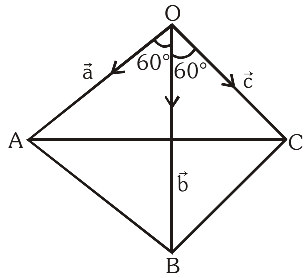
SCORE JEE (Advanced)
HOME ASSIGNMENT # 04

SOLUTION MATHEMATICS

1. Ans. (C)

Let centroid of ΔOAB be $G_1(\vec{g}_1)$

& ΔOBC be $G_2(\vec{g}_2)$



$\vec{g}_1 = \frac{\vec{a}+\vec{b}}{3}$

$\vec{g}_2 = \frac{\vec{b}+\vec{c}}{3}$

$|\vec{g}_1 - \vec{g}_2| = \frac{\vec{a}-\vec{c}}{3}$

$|\vec{g}_1 - \vec{g}_2| = \frac{\sqrt{\vec{a}^2+\vec{c}^2-2\vec{a}.\vec{c}}}{3} = \frac{1}{3}$ $\left\{\because \vec{a}.\vec{c} = \frac{1}{2}\right\}$

2. Ans. (B)

$\frac{3\vec{a}+4\vec{b}}{7} = \frac{6\vec{c}+\vec{d}}{7} = \frac{4\vec{e}+3\vec{f}}{7} = \frac{\vec{x}}{7}$

$\Rightarrow$ The join of points AB, CD & EF meets at a point whose position vector is $\frac{\vec{x}}{7}$.

3. Ans. (B)

$|\omega z - 1 - \omega^2| = a \quad \Rightarrow \quad |z+1| = a$

$\Rightarrow \quad |z-3+4| = a \quad \Rightarrow \quad |z-3|+4 \geq a$

$\Rightarrow \quad 0 \leq a \leq 8$

9. Ans. (B)

Since line joining AP is not parallel to the plane
So, it intersects the plane.

$\frac{x-1}{0} = \frac{y-0}{5} = \frac{z+3}{-10} = r$

$x = 1,\ y = 5r,\ z = -10r-3$

which lies on the plane.

$2x+3y+5z = 1$

$\Rightarrow \ 2+15r-50r-15 = 1 \quad \Rightarrow \quad r = \frac{-14}{35} = -\frac{2}{5}$

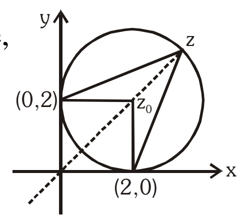
$\therefore \ P \equiv (1, -2, 1)$

$AP = \sqrt{0+4+16} = 2\sqrt{5}$

11. Ans. (D)

z lies on major arc of circle,
obviously,
minimum values of $|z|$ is 2.
maximum value of $|z|$ is

$2\sqrt{2}+2$



12. Ans. (D)

log (a + c), log(a + b), log(b + c) are in A.P.

$(a+b)^2 = (a+c)(b+c)$

$(a+b)^2 = ab + c(a+b) + c^2$ ...... (i)

a, c, b are in H.P. $\therefore \ c = \frac{2ab}{a+b}$

$\Rightarrow \ ab = \frac{c(a+b)}{2} = \frac{kc^2}{8}$

from (i), $\frac{k^2c^2}{16} = \frac{3kc^2}{8} + c^2$

$\Rightarrow \ k^2 - 6k - 16 = 0 \Rightarrow \ k = 8$ or -2

$\therefore \ k = 8 \quad \{k > 0\}$

13. Ans. (D)

$\vec{a}\times\vec{b} = p\vec{a}+q\vec{b}+r\vec{c}$,

Taking dot product with $\vec{a},\ \vec{b}\ \&\ \vec{c}$

$0 = p + \frac{3}{5}q + \frac{3}{5}r$ .... (i)

$0 = \frac{3}{5}p + q + \frac{3}{5}r$ .... (ii)

$[\vec{a}\,\vec{b}\,\vec{c}] = \frac{3}{5}p + \frac{3}{5}q + r$ .... (iii)

$$[\vec{a}\,\vec{b}\,\vec{c}]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix} = \begin{vmatrix} 1 & \frac{3}{5} & \frac{3}{5} \\ \frac{3}{5} & 1 & \frac{3}{5} \\ \frac{3}{5} & \frac{3}{5} & 1 \end{vmatrix} = \frac{44}{125}$$

From (i), (ii) & (iii)

$\frac{5}{11}[\vec{a}\,\vec{b}\,\vec{c}] = -\frac{2}{3}q$

$\frac{25}{121}.\frac{44}{125} = \frac{4}{9}q^2 \ \Rightarrow \ 55q^2 = 9$

14. Ans. (A)

$(\sin^2 a_1 - \sin^2 a_2) + (\sin^2 a_3 - \sin^2 a_4) + \ldots\ldots\ldots\ldots + (\sin^2 a_{(2n-1)} - \sin^2 a_{2n})$

$= (\sin(a_1+a_2)\sin(a_1-a_2))$
$+(\sin(a_3+a_4)\sin(a_3-a_4)) + \ldots\ldots\ldots$
$+ \sin(a_{2n-1} - a_{2n})\sin(a_{2n-1} + a_{2n})$

$= \sin(a_1+a_2)\sin(-d) + \sin(a_3+a_4)\sin(-d) + \ldots\ldots\ldots$
$+ \sin(a_{2n-1)} + a_{2n})\sin(-d)$

$= \sin(-d)((\sin(a_1+a_2) + \sin(a_3+a_4) + \ldots\ldots\ldots\ldots$

$+ \sin(a_{2n-1} + a_{2n}))$

The angles of the series are in A.P.

where $\alpha = a_1 + a_2$; $\beta = 4d$; no. of terms = n

$$= -\sin d\left(\frac{\sin\left((a_1+a_2)+(n-1)\frac{(4d)}{2}\right)\sin\left(n\frac{(4d)}{2}\right)}{\sin\left(\frac{4d}{2}\right)}\right)$$

$$= -\sin d\left(\frac{\sin\left(a_1+(a_1+d)+2nd-2d\right)\sin(2nd)}{\sin 2d}\right)$$

$$= -\frac{\sin d\left(\sin\left(2a_1+(2n-1)d\right)\right)\sin(2nd)}{2\sin d\cos d}$$

$(2n-1)d = \frac{\pi}{2} \quad \Rightarrow \quad 2nd = \frac{\pi}{2} + d$

$$= -\frac{\sin\left(\frac{\pi}{2}+2a_1\right)}{2}\frac{\sin\left(\frac{\pi}{2}+d\right)}{\cos d} = -\frac{\cos 2a_1}{2}$$

15. Ans. (B)

$T_6 = ar^5 = 96$

$a = \frac{96}{r^5}$

$500 < T_n < 780 \quad \Rightarrow \quad 500 < a_r^{(n-1)} < 780$

$\Rightarrow \quad 500 < 96r^{n-6} < 780$

$\Rightarrow \quad \frac{500}{96} < r^{(n-6)} < \frac{780}{96} \quad \Rightarrow \quad 5.2 < r^{(n-6)} < 8\,.\,1$

$r^{n-6} = 6$; $r^{n-6} = 7$; $r^{n-6} = 8$

$r = 6, n = 7$; $r = 7, n = 7$; $r = 8, n = 7$; $r = 2, n = 9$

for r = 6, 7, 8

$0 < a < 1 \Rightarrow [a] = 0$

r = 2 is possible

n – 6 = 3

n = 9

17. Ans. (B)

$f'(x) = 0 \Rightarrow x = \pm 1/3$

max. $\{f(0), f(1/3), f(3)\} = 2$

let G.P. is $r^2, r^3, r^4, \ldots\ldots$

so $\frac{r^2}{1-r} = 2$

$\Rightarrow \quad r = \sqrt{3}-1,\ -(\sqrt{3}+1)$ (Rejected)

$\Rightarrow \quad r = \sqrt{3}-1$

18. Ans. (B)

Let $\vec{c} = x\vec{a} + y\vec{b} + z(\vec{a}\times\vec{b})$

Put in $2(\vec{a}+\vec{b}) + \vec{c} = \vec{b}\times\vec{c}$

$\Rightarrow 2(\vec{a}+\vec{b}) + x\vec{a} + y\vec{b} + z(\vec{a}\times\vec{b}) = x(\vec{b}\times\vec{a}) + z(\vec{b}\times(\vec{a}\times\vec{b}))$

or $(2+x)\vec{a} + (2+y)\vec{b} + (z+x)(\vec{a}\times\vec{b})$

$= z((\vec{b}\,.\,\vec{b})\vec{a} - (\vec{b}\,.\,\vec{a})\vec{b})$

$\therefore \quad z + x = 0,\ 2 + x = z|\vec{b}|^2,\ 2 + y = -z(\vec{b}\,.\,\vec{a})$

$\Rightarrow \quad z = -x,\ 2 + x = z,\ 2 + y = -z \quad (\because\ \vec{b}.\vec{a} = 1)$

$\therefore \quad x = -1,\ z = 1,\ y = -3$

$\Rightarrow \quad \vec{c} = -\vec{a} - 3\vec{b} + (\vec{a}\times\vec{b})$

$(\vec{a}\times\vec{c}).\vec{b} = \{-3(\vec{a}\times\vec{b}) + (\vec{a}\times(\vec{a}\times\vec{b}))\}.\vec{b}$

$= (\vec{a}.\vec{b})^2 - |\vec{a}|^2|\vec{b}|^2 = 1 - 4 = -3$

22. Ans. (A)

Clearly t = 10

Since T_4 is the numerically greatest term

$\therefore \quad \left|\frac{T_4}{T_3}\right| \ge 1$ and $\left|\frac{T_5}{T_4}\right| \le 1$

$$\Rightarrow \quad \left|\frac{{}^{10}C_3.3^7\left(\frac{-2x}{5}\right)^3}{{}^{10}C_2.3^8\left(\frac{-2x}{5}\right)^2}\right| \ge 1 \text{ and } \left|\frac{{}^{10}C_4.3^6\left(\frac{-2x}{5}\right)^4}{{}^{10}C_3.3^7\left(\frac{-2x}{5}\right)^3}\right| \le 1$$

or $\left|\frac{8}{3.3}\left(-\frac{2x}{5}\right)\right| \ge 1$ and $\left|\frac{7}{4.3}\left(-\frac{2x}{5}\right)\right| \le 1$

or $|x| \ge \frac{45}{16}$ & $|x| \le \frac{30}{7}$

23. Ans. (B)

$\text{Arg}(z_1) = \frac{\pi}{6}$

$\Rightarrow \quad \sin\left(\frac{\pi}{4} + \arg z_1\right) + \cos\left(\frac{3\pi}{4} - \arg z_1\right) = \frac{1}{\sqrt{2}}$

$\therefore \quad \left|\sqrt{2}z - 3 + 2i\right| = |z|\frac{1}{\sqrt{2}}$ or $\left|\frac{z - \frac{3-2i}{\sqrt{2}}}{z}\right| = \frac{1}{2}$

which represents a circle.

