^2\frac{6\pi}{24}\right)$$

$$= \left(\tan^2\frac{\pi}{24}+\cot^2\frac{\pi}{24}\right)$$

$$+\left\{\left(2-\sqrt{3}\right)^2+\left(2+\sqrt{3}\right)^2+\left(\sqrt{2}-1\right)^2+\left(\sqrt{2}+1\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2+\left(\sqrt{3}\right)^2\right\}$$

$$+\left(\tan^2\frac{5\pi}{24}+\cot^2\frac{5\pi}{24}\right)+1$$

Applying $(\tan^2\theta + \cot^2\theta) = 2 + 4\cot^2 2\theta$

$$= 2 + 4\cot^2\frac{\pi}{12} + \frac{70}{3} + 2 + \cot^2\frac{5\pi}{12} + 1$$

$$= \frac{85}{3}+4\left[\left(2+\sqrt{3}\right)^2+\left(2-\sqrt{3}\right)^2\right]$$

$$= \frac{85}{3}+4(2)(4+3) = \frac{253}{3}$$

|||ly $b = \left(\tan^2\frac{\pi}{24}+\tan^2\frac{11\pi}{24}\right)-\left(\tan^2\frac{2\pi}{24}+\tan^2\frac{10\pi}{24}\right)$

$$+\left(\tan^2\frac{3\pi}{24}+\tan^2\frac{9\pi}{24}\right)$$

$$-\left(\tan^2\frac{4\pi}{24}+\tan^2\frac{8\pi}{24}\right)+\left(\tan^2\frac{5\pi}{24}+\tan^2\frac{7\pi}{24}\right)$$

$$-\left(\tan^2\frac{6\pi}{24}\right)$$

$$=\left(\tan^2\frac{\pi}{24}+\cot^2\frac{\pi}{24}\right)-\left\{\left(2-\sqrt{3}\right)^2+\left(2+\sqrt{3}\right)^2\right\}$$

$$+\left\{\left(\sqrt{2}-1\right)^2+\left(\sqrt{2}+1\right)^2\right\}$$

$$-\left\{\left(\frac{1}{\sqrt{3}}\right)^2+\left(\sqrt{3}\right)^2\right\}+\left(\tan^2\frac{5\pi}{24}+\cot^2\frac{5\pi}{24}\right)-1$$

$$=\left(2+4\cot^2\frac{\pi}{12}\right) - [2\times(4+3)] + 2(2+1)$$

$$-\left(\frac{1}{3}+3\right)+\left(2+4\cot^2\frac{5\pi}{12}\right) - 1$$

$$= 2 + 4\left(2+\sqrt{3}\right)^2 - 14 + 6 - \frac{10}{3} + 2$$

$$+ 4\left(2-\sqrt{3}\right)^2 - 1$$

$$= 4(2)(4+3) - 5 - \frac{10}{3} = 51 - \frac{10}{3}$$

$$= \frac{153-10}{3} = \frac{143}{3}$$

$\therefore$ $(2a-b) = 2\left(\frac{253}{3}\right) - \frac{143}{3} = \frac{506-143}{3}$

$$= \frac{363}{3} = 121$$

and $(2b-a) = 2\left(\frac{143}{3}\right)-\frac{253}{3}$

$$= \frac{286-253}{3} = \frac{33}{3} = 11$$

Hence $\log_{(2b-a)}(2a-b) = 2$ **Ans.**

120. Ans. 26

We have $(a\cot A + b\cot B + c\cot C)$

$$= 2R\sin A\left(\frac{\cos A}{\sin A}\right) + 2R\sin B\left(\frac{\cos B}{\sin B}\right)$$

$$+ 2R\sin C\left(\frac{\cos C}{\sin C}\right)$$

$$= 2R(\cos A + \cos B + \cos C) = 2R\left(1+\frac{r}{R}\right)$$

$$= 2(R + r) = 2(10 + 3) = 26.$$

121. Ans. 5

$$y = \frac{1}{\sqrt{2}}\left[\sqrt{1+\cos 2x}+\sqrt{1+\cos 4x}+....+\sqrt{1+\cos 12x}\right]$$

$$= [|\cos x| + |\cos 2x| + |\cos 3x| + ... + |\cos 6x|]$$

at $x = \frac{\pi}{5}$

$$=\left[\left|\cos\frac{\pi}{5}\right|+\left|\cos\frac{2\pi}{5}\right|+\left|\cos\frac{3\pi}{5}\right|+\left|\cos\frac{4\pi}{5}\right|+\left|\cos\frac{5\pi}{5}\right|+\left|\cos\frac{6\pi}{5}\right|\right]$$