24. Ans. (D)

$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin 30°\,\hat{n} = 3\hat{n}$

Since $\vec{d}$ is perpendicular to both $\vec{a}$ & $\vec{b}$

$\Rightarrow \quad \hat{n} = \pm\vec{d}$

$\therefore \quad \vec{a}\times\vec{b} = \pm 3\vec{d}$

Now $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \pm 3\vec{d}\times(\vec{c}\times\vec{d})$

$\Rightarrow \quad \pm 3\{(\vec{d}.\vec{d})\vec{c} - (\vec{d}.\vec{c})\vec{d}\} = \frac{\hat{i}}{6} - \frac{\hat{j}}{3} + \frac{\hat{k}}{3}$

or $\quad \pm 3\vec{c} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{6}$

$\therefore \quad |\vec{c}.\hat{i}| = \frac{1}{18}, |\vec{c}.\hat{j}| = \frac{1}{9}, |\vec{c}.\hat{k}| = \frac{1}{9}$

30. Ans. (C)

$\sin\alpha = -\frac{3}{5}, \cos\beta = \frac{2+2}{\sqrt{5}\sqrt{5}} = \frac{4}{5}$

$\therefore \quad \frac{\cos^2\alpha}{\sin^2\beta} = \frac{1 - \frac{9}{25}}{1 - \frac{16}{25}} = \frac{16}{9}$

32. Ans. (A)

x = 1 satisfies the equation

$(a - 3b)x^2 + (2b + 5a)x + (b - 6a) = 0$

$\therefore$ x = 1 is a root of the equation

Now $T_m = \frac{1}{n}, T_n = \frac{1}{m}$

Let A & D be first term and common difference respectively of the AP

then $A + (m-1)D = \frac{1}{n}$ ..... (i)

$A + (n-1)D = \frac{1}{m}$ ..... (ii)

$\Rightarrow \quad D = \frac{1}{mn}, A = \frac{1}{mn}$

$\therefore \quad T_{mn} = \frac{1}{mn} + (mn-1)\frac{1}{mn} = 1$

35. Ans. (C)

$2\vec{a} + 3\vec{b} + 6\vec{c} = 0$ .... (i)

Taking cross product of (i) with $\vec{b}$

$\Rightarrow \quad 2(\vec{a} \times \vec{b}) + 6(\vec{c} \times \vec{b}) = 0$

$\Rightarrow \quad \vec{a} \times \vec{b} = 3(\vec{b} \times \vec{c})$ .... (ii)

Taking cross product of (i) with $\vec{c}$

$\Rightarrow \quad 2(\vec{a} \times \vec{c}) + 3(\vec{b} \times \vec{c}) = 0$

$\Rightarrow \quad \vec{c} \times \vec{a} = \frac{3}{2}(\vec{b} \times \vec{c})$ .... (iii)

from (ii) & (iii),

$(\vec{a} \times \vec{b}) - (\vec{b} \times \vec{c}) + 2(\vec{c} \times \vec{a}) = 5(\vec{b} \times \vec{c})$

36. Ans. (D)

The given line will be parallel to one of the bisector planes of the given planes. Hence equation of bisector planes are

$\frac{x+y+z-3}{\sqrt{3}} = \pm\frac{x-y+z-5}{\sqrt{3}}$

y + 1 = 0 and x + z = 4

$\therefore$ k = 0

Now any point on the line L is $(3\lambda + 1, -2, 3\lambda + 1)$.

It intersects the plane x – 2y + 3z + 4 = 0

$\Rightarrow 4(3\lambda + 1) + 8 = 0$ or $\lambda = -1$

$\therefore$ Point of intersection of line and plane is (–2, –2, –2) which is at a distance $p = \sqrt{12}$ from origin.

37. Ans. (C)

a, b, c are in A.P.

$\therefore \quad e^a, e^b, e^c$ are in G.P.

$e^a + e^b, 2e^b, e^b + e^c$ are in H.P.

(If three numbers are in G.P. and middle term is added to all terms then terms becomes in H.P.)

38. Ans. (C)

$x^8 + y^8 + z^8 + 5 = 8xyz$

Applying A.M. ≥ G.M. for the numbers

x^8, y^8, z^8, 1, 1, 1, 1, 1

$\frac{x^8 + y^8 + z^8 + 1 + 1 + 1 + 1 + 1}{8} \geq \left(x^8.y^8.z^8.1.1.1.1.1\right)^{1/8}$

$x^8 + y^8 + z^8 + 5 \geq 8xyz$

Equality holds when numbers are equal

$x^8 = y^8 = z^8 = 1$

Possible set of values of (x, y, z) are 4

i.e. (1, 1, 1), (–1, –1, 1), (–1, 1, –1) (1, –1, –1)

39. Ans. (C)

$\vec{a} + 2\vec{b} = \lambda\vec{c}$ ...... (i)

$\vec{b} + 3\vec{c} = \mu\vec{a}$ ...... (ii)

(i) – 2x(ii)

$\vec{a} - 6\vec{c} = \lambda\vec{c} - 2\mu\vec{a} \quad \Rightarrow \quad (1 + 2\mu)\vec{a} = (6 + \lambda)\vec{c}$

$\because \quad \vec{a}$ is not parallel to $\vec{c}$

$\therefore \quad 1 + 2\mu = 0 \;\; \& \; \lambda + 6 = 0$

$\Rightarrow \quad \mu = -\frac{1}{2} \quad \& \quad \lambda = -6$

putting $\lambda = -6$ in (i)

$\vec{a} + 2\vec{b} + 6\vec{c} = 0$

$\therefore \quad \left|\vec{a} + 2\vec{b} + 6\vec{c} + 2\vec{c}\right| = \left|2\vec{c}\right| = 2$

$\{\because \; \vec{c}$ is a unit vector$\}$

40. Ans. (A)

$\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{5^n}\,{}^nC_r\sum_{k=1}^{r-1}{}^rC_k.3^k\right)$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{1}{5^n} . {}^nC_r \left\{ (1+3)^r - 1 - 3^r \right\} \right)$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} {}^nC_r . \frac{4^r - 3^r - 1}{5^n}$$

$$= \lim_{n\to\infty} \frac{(5^n - 1) - (4^n - 1) - (2^n - 1)}{5^n}$$

$$= \lim_{n\to\infty} \frac{5^n - 4^n - 2^n + 1}{5^n} = 1$$

42. Ans. (C)

$$\frac{\frac{a}{3} + \frac{a}{3} + \frac{a}{3} + \frac{b}{2} + \frac{b}{2} + c}{6} \geq \left(\frac{a^3}{27} . \frac{b^2}{4} . c \right)^{1/6}$$

$$\Rightarrow \left(\frac{a^3}{27} . \frac{b^2}{4} . c \right) \leq 1 \Rightarrow a^3 b^2 c \leq 3^3 . 2^2$$

44. Ans. (B)

$$a^{\left(\frac{1}{a} + \frac{1}{2a} + \frac{1}{4a} + ... \infty\right)} . 2^{\frac{1}{2a} + \frac{1}{4a} + \frac{1}{8a} + ... \infty} = \frac{8}{27}$$

now $\frac{1}{a}\left(1 + \frac{1}{2} + \frac{1}{2^2} + ... \infty\right) = \frac{2}{a}$

and $\frac{1}{2a} + \frac{2}{4a} + \frac{3}{8a} + ... \infty = \frac{2}{a}$ (use AGP)

$\therefore \quad a^{\frac{1}{a}} . 2^{\frac{1}{a}} = \frac{8}{27} = \left(\frac{1}{3}\right)^3 . 2^3 \Rightarrow a = \frac{1}{3}$

47. Ans. (C)

Let $b = ar$, $c = ar^2$

$x = \frac{a(1+r)}{2}$

$y = \frac{ar(1+r)}{2}$

$$\left(\frac{a}{x} + \frac{c}{y}\right)\left(\frac{b}{x} + \frac{b}{y}\right)$$

$$= \left\{ \frac{a \times 2}{a(1+r)} + \frac{ar^2 \times 2}{ar(1+r)} \right\} \left\{ \frac{ar \times 2}{a(1+r)} + \frac{ar \times 2}{ar(1+r)} \right\}$$

$$= \left\{ \frac{2}{1+r} + \frac{2r}{1+r} \right\} \left\{ \frac{2r}{1+r} + \frac{2}{1+r} \right\} = 4$$

48. Ans. (C)

Consider line $3x + 4y = 25$. Let $P(a, b)$ lies on this line. The minimum value of $a^2 + b^2$ is equal to square of perpendicular distance of this line from origin.

50. Ans. (A)

$\vec{a}.\vec{b} + \vec{a}.\vec{c} = 4$ ....(i)

& $(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = (x^2 - 2x + 6)\vec{b} + (\sin y)\vec{c}$

$\because$ $\vec{b}$ & $\vec{c}$ are non-collinear vectors

$\Rightarrow$ $\vec{a}.\vec{c} = x^2 - 2x + 6$ & $\vec{a}.\vec{b} = -\sin y$

Putting in (i), we get

$x^2 - 2x + 6 - \sin y = 4$

or $x^2 - 2x + 2 - \sin y = 0$

or $(x - 1)^2 + (1 - \sin y) = 0$

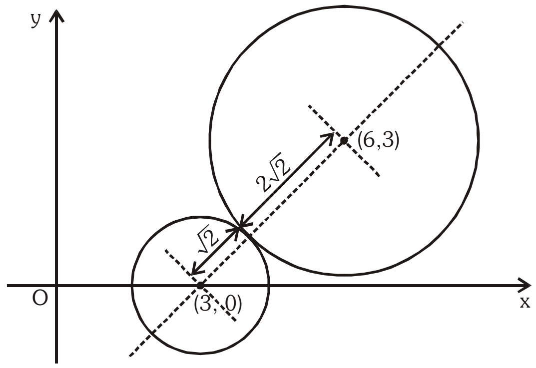
$\therefore$ $x = 1$ & $\sin y = 1$

52. Ans. (B)

$|z - 3| \leq \sqrt{2}$; centre : (3, 0), radius $= \sqrt{2}$

$|z - (6 + 3i)| \leq 2\sqrt{2}$; centre : (6, 3),

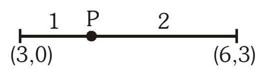
radius = $2\sqrt{2}$



Now, $C_1C_2 = \sqrt{9 + 9} = 3\sqrt{2} = r_1 + r_2$

$\therefore$ circles touching each other externally. Their point of contact is only the point which satisfies both inequalities.

P divides $C_1(3, 0)$ and $C_2(6, 3)$ in the ratio 1 : 2



$\therefore$ co-ordinates of P are (4, 1)

$z = 1 + 4i \Rightarrow \theta = \tan^{-1}\frac{1}{4} \Rightarrow \tan\theta = \frac{1}{4}$

$$\tan 2\theta = \frac{2.\frac{1}{4}}{1 - \frac{1}{16}} = \frac{8}{15}$$

53. Ans. (C)

$z - 2 = re^{i\alpha}$

$\therefore$ $(x - 2) + iy = r(\cos\alpha + i \sin\alpha)$

$r^2 = (x - 2)^2 + y^2$ and $\tan\alpha = \frac{y}{x - 2}$

The expression $\frac{z-2}{e^{i\theta}}+\frac{e^{i\theta}}{z-2}=re^{i(\alpha-\theta)}+\frac{1}{r}e^{-i(\alpha-\theta)}$

which is given as real

$\therefore \quad r\sin(\alpha-\theta)-\frac{1}{r}\sin(\alpha-\theta)=0$

$\Rightarrow \quad r-\frac{1}{r}=0 \Rightarrow r^2=1 \Rightarrow (x-2)^2+y^2=1$

55. Ans. (A)

$z_1+z_2=-6,\ z_1z_2=b$

If z_1, z_2 & origin are the vertices of an equilateral triangle then $z_1^2+z_2^2=z_1z_2$

$\Rightarrow \quad (z_1+z_2)^2=3z_1z_2 \Rightarrow 36=3b \Rightarrow b=12$

56. Ans. (D)

Now $\frac{1}{a}+\frac{1}{a-2b}+\frac{1}{c}+\frac{1}{c-2b}$

$=\left(\frac{1}{a}+\frac{1}{c-2b}\right)+\left(\frac{1}{c}+\frac{1}{a-2b}\right)$

$=(a+c-2b)\left(\frac{1}{a(c-2b)}+\frac{1}{c(a-2b)}\right)=0$

As $\quad a+c-2b\neq 0 \Rightarrow \frac{1}{b}=\frac{1}{a}+\frac{1}{c}$

i.e. a, 2b, c are in H.P.

59. Ans. (B)

Let m^{th} term of the first sequence be equal to the n^{th} term of the second sequence. Then,

$17+4(m-1)=16+5(n-1)$

$\Rightarrow \quad 4m+13=5n+11 \Rightarrow 4m+2=5n$

$\Rightarrow \quad n=m-(1/5)(m-2)$

Since n is an integer, m – 2 must be a multiple of 5, i.e., m must be of the form 5k + 2, with $k\geq 0$. Then, n = 4k + 2.

The first sequence has 101 terms and the second has 91 terms.

$\therefore \quad 0\leq 5k+2<101$ and $0\leq 4k+2\leq 91$

$\Rightarrow \quad 0\leq k\leq 19$

Hence, the given sequences have 20 common terms.

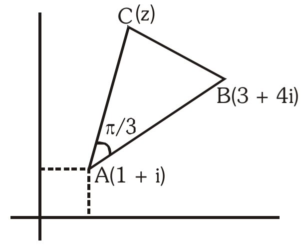
60. Ans. (A)

$$\sum_{\substack{r=0\\ r\leq s}}^{s}\sum_{s=1}^{n}{}^nC_s\,{}^sC_r=\sum_{s=1}^{n}{}^nC_s\left({}^sC_0+{}^sC_1+{}^sC_2+......+{}^sC_s\right)$$

$$=\sum_{s=1}^{n}{}^nC_s2^s=\sum_{s=0}^{n}{}^nC_s2^s-{}^nC_02^0$$

$=(1+2)^n-1=3^n-1$

62. Ans. (A)



$$\frac{z-(1+i)}{AC}=\frac{(2+3i)}{AB}e^{i\pi/3}$$

$\Rightarrow \quad z-1-i=2(2+3i)\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$

$\Rightarrow \quad z-1-i=\frac{4-6\sqrt{3}+i(6+4\sqrt{3})}{2}$

$\Rightarrow \quad z=3+4i+i\sqrt{3}(2+3i)$

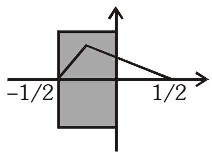
63. Ans. (B)

$\log_3|2z-1|>\log_3|2z+1|$

$\Rightarrow \quad (2z-1)(2\bar{z}-1)>(2z+1)(2\bar{z}+1)$

$\Rightarrow \quad 4|z|^2-2(z+\bar{z})+1>4|z|^2+2(z+\bar{z})+1$

$\Rightarrow \quad 4.2\text{Re}(z)<0$



$\Rightarrow \quad \text{Re}(z)<0$

64. Ans. (A)

$(z_1-3z_2)(\bar{z}_1-3\bar{z}_2)=(3-z_1\bar{z}_2)(3-\bar{z}_1z_2)$

$\Rightarrow \quad |z_1|^2-3z_1\bar{z}_2-3z_2\bar{z}_1+9|z_2|^2$

$=9-3\bar{z}_1z_2-3z_1\bar{z}_2+|z_1|^2|z_2|^2$

$\Rightarrow \quad |z_1|^2-|z_1|^2|z_2|^2+9|z_2|^2-9=0$

$\Rightarrow \quad |z_1|^2(1-|z_2|^2)-9(1-|z_2|^2)=0$

$\Rightarrow \quad (1-|z_2|^2)(|z_1|^2-9)=0 \Rightarrow |z_1|=3$

66. Ans. (B)

Put x = i

$(1+i)^5=(a_0-a_2+a_4)+i(a_1-a_3+a_5)$

$\Rightarrow \quad |1+i|^5=|(a_0-a_2+a_4)+i(a_1-a_3+a_5)|$

$\Rightarrow 2^5=(a_0-a_2+a_4)^2+(a_1-a_3+a_5)^2$

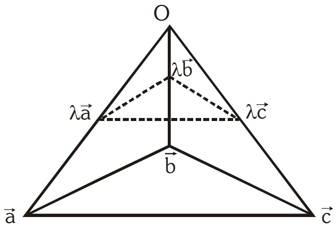
67. Ans. (D)

Let $z'=-2i+\frac{4-2\sqrt{5}i}{z} \Rightarrow z'+2i=\frac{4-2\sqrt{5}i}{z}$

$\Rightarrow \quad |z'+2i|=\left|\frac{4-2\sqrt{5}i}{z}\right| \Rightarrow \quad |z'+2i|=3$

$\therefore$ locus of z' is a circle of centre (0, –2) and radius 3

68. Ans. (B,C)



$$\frac{[\lambda\vec{a}\ \lambda\vec{b}\ \lambda\vec{c}]}{6} = \frac{1}{6}\frac{[\vec{a}\ \vec{b}\ \vec{c}]}{2}$$

$$\lambda = \frac{1}{2^{1/3}}$$

$$\frac{x}{a\lambda} + \frac{y}{b\lambda} + \frac{z}{c\lambda} = 1$$

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = \frac{1}{2^{1/3}}$$

$$\frac{\text{area }(\Delta ABC)}{\text{area }(\Delta A'C'C')} = \frac{\frac{1}{2}\sqrt{(ab)^2 + (bc)^2 + (ca)^2}}{\frac{1}{2}\sqrt{\lambda^2[(ab)^2 + (bc)^2 + (ca)^2]}} = 2^{1/3}$$