$$=\left[\left|\cos\frac{\pi}{5}\right|+\left|\cos\frac{2\pi}{5}\right|+\left|\cos\frac{2\pi}{5}\right|+\left|\cos\frac{\pi}{5}\right|+\left|\cos\pi\right|+\left|\cos\frac{\pi}{5}\right|\right]$$

$$=3\left(\frac{\sqrt{5}+1}{4}\right)+2\left(\frac{\sqrt{5}-1}{4}\right)+1=\frac{5}{4}\left[\sqrt{5}+1\right]$$

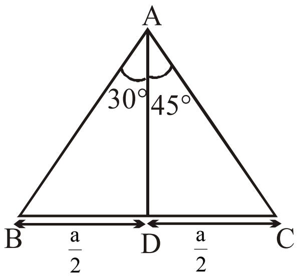
$$=\frac{5}{4}\left[\sqrt{p}+q^2\right] \Rightarrow p=5,\ q=1$$

then $\frac{5}{6}\,[p+q^2]=\frac{5}{6}\times 6=5$

122. Ans. 2

Using (m,n) theorem in Δ ABC $(1+1)\cot\theta$ $=1\cdot\cot30°-1\cdot\cot45°$

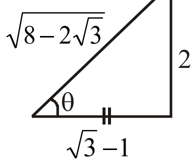
$$\Rightarrow \boxed{\cot\theta=\frac{\sqrt{3}-1}{2}}$$



Using Δ ADC, $\frac{a/2}{\sin 45°}=\frac{AD}{\sin(\theta+45°)}$

$$\therefore \frac{AD}{\frac{1}{\sqrt{2}}(\sin\theta+\cos\theta)}=\frac{a/2}{1/\sqrt{2}}$$

$$\therefore \frac{a}{2}=\frac{\frac{1}{\sqrt{11-6\sqrt{3}}}}{\frac{\sqrt{3}+1}{\sqrt{8-2\sqrt{3}}}}$$



$$\Rightarrow \frac{a}{2}=\frac{\sqrt{8-2\sqrt{3}}}{(\sqrt{3}+1)\sqrt{11-6\sqrt{3}}}$$

$$=\frac{\sqrt{8-2\sqrt{3}}}{\sqrt{(\sqrt{3}+1)^2(11-6\sqrt{3})}}$$

$$\frac{a}{2}=\frac{\sqrt{8-2\sqrt{3}}}{\sqrt{(4+2\sqrt{3})(11-6\sqrt{3})}}$$

$$=\frac{\sqrt{8-2\sqrt{3}}}{\sqrt{8-2\sqrt{3}}}=1 \qquad \boxed{\therefore a=2}$$

123. Ans. 2002

$f(-1)=0 \Rightarrow a-2b+c=0 \Rightarrow 2b=a+c$

$\Rightarrow$ a, b, c in A.P.

Now 1, 3, 5,...........2001 (1001 numbers)

2, 4, 6,..............2002 (1001 numbers)

number of triplets of a, b, c in A.P. are

$({}^{1001}C_2+{}^{1001}C_2)=2\cdot{}^{1001}C_2$

However a and c can be interchanged

no. of polynomials = $4\cdot{}^{1001}C_2$

$$=\frac{4\times1001\times1000}{2}$$

$=2002000 \qquad \Rightarrow 2002$ **Ans.**

124. Ans. 2275

We have number of quadrilateral ${}^{17}C_4-{}^{15}C_2$

$$=\frac{17\cdot16\cdot15\cdot14}{1\cdot2\cdot3\cdot4}-\frac{15\cdot14}{1\cdot2}=(10)(17)(14)-(15)(7)$$

$=2380-105=2275$ **Ans.**

Alternatively :

Number of quadrilateral fromed $=\frac{{}^{20}C_1\times{}^{15}C_3}{4}$

$=25\times91=2275$

125. Ans. 12

$\sin x+2\cos\alpha\cos x=2$

solution is possible if $1+4\cos^2\alpha\ge4$

$$\Rightarrow \cos^2\alpha\ge\frac{3}{4} \Rightarrow \sin^2\alpha\le\frac{1}{2}$$

$$\Rightarrow n\pi-\frac{\pi}{6}\le\alpha\le n\pi+\frac{\pi}{6}$$