69. Ans. (A,B,C,D)

Let $z_1 = \cos\theta_1 + i\sin\theta_1$ and $z_2 = \cos\theta_2 + i\sin\theta_2$

$(\because\ |z_1| = |z_2| = 1)$

Now $\quad z_1\bar{z}_2 = \cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)$

$\text{Im}(z_1\bar{z}_2) = 0 \Rightarrow \theta_1 - \theta_2 = n\pi,\ n \in I$

Now $\quad w_1 = \cos\theta_1 + i\cos\theta_2,\ w_2 = \sin\theta_1 + i\sin\theta_2$

(A) $w_1\bar{w}_2 = \cos\theta_1\sin\theta_1 + \cos\theta_2\sin\theta_2 + i\sin(\theta_1 - \theta_2)$

$\Rightarrow \quad \text{Im}(w_1\bar{w}_2) = 0$

(B) $w_2\bar{w}_1 = \cos\theta_1\sin\theta_1 + \cos\theta_2\sin\theta_2 + i\sin(\theta_1 - \theta_2)$

$\Rightarrow \quad \text{Im}(w_2\bar{w}_1) = 0$

(C) $\dfrac{w_1}{w_2} = \dfrac{\cos\theta_1 + i\cos\theta_2}{\sin\theta_1 + i\sin\theta_2} \times \dfrac{\sin\theta_1 - i\sin\theta_2}{\sin\theta_1 - i\sin\theta_2}$

$$= \frac{\cos\theta_1\sin\theta_1 + \cos\theta_2\sin\theta_2 + i\sin(\theta_1 - \theta_2}{\sin^2\theta_1 + \sin^2\theta_2}$$

$\Rightarrow \quad \text{Im}\left(\dfrac{w_1}{w_2}\right) = 0$

(D) Similarly $\text{Re}\left(\dfrac{w_1}{\bar{w}_2}\right) = 0$

70. Ans. (B,D)

Let the four numbers be a, ar, ar^2, ar^3

Now $x^{2+\log_{10}x} = (0.001)^{-\frac{8}{3}} = 10^8$

Taking logarithm of both sides to base 10

$(2 + \log_{10}x)\log_{10}x = 8$

$\therefore \quad y^2 + 2y - 8 = 0$ where $y = \log_{10}x$

$\Rightarrow \quad y = 2, -4 \quad$ or $\quad x = 10^2, 10^{-4}$

$\therefore \quad$ Greater root $= 10^2 = a.\ ar^3$

$\Rightarrow \quad a^2r^3 = 100 \quad$ ....(i)

Also $(ar)^2 + (ar^2)^2 = 250$

$\Rightarrow \quad a^2r^2(1 + r^2) = 250 \quad$ ....(ii)

Dividing (i) by (ii) $\quad \Rightarrow \quad \dfrac{1 + r^2}{r} = \dfrac{5}{2}$

or $\quad 2r^2 - 5r + 2 = 0$ gives $r = \dfrac{1}{2}, 2$

when $r = 2$, $\quad a^2 = \dfrac{100}{8} = \dfrac{25}{2}$ and the four numbers are $\dfrac{5}{\sqrt{2}}, 5\sqrt{2}, 10\sqrt{2}$ and $20\sqrt{2}$

when $r = \dfrac{1}{2}$, we get the same four irrational numbers.

71. Ans. (A,B,D)

Any point on line x = 2y = 3z is $\left(\lambda, \dfrac{\lambda}{2}, \dfrac{\lambda}{3}\right)$

line meets plane x + y + z = 11 at P

$\Rightarrow$ P is (6, 3, 2)

Similarly it meets sphere given by

$x^2 + y^2 + z^2 = 196$ at R & S.

$\therefore \quad \lambda^2 + \left(\dfrac{\lambda}{2}\right)^2 + \left(\dfrac{\lambda}{3}\right)^2 = 196$

or $\lambda = \pm 12$

$\therefore \quad$ R(12, 6, 4) & S(–12, –6, –4)

Now $\quad PR = \sqrt{36 + 9 + 4} = 7$,

$PS = \sqrt{324 + 81 + 36} = 21$

& $\quad RS = \sqrt{576 + 144 + 64} = 28$

72. Ans. (A,D)

$\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} + \hat{k}$

$\vec{p} - \vec{a} = \lambda(\vec{b} - \vec{a}) = \lambda(-4\hat{j} + 3\hat{k})$

$|\vec{p} - \vec{a}| = \lambda\sqrt{16 + 9}$

$25\lambda^2 = 1 \Rightarrow \lambda = \pm\dfrac{1}{5}$

$$\therefore \quad \vec{p} = \vec{a} + \lambda(\vec{b} - \vec{a}) = \begin{cases} \dfrac{1}{5}(5\hat{i} + \hat{j} - 7\hat{k}) \\ \dfrac{1}{5}(5\hat{i} + 9\hat{j} - 13\hat{k}) \end{cases}$$

74. Ans. (A,C,D)

(A) Number of location = $^5C_3$

(C) Number of location = $^{10}C_3$

(D) Number of location = $^{10}C_3 - {^6C_3}$

75. Ans. (A,B,C,D)

(A) Line of intersection of

$\vec{r}.\vec{n}_1 = q_1$ & $\vec{r}.\vec{n}_2 = q_2$ is along $\vec{n}_1 \times \vec{n}_2$.

line of intersection of

$\vec{r}.\vec{n}_3 = q_3$ & $\vec{r}.\vec{n}_4 = q_4$ is along $\vec{n}_3 \times \vec{n}_4$.

$\Rightarrow$ The two lines are perpendicular when

$(\vec{n}_1 \times \vec{n}_2).(\vec{n}_3 \times \vec{n}_4) = 0$

$\Rightarrow (\vec{n}_1.\vec{n}_3)(\vec{n}_2.\vec{n}_4) - (\vec{n}_1.\vec{n}_4)(\vec{n}_2.\vec{n}_3) = 0$

(B) $(\vec{n}_1 \times \vec{n}_2).\vec{n}_4 = 0 = (\vec{n}_2 \times \vec{n}_3).\vec{n}_4 = (\vec{n}_1 \times \vec{n}_3).\vec{n}_4$

(C) $(\vec{n}_1 \times \vec{n}_2) \times (\vec{n}_3 \times \vec{n}_4) = 0$

$\Rightarrow \quad [\vec{n}_1\ \vec{n}_2\ \vec{n}_4]\vec{n}_3 = [\vec{n}_1\ \vec{n}_2\ \vec{n}_3]\vec{n}_4$

(D) Plane contains line along $\vec{n}_1 \times \vec{n}_2$.

Also plane is parallel to the line along $\vec{n}_3 \times \vec{n}_4$.

But does not imply that $\vec{n}_1 \times \vec{n}_2$ is parallel to $\vec{n}_3 \times \vec{n}_4$.

76. Ans. (A,B,D)

(A) $\dfrac{(a_3 - 2d) + (a_3 - d) + a_3 + (a_3 + d) + (a_3 + 2d)}{5}$

$> \left((a_3 - 2d)(a_3 - d)(a_3)(a_3 + d)(a_3 + 2d)\right)^{1/5}$

$\Rightarrow \quad a_3^5 > (a_3^2 - 4d^2)(a_3^2 - d^2)a_3$

$\Rightarrow \quad a_3^4 > a_3^4 - 5a_3^2d^2 + 4d^4 \Rightarrow 5a_3^2 > 4d^2$

(B) $a_1 + a_2 + a_3 + a_4 + a_5$

$a_1 + a_5 = 2a_3$ & $a_2 + a_4 = 2a_3$

(C) Not possible

(D) $(a_3 - 2d)(a_3 + 2d) < (a_3 - d)(a_3 + d)$

$\Rightarrow \quad d^2 > 0$ which is true.

82. Ans. (A,C)

$z = \exp\left\{\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}\right\} = e^{1/2} . e^{i\frac{\sqrt{3}}{2}}$

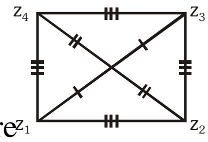
$z = e^{1/2}\left\{\cos\dfrac{\sqrt{3}}{2} + i\sin\dfrac{\sqrt{3}}{2}\right\}$

$\Rightarrow \quad \ell n(\text{Re}\, z) = \dfrac{1}{2} + \ell n\left(\cos\dfrac{\sqrt{3}}{2}\right)$

and $\quad \ell n(\text{Im}\, z) = \dfrac{1}{2} + \ell n\left(\sin\dfrac{\sqrt{3}}{2}\right)$

83. Ans. (A,B,C)

As it is a rhombus



$|z_1 - z_2| = |z_2 - z_3|$

& $z_1 + z_3 = z_2 + z_4$

$\because$ Diagonals of rhombus are perpendicular bisectors

$\Rightarrow \quad \arg\dfrac{z_1 - z_3}{z_2 - z_4} = \pm\dfrac{\pi}{2}$

$\Rightarrow \quad \dfrac{z_1 - z_3}{z_2 - z_4}$ is purely imaginary.

84. Ans. (A,C,D)

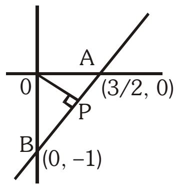
Put $z = x + iy$

$(2 + 3i)(x + iy) + (2 - 3i)(x - iy) - 6 = 0$

$2(2x - 3y) - 6 = 0$

$2x - 3y = 3$

Option A : In figure P represents the complex number with minimum modulus.



Equation of OP is $y = -\dfrac{3}{2}x$

Solving OP with AB, we get $\dfrac{6}{13} - \dfrac{9}{13}i$.

Option B is incorrect.

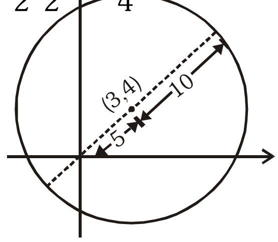
Option C : $\dfrac{z - (3+i)}{3 + 2i}$ is purely real

$\Rightarrow \quad \dfrac{z - (3+i)}{3 + 2i} = \dfrac{\bar{z} - (3-i)}{3 - 2i}$

only solving it gives same line.

Option D : $\dfrac{1}{2}|z_1||z_2| = \dfrac{1}{2}.\dfrac{3}{2}.1 = \dfrac{3}{4}$.

86. Ans. (B,C)



$|z - 3 - 4i| \le 10$

$|z|_{max} = 15$

$|z|_{min} = 0$

$\alpha = \sin^{-1}(\sin(|z|_{max}))$

$= \sin^{-1}(\sin 15) = 5\pi - 15$

$\beta = \cos^{-1}\left(\cos\left(-\dfrac{|z|_{max} + |z|_{min}}{3}\right)\right)$

$= \cos^{-1}\cos(-5) = \cos^{-1}\cos 5 = 2\pi - 5$

$\therefore \quad \alpha - 3\beta = 5\pi - 15 - 6\pi + 15 = -\pi$

$\sin^2\alpha + \cos^2 3\beta = \sin^2 15 + \cos^2 15 = 1$

87. Ans. (A,B)

Let sides be 3k, 4k & 5k

Now, $3k + 4k + 5k = 24 \Rightarrow k = 2$

$\therefore$ sides are 6, 8, 10 units

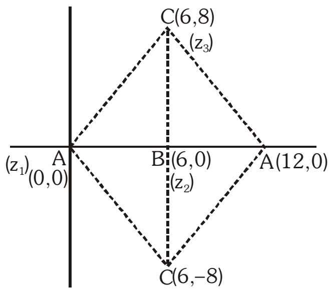
$B(z_2) \equiv (6, 0)$

$\because$ AB coincides with real axes

$\therefore$ Possible value of z_1 are $0 + i\,.\,0$ or $12 + i.0$

$|z_1|_{max} = 12$

$|z_1|_{min} = 0$



Inradius of the Δ is

$$= \frac{\text{Area}}{\frac{1}{2}(\text{perimeter})}$$

$$= \frac{\frac{1}{2}\times 6\times 8}{12} = 12$$

Circumcircle of the ΔABC is $\left|z - \frac{z_1 + z_3}{2}\right| = 5$

88. Ans.(A,B)

$\vec{x}\times\vec{z} = \vec{x}\times\vec{y}$

$\vec{x}\times(\vec{z}-\vec{y}) = 0$

$\vec{z}-\vec{y} = \lambda\vec{x}$

$\vec{z} = \vec{y}+\lambda\vec{x} = 2\hat{i}+4\hat{j}+\hat{k}+\lambda\left(\hat{i}+3\hat{j}-2\hat{k}\right)$

$\vec{z} = (2+\lambda)\hat{i}+\hat{j}(4+3\lambda)+\hat{k}(1-2\lambda)$

$\vec{z}.\vec{x} = 0 \Rightarrow 2+\lambda+3(4+3\lambda)-2(1-2\lambda) = 0$

$2 + \lambda + 12 + 9\lambda - 2 + 4\lambda = 0$

$14\lambda = -12$

$\lambda = -\frac{6}{7}$

so $\vec{z} = \frac{8}{7}\hat{i}+\frac{10}{7}\hat{j}+\frac{19}{7}\hat{k}$

$|\vec{z}| = \frac{1}{7}\sqrt{8^2+10^2+19^2}$

$[|\vec{z}|] = 3$

89. Ans. (A,B,C)

$$\sum_{r=1}^{n}\left(r.\sum_{p=1}^{r}(\omega^{P-1})\right) - 155\omega$$

$$= \sum_{r=1}^{n}\left(r.\left(\omega^0+\omega^1+\omega^2+\omega^2+.....+\omega^{r-1}\right)\right) - 155\omega$$

$= 1.\omega^0 + 2(\omega^0 + \omega^0) + 3(\omega^0 + \omega^1 + \omega^2)$
$+ 4(\omega^0 + \omega^1 + \omega^2 + \omega^3).....$
$n(\omega^0 + \omega^1 + \omega^2 + ...+ \omega^{4-1}) - 155\omega$

$= 1 + 2(-\omega^2) + 3(0) + 4(1) + 5(-\omega^2) + ...$

upto n terms $- 155\omega$

will be real if n = 29, 30, 31

90. Ans. (A,B,C,D)

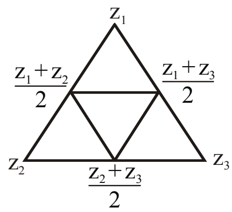
For equilateral Δ

$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$

(A) Which is satisfied by $-z_1 - z_2$ & z_3 also similarly $|z_1| = |z_2| = |z_3| = 1$

(B) $z_1 + 1, z_2 + 1, z_3 + 1$ are also satisfying the relation

(C)



are also vertices of equilateral Δ.

(D) $\frac{z_1}{2}, \frac{z_2}{2}, \frac{z_3}{2}$ are also vertices of an equilateral triangle.

Paragraph for Question 91 and 92

$S = 3^{20}$

unit's digit of S is b = 1

Also $p = \frac{^{20}C_1}{^{41}C_2} = \frac{1}{41}$

$\therefore$ a = 4

91. Ans. (C)

$\left(\sqrt{3}+1\right)^6 = I + F$ where $0 \le F < 1$

Let $\left(\sqrt{3}-1\right)^6 = G$ where $0 \le G < 1$

$\therefore$ $I + F + G = \left(\sqrt{3}+1\right)^6 + \left(\sqrt{3}-1\right)^6$

$= 2\left\{{}^6C_0(\sqrt{3})^6 + {}^6C_2(\sqrt{3})^4 + {}^6C_4(\sqrt{3})^2 + {}^6C_6\right\}$

$= 2\{1.27 + 15.9 + 15.3 + 1\} = 416$

or $I = 416 - (F + G)$

But $0 \le F + G < 2$ and $F + G$ has to be an integer

$\therefore$ $I = 416 - 1 = 415$

92. Ans. (B)

The general term in the expansion of $(1+x+x^2)^{20}$

is $\frac{20!}{r!\,s!\,t!}1^r x^s x^{2t}$

where r + s + t = 20

for coefficient of x^3, s + 2t = 3

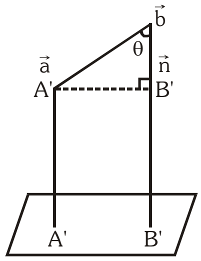
$\Rightarrow$ t = 0, s = 3, r = 17

or t = 1, s = 1, r = 18

$\therefore$ coefficient of $x^3 = \dfrac{20!}{17!\,3!} + \dfrac{20!}{18!\,1!} = \dfrac{4.20!}{18!}$

Paragraph for Question 93 to 95

93. Ans. (C)

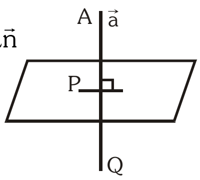


$A'B' = |\vec{a} - \vec{b}| \sin\theta$

$= |\vec{a} - \vec{b}| \left| \dfrac{(\vec{a} - \vec{b}) \times \vec{n}}{|\vec{a} - \vec{b}|\,|\vec{n}|} \right|$

$= \dfrac{|(\vec{a} - \vec{b}) \times \vec{n}|}{|\vec{n}|}$

94. Ans. (A)



Equation of line AP is $\vec{r} = \vec{a} + \lambda\vec{n}$

Intersection of this line with the plane $\vec{r}.\vec{n} = d$

$(\vec{a} + \lambda\vec{n}).\vec{n} = d$

$\lambda = \dfrac{d - \vec{a}.\vec{n}}{|n|^2}$

P has position vector $\vec{a} + \vec{n}\dfrac{(d - \vec{a}.\vec{n})}{n^2} = \vec{p}$

$\dfrac{\vec{a} + \vec{q}}{2} = \vec{p}$

$\vec{q} = 2\vec{a} + 2\vec{n}\dfrac{(d - \vec{a}.\vec{n})}{n^2} - \vec{a}$

95. Ans. (C)

Plane parallel to $\vec{r}.\vec{n} = d$ is $\vec{a}.\vec{n} = \mu$

if it passes through $\vec{a}$

$\vec{a}.\vec{n} = \mu$

$\vec{r}.\vec{n} - \vec{a}.\vec{n} = 0$ ....(i)

& second plane passes through $\vec{b}$ is having the equation $\vec{r}.\vec{n} - \vec{b}.\vec{n} = 0$ ....(ii)

distance between the planes (i) and (ii) is

$\dfrac{(\vec{a} - \vec{b}).\vec{n}}{|\vec{n}|} = d_1$

Paragraph for Question 96 to 98

96. Ans. (B)

$f(x) = ae^{2x} + be^x + cx$

$f(0) = a + b = -1$ ......(i)

$f'(x) = 2ae^{2x} + be^x + c$

$f'(\ell n2) = 8a + 2b + c = 31$ ......(ii)

$\displaystyle\int_0^{\ell n4} (ae^{2x} + be^x)dx = \left(a\frac{e^2x}{2} + be^x \right)_0^{\ell n4}$

$= 8a + 4b - \dfrac{a}{2} - b \quad \Rightarrow \quad \dfrac{15a}{2} + 3b = \dfrac{39}{2}$

$\Rightarrow$ 15a + 6b = 39 ....(iii)

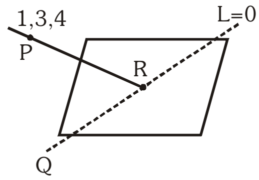
Solving (i), (ii) & (iii) we get

a = 5, b = –6, c = 3

a + b + c = 5 – 6 + 3 = 2

97. Ans. (D)

Given line is $\dfrac{x-1}{3} = \dfrac{y-3}{5} = \dfrac{z-4}{2}$



Given plane is

2x – y + z + 3 = 0

Image of point P in the plane is

$\dfrac{x-1}{2} = \dfrac{y-3}{-1} = \dfrac{z-4}{1}$

$= -2\left(\dfrac{2\times1 - 3\times1 + 4\times1 + 3}{4+1+1}\right)$

$\dfrac{x-1}{2} = \dfrac{y-3}{-1} = \dfrac{z-4}{1} = -2$

x = – 3, y = 5, z = 2

$\therefore$ Q ≡ (–3, 5, 2)

Point of intersection of the line with the plane

$\dfrac{x-1}{3} = \dfrac{y-3}{5} = \dfrac{z-4}{2} = r$

x = 3r + 1, y = 5r + 3, z = 2r + 4

R = (3r + 1, 5r + 3, 2r + 4)

2(3r + 1) – (5r + 3) + 2r + 4 + 3 = 0

3r = – 6 $\Rightarrow$ r = –2

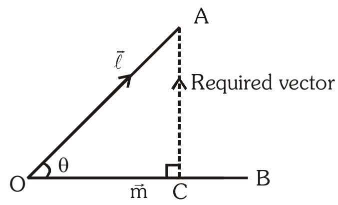
$\therefore$ R ≡ (–5, –7, 0)

Dr's of L = 0 are 2, 12, 2 or 1, 6, 1

$\therefore$ p = 6 and q = 1

$p^2 + 4q = 36 + 4 = 40$

98. Ans. (D)



$\vec{\ell} = 5\hat{i} - 6\hat{j} + 3\hat{k}$

$\vec{m} = 4\hat{i} + 2\hat{j} + 4\hat{k}$

Projection of $\vec{\ell}$ on $\vec{m}$ is $OC = \dfrac{\vec{\ell}.\vec{m}}{|\vec{m}|}$

$\overrightarrow{OC} = \dfrac{\vec{\ell} - \vec{m}}{|\vec{m}|}.\hat{m}$

$\therefore \quad \overrightarrow{CA} = \vec{\ell} - \dfrac{\vec{\ell}.\vec{m}}{|\vec{m}|}.\hat{m}$

$= 5\hat{i} - 6\hat{j} + 3\hat{k} - \dfrac{20 - 12 + 12}{6} \times \dfrac{4\hat{i} + 2\hat{j} + 4\hat{k}}{6}$

$= 5\hat{i} - 6\hat{j} + 3\hat{k} - \frac{5}{9}\left(4\hat{i} + 2\hat{j} + 4\hat{k}\right) = \frac{25\hat{i} - 64\hat{j} + 7\hat{k}}{9}$

Paragraph for Question 99 to 101

99. Ans. (B)

A(2, 2, –1), B(3, 1, 2), C(1, 1, 1)

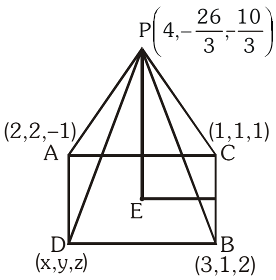
$\overline{AB} = \hat{i} - \hat{j} + 3\hat{k}$

$\overline{AC} = -\hat{i} - \hat{j} + 2\hat{k}$

$\overline{BC} = -2\hat{i} - \hat{k} \quad \therefore \quad AC \perp BC$

mid point of AB

= mid point of CD



$\frac{x+1}{2} = \frac{5}{2} \Rightarrow x = 4$

$\frac{y+1}{2} = \frac{3}{2} \Rightarrow y = 2$

$\frac{z+1}{2} = \frac{1}{2} \Rightarrow z = 0$

$\therefore$ Co-ordinates of D are (4, 2, 0)

100. Ans. (A)

Equation of the base

a(x – 1) + b(y – 1) + c(z – 1) = 0

which also passes through A(2, 2, –1) and B(3, 1, 2)

$\therefore$ a + b – 2c = 0 ......(i)

2a + c = 0 ......(ii)

c = –2a, b = –5a

a(x – 1) – 5a(y – 1) – 2a(z – 1) = 0

$\Rightarrow$ x – 5y – 2z + 6 = 0

foot of the normal from P

$$\frac{x-4}{1} = \frac{y+\frac{26}{3}}{-5} = \frac{z+\frac{10}{3}}{-2}$$

$$= -\left(\frac{4-5\left(-\frac{26}{3}\right)-2\left(-\frac{10}{3}\right)+6}{1+25+4}\right)$$

$$\Rightarrow \frac{x-4}{1} = \frac{y+\frac{26}{3}}{-5} = \frac{z+\frac{10}{3}}{-2} = -2$$

$\Rightarrow$ x = 2, $y = \frac{4}{3}$, $z = \frac{2}{3}$

$\therefore \left(2, \frac{4}{3}, \frac{2}{3}\right)$

101. Ans. (A)

Volume of the pyramid

$= \frac{1}{3}(\text{Base area}) \times \text{Height} = \frac{1}{3}\left|\overline{AB} \times \overline{AC}\right| \times EP$

$= \frac{1}{3}\left|\hat{i} - 5\hat{j} - 2\hat{k}\right| \times \sqrt{120}$

$= \frac{1}{3}\sqrt{30}.\sqrt{120} = 20$ cubic units

Paragraph for Question 102 to 104

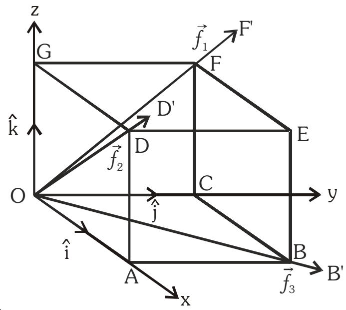
102. Ans. (B)

Let OA, OC & OG represents x, y & z axis respectively & let ℓ(OA) = ℓ(OC) = ℓ(OG) = 1

$\vec{f}_1 = 2\frac{(\hat{j} + \hat{k})}{\sqrt{2}}$

$\vec{f}_2 = 4\frac{(\hat{i} + \hat{k})}{\sqrt{2}}$

$\vec{f}_3 = 6\frac{(\hat{i} + \hat{j})}{\sqrt{2}}$



$\vec{f} = \frac{1}{\sqrt{2}}(10\hat{i} + 8\hat{j} + 6\hat{k}) = \sqrt{2}(5\hat{i} + 4\hat{j} + 3\hat{k})$

$|\vec{f}| = 10$

103. Ans. (C)

$$v = 2\sqrt{2}\begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 2 \\ 3 & 3 & 0 \end{vmatrix} = 2\sqrt{2}\{0 - (0-6) + 6\} = 24\sqrt{2}$$

104. Ans. (A)

$$\overline{OX} = \frac{\vec{f}_1 + \vec{f}_2 + \vec{f}_3}{2} = \frac{\sqrt{2}(5\hat{i} + 4\hat{j} + 3\hat{k})}{2}$$

$\left|\overline{OX}\right| = 5$

Paragraph for Question 105 to 108

105. Ans. (C)

$1 + \alpha^{10} + \alpha^{20} + \alpha^{30} + \ldots\ldots + \alpha^{190} = 0$

as 10 is not an integral multiple of n.

106. Ans. (A)

$\because \; z^n - 1 = (z-1)(z-\alpha)(z-\alpha^2)\ldots\ldots(z-\alpha^{n-1})$

$\Rightarrow \frac{z^n - 1}{z-1} = (z-\alpha)(z-\alpha^2)\ldots\ldots(z-\alpha^{n-1})$

$\Rightarrow \lim_{z\to1}\frac{z^n-1}{z-1} = \lim_{z\to1}(z-\alpha)(z-\alpha^2)\ldots\ldots(z-\alpha^{n-1})$

$\Rightarrow n = \prod_{r=1}^{n-1}(1-\alpha^r)$

$= \prod_{r=1}^{n-1}(1-\alpha^{\frac{2\pi ri}{n}})$

$$= \prod_{r=1}^{n-1}\left\{-e^{\frac{\pi ri}{n}}\left(e^{\frac{\pi ri}{n}} - e^{-\frac{\pi ri}{n}}\right)\right\} = \prod_{r=1}^{n-1} -e^{\frac{\pi ri}{n}} 2i\sin\left(\frac{r\pi}{n}\right)$$

$$\Rightarrow |n| = \prod_{r=1}^{n-1} 2\sin\left(\frac{r\pi}{n}\right)$$

$$\Rightarrow |n| = 2^{n-1}\prod_{r=1}^{n-1}\sin\left(\frac{r\pi}{n}\right)$$

$$\Rightarrow \prod_{r=1}^{n-1}\sin\left(\frac{r\pi}{n}\right) = \frac{n}{2^{n-1}} \qquad (\because\ n > 1)$$

So $\sum_{r=1}^{n-1} \ell n\sin\frac{r\pi}{n} = \ell n\left(\prod_{r=1}^{n-1}\sin\frac{r\pi}{n}\right)$

$$= \ell n\frac{n}{2^{n-1}} = \ell n n - (n-1)\ell n 2$$

107. Ans. (D)

$n = 7$, $\alpha^7 = 1$ and $1 + \alpha + \alpha^2 + \dots\dots\ \alpha^6 = 0$

Sum of the roots $= \alpha + \alpha^2 + \alpha^4 + \alpha^3 + \alpha^5 + \alpha^6 = -1$

Product of the roots $= (\alpha + \alpha^2 + \alpha^4)(\alpha^3 + \alpha^5 + \alpha^6)$

$= \alpha^4 + \alpha^6 + \alpha^7 + \alpha^5 + \alpha^7 + \alpha^8 + \alpha^7 + \alpha^9 + \alpha^{10}$

$= 2 + (1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \alpha^6) = 2$

So the required equation is

$x^2 - (-1)x + 2 = 0 \quad \Rightarrow \quad x^2 + x + 2 = 0$

108. Ans. (C)

$n = 5$, $\alpha^5 = 1$

and $1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 = 0$

$(1 + \alpha)(1 + \alpha^2)(1 + \alpha^3)(1 + \alpha^4)$

$= (1 + \alpha + \alpha^2 + \alpha^3)(1 + \alpha^3 + \alpha^4 + \alpha^7)$

$= (-\alpha^4)(1 + \alpha^2 + \alpha^3 + \alpha^4)$

$= (-\alpha^4)(-\alpha) = \alpha^5 = 1$

Paragraph for Question 109 to 111

$\sqrt{17(1+\tan^2 x) + 8\tan x\sec x - 16} = -2\tan x(1 + 4\sin x)$

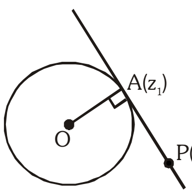
or $|4\tan x + \sec x| = -2\tan x\ (1 + 4\sin x)$

or $(4\sin x + 1) = 2\sin x\ (1 + 4\sin x)$, $\cos x < 0$

For $x \in \left(\frac{\pi}{2}, \pi\right)$, $\sin x = 1/2$ is the only solution.

$\Rightarrow \theta = \frac{5\pi}{6}$

109. Ans. (D)



$OA \perp AP \Rightarrow \arg(z - z_1) - \arg(z_1) = \pm\frac{\pi}{2}$

or $\arg\left(\frac{z - z_1}{z_1}\right) = \pm\frac{\pi}{2}$

$\Rightarrow \frac{z - z_1}{z_1}$ is purely imaginary.

or $\frac{z - z_1}{z_1} = \lambda i \Rightarrow z = z_1(1 + \lambda i)$, $\lambda \in R_0$.

Also complex slope of AP + complex slope of OA = 0

$\Rightarrow \frac{z - z_1}{\bar{z} - \bar{z}_1} + \frac{z_1}{\bar{z}_1} = 0$ or $\frac{z - z_1}{z_1} + \frac{\bar{z} - \bar{z}_1}{\bar{z}_1} = 0$

110. Ans. (C)

Let $|z_1| = |z_2| = |z_3| = a$

If z be the complex representation of F, then

$$\frac{z - z_1}{z_1} + \frac{\bar{z} - \bar{z}_1}{\bar{z}_1} = 0$$

i.e., $\bar{z}_1 z + \bar{z} z_1 = 2a^2$ ....(i)

similarly, $\bar{z}_2 z + \bar{z} z_2 = 2a^2$ ....(ii)

From (i) and (ii), we have

$(\bar{z}_1 z_2 - \bar{z}_2 z_1)z = 2a^2(z_2 - z_1)$

$a^2\left(\frac{z_2}{z_1} - \frac{z_1}{z_2}\right)z = 2a^2(z_2 - z_1)$ or $z = \frac{2z_1 z_2}{z_1 + z_2}$

111. Ans. (A)

$z_2 = z_1 . e^{\frac{i5\pi}{6}}$ and $z_3 = z_2 . e^{\frac{i5\pi}{6}}$

$\Rightarrow z_1, z_2, z_3$ are in G.P. with common ratio $e^{\frac{i5\pi}{6}}$.

$\therefore i\ \ell nr = -\frac{5\pi}{6}$

$\Rightarrow [i\ \ell nr] = -3$

Paragraph for Question 112 to 114

$$\phi(\theta) = \int_{-\cos^2\theta}^{\sin^2\theta} (f(x))^2 dx$$

Also $f(x) + f(y) = \frac{x + y}{xy}$

Put y = x

$2f(x) = \frac{2}{x} \Rightarrow f(x) = \frac{1}{x}$

$$\therefore \phi(\theta) = \int_{-\cos^2\theta}^{\sin^2\theta} \frac{1}{x^2}dx = \left[-\frac{1}{x}\right]_{-\cos^2\theta}^{\sin^2\theta}$$

$= -\left[\frac{1}{\sin^2\theta} + \frac{1}{\cos\theta}\right] = -4\text{cosec}^2 2\theta$

112. Ans. (A)

Fundamental period of $-4\text{cosec}^2 2\theta$ will be $\frac{\pi}{2}$

113. Ans. (B)

Now $\quad h(\theta) = -\phi(\theta) + \left|\vec{a}\times\vec{b}_1\right|^2 \left|\vec{a}.\vec{b}_1\right|^2$

$= 4\text{cosec}^2 2\theta + 16\sin^2\theta\cos^2\theta$

$= 4\left(\text{cosec}^2 2\theta + \frac{1}{\text{cosec}^2 2\theta}\right)$

It will be minimum when $\text{cosec}^2 2\theta = 1$

$\Rightarrow \theta = \frac{\pi}{4}$

So volume of the parallelopiped will be

$\left[\vec{a}\ \vec{b}\ (\vec{a}\times\vec{b})\right] = (\vec{a}\times\vec{b})^2 = \frac{1}{2}$

114. Ans. (B)

$4\phi(\theta) + \sqrt{3}\phi'(\theta) = 0$

$-16\text{cosec}^2 2\theta + 16\sqrt{3}\text{cosec}^2 2\theta\cot 2\theta = 0$

$16\text{cosec}^2 2\theta(-1 + \sqrt{3}\cot 2\theta) = 0$

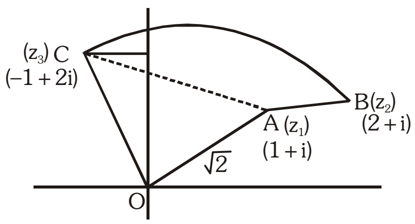
$\Rightarrow \quad \cot 2\theta = \frac{1}{\sqrt{3}} \qquad$ (as $\text{cosec}^2 2\theta \neq 0$)

$\Rightarrow \quad \tan 2\theta = \sqrt{3}$

$\Rightarrow \quad 2\theta = n\pi + \frac{\pi}{3} \quad \Rightarrow \quad \theta = \frac{n\pi}{2} + \frac{\pi}{6},\ n \in I$

So number of solutions in $[0, 2\pi]$ will be 4.

Paragraph for Question 115 to 117



115. Solving $|z - z_1| = |z - z_3|$, we get

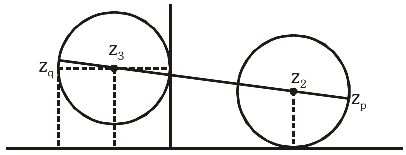
$z(-2 - i) + \bar{z}(-2 + i) = 3 \qquad$ ....(i)

Let z_p be the image of z_2 in (i)

$\Rightarrow \quad z_p(-2 - i) + \bar{z}_2(-2 + i) = 3$

$\Rightarrow \quad z_p = \frac{-8}{5} + \frac{14}{5}i$

116.



From figure maximum value of $|z_p - z_q| = \sqrt{9+1} + 2 = 2 + \sqrt{10}$

117. Area enclosed

= Area of quarter circle OBC – A(ΔOAB)

$= \frac{\pi.5}{4} - \left|\frac{1}{2}(2-1)\right| = \frac{5\pi}{4} - \frac{1}{2}$

Paragraph for Question 118 to 120

$z^{12} + z^6 + 1 = 0$ .... (i)

$(z^6 - 1)(z^{12} + z^6 + 1) = 0$

$z^{18} - 1 = D$

Let roots of the above equation are

$z_1, z_2, z_3, \ldots\ldots\ldots z_{18}$

$z = (1)^{1/18}$

$z = (\cos 2n\pi + i\sin 2n\pi)^{1/18}$

$= \cos\frac{2n\pi}{18} + i\sin\frac{2n\pi}{18}$

where, n = 0, 1, 2, ..., 17

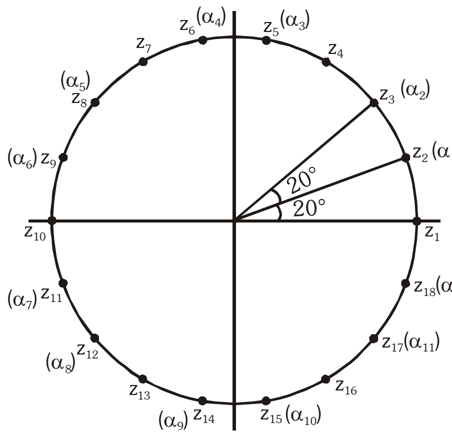
$z_1 = \cos 0° + i\sin 0° = 1$

$z_2 = \cos\frac{2\pi}{18} + i\sin\frac{2\pi}{18} = \cos 20° + i\sin 20°$

$z_3 = \cos\frac{4\pi}{18} + i\sin\frac{4\pi}{18} = \cos 40° + i\sin 40°$

⋮

$z_{18} = \cos\frac{34\pi}{18} + i\sin\frac{34\pi}{18} = \cos 340° + i\sin 340°$



$z^6 - 1 = 0$

$z = (1)^{1/6}$

$z = \cos\frac{2n\pi}{6} + i\sin\frac{2n\pi}{6}$, n = 0, 1, 2, 3, 4, 5

$z'_1 = \cos 0° + i\sin 0° = z_1$

$z'_2 = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = \cos 60° + i\sin 60° = z_4$

$z'_3 = \cos\frac{4\pi}{6} + i\sin\frac{4\pi}{6} = \cos 120° + i\sin 120° = z_7$

$z'_4 = \cos\frac{6\pi}{6} + i\sin\frac{6\pi}{6} = \cos 180° + i\sin 180° = z_{10}$

$z'_5 = \cos\frac{8\pi}{6} + i\sin\frac{8\pi}{6} = \cos 240° + i\sin 240° = z_{13}$

$z'_6 = \cos\frac{10\pi}{6} + i\sin\frac{10\pi}{6} = \cos 300° + i\sin 300° = z_{16}$

$\therefore$ Roots of the equation $z^{12} + z^6 + 1 = 0$ are $z_2, z_3, z_5, z_6, z_8, z_9, z_{11}, z_{12}, z_{14}, z_{15}, z_{17}, z_{18}$ which are respectively $\alpha_1, \alpha_2, \alpha_3, ...\alpha_{12}$

118. Ans. (C)

$\cos\left(\sum_{i=1}^{12}\theta_i\right) + i\sin\left(\sum_{i=1}^{12}\theta_i\right)$

$= \cos(\theta_1 + \theta_2 + ... + \theta_{12}) + i\sin(\theta_1 + \theta_2 + ... + \theta_{12})$

$= e^{i\theta_1}.e^{i\theta_2}.e^{i\theta_3}........e^{i\theta_{12}} = \alpha_1.\alpha_2.\alpha_3.........\alpha_{12} = 1$

(product of the roots of the equation (i))

119. Ans. (B)

$\arg(\alpha_1) = 20°$ & $\arg(\alpha_{12}) = 340°$

$\therefore$ $\sec(3\tan^{-1}(\tan 20°) + 6\cos^{-1}(\cos 340°))$

$\sec(3 \times 20° + 6 \times (360° - 340°)) = \sec 180° = -1$

120. Ans. (D)

$\alpha_7, \alpha_8, \alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{12}$ are the roots whose imaginary parts are negative.

Sum of their real parts

$= \cos 200° + \cos 220° + \cos 240° + \cos 260° + \cos 280° + \cos 300° + \cos 320° + \cos 340° - \cos 240° - \cos 300°$

$= \frac{\sin 8.10}{\sin 10}\cos(200° + 7.10) + \frac{1}{2} - \frac{1}{2} =$

$\frac{\sin 80°}{\sin 10}\cos 270° = 0$

121. Ans. (A)→(S), (B)→(R), (C)→(P), (D)→(P)

(A) $\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{r} = \vec{b} + \mu\vec{a}$

solving two lines $\lambda = \mu = 1$

$\therefore$ $\overline{OP} = 3\hat{i} + \hat{j} - \hat{k}$

$\therefore$ $\ell^2(OP) = 11$

(B) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = 10\vec{b} - 3\vec{c} + 0.\vec{a}$

$\therefore$ x + y + z = 7

(C) $\vec{a}.\vec{b} < 0$

$\therefore$ $x^9(x^3 - 1) + x(x^3 - 1) + 1 < 0$

$x^{12} + x^4 - x^9 - x + 1 < 0$

LHS > 0 $\forall\, x \in R$

(D) Obviously points P, Q & R are collinear points

$\therefore$ $A(\Delta PQR) = 0$

122. Ans. (A)→(R); (B)→(P); (C)→(Q); (D)→(S)

(A) According to given information the system has infinite solutions

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ \alpha & -1 & 3 \end{vmatrix} = 0$$

$2(-3 + 1) - (3 - \alpha) + (-1 + \alpha) = 0$

$\Rightarrow$ $\alpha = 4$

(B) Put z = 0 in P_1 & P_2

$\Rightarrow$ 2x + y = 1

x – y = 2

$\therefore$ $P \equiv (1, -1, 0)$

Put x = 0 in P_1 & P_2

y + z = 1

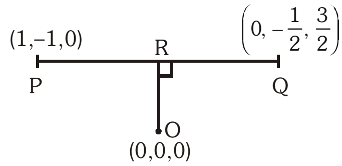
–y + z = 2

$Q \equiv \left(0, -\frac{1}{2}, \frac{3}{2}\right)$

$\overline{PQ} = -\hat{i} + \frac{1}{2}\hat{j} + \frac{3}{2}\hat{k}$

Projection of PQ on x-axis = $|\overline{PQ}.\hat{i}| = 1$

(C)



Equation of line PQ is

$\frac{x-1}{-1} = \frac{y+1}{1/2} = \frac{z}{3/2} = \lambda$

Let $R \equiv \left(-\lambda + 1, \frac{\lambda}{2} - 1, \frac{3\lambda}{2}\right)$

$\overline{OR} . \overline{PQ} = 0$

$\Rightarrow \quad -1(-\lambda + 1) + \frac{1}{2}\left(\frac{\lambda}{2} - 1\right) + \frac{9\lambda}{4} = 0$

$\Rightarrow \quad \lambda = \frac{3}{7}$

$\therefore \quad R \equiv \left(\frac{4}{7}, -\frac{11}{14}, \frac{9}{14}\right)$

$\Rightarrow \quad 7a + 14b + 14c = 2$

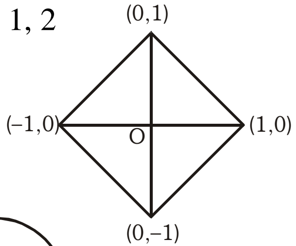
(D) Area $= \frac{1}{2}\left|\overline{OP} \times \overline{OQ}\right| = \sqrt{\frac{19}{16}}$

124. Ans. (A)→(Q), (B)→(Q), (C)→(P), (D)→(S)

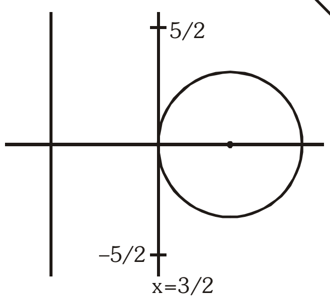
(A) $2a < 6 \Rightarrow a < 3 \Rightarrow 0 < a < 3$

integral values are 1, 2

(B) Area $= \left(\sqrt{2}\right)^2 = 2$



(C)



$f(z) = ((2x - 3) + iy. 2)i$

$f(z) = -2y + i(2x - 3)$

for purely real $x = \frac{3}{2}$ & $-5 \le 2y \le 5$

$\Rightarrow \quad -\frac{5}{2} \le y \le \frac{5}{2}$

Number of solution is 1

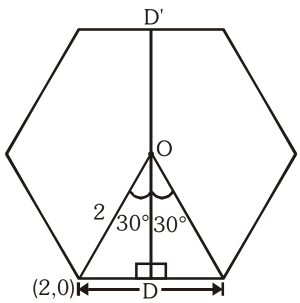
(D) Shifting origin will never effect the distances

$z^6 = 64$

$OD = \sqrt{3}$

$OD' = 2\sqrt{3} = \lambda$

$\frac{\lambda^2}{2} = \frac{4.3}{2} = 6$



125. Ans. (A)→(S), (B)→(P), (C)→(Q), (D)→ (P,R)

(A) $\frac{z_3 - z_1}{z_2 - z_1} = \frac{1 + i\sqrt{3}}{2} \quad \Rightarrow \quad \frac{z_3 - z_1}{z_2 - z_1} = e^{i\frac{\pi}{3}}$

$\Rightarrow$ Triangle is equilateral

(B) $\frac{z_3 - z_1}{z_3 - z_2}$ is purely imaginary

$\Rightarrow$ triangle is right angled

(C) Let $\frac{z_3 - z_1}{z_3 - z_2} = r(\cos\theta + i\sin\theta)$

as $\cos\theta < 0$

$\Rightarrow$ triangle is obtuse angled

(D) $|z_3 - z_1| = |z_3 - z_2|$ and $\frac{z_3 - z_1}{z_3 - z_2}$ is purely imaginary so triangle is isosceles rightangled.

126. Ans. (A)→(Q); (B)→(R); (C)→(S); (D)→(P)

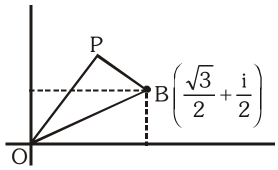
(A) $z_2 - z_3 = \lambda(z_3 - z_1)$

$\Rightarrow$ z_3 lies on the line joining z_1 & z_2 in the first quadrant.

$\tan(\arg(z_3))$ = slope of line joining z_1 & z_2

$\tan(\arg(z_3)) = \frac{1}{2}$

(B)



$|z| + \left|z - \frac{\sqrt{3}}{2} - \frac{i}{2}\right| \ge OB$

$|z| + \left|z - \frac{\sqrt{3}}{2} - \frac{i}{2}\right| \ge 1$

(C) $z_3 - (5 + 2i) = \left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right)(-1 + i)$

$z_3 - (5 + 2i) = \frac{1}{2} - \frac{\sqrt{3}}{2} + i\left(\frac{-1}{2} - \frac{\sqrt{3}}{2}\right)$

$z_3 = 5 + 2i + \frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{i}{2} - \frac{i\sqrt{3}}{2}$

$z_3 = \frac{11}{2} - \frac{\sqrt{3}}{2} + i\left(\frac{3}{2} - \frac{\sqrt{3}}{2}\right)$

$\text{Re}(z_3) - \text{Im}(z_3) = \frac{11 - 3}{2} = 4$

(D) Let $z_1 = a + ib$

& $z_2 = c + id$

$\text{Im}(z_1 z_2) = 0$

$\Rightarrow$ ad + bc = 0 $\Rightarrow \frac{a}{b} = -\frac{c}{d}$ ....(i)

$\tan(\arg(z_1 + z_2)) = \frac{b+d}{a+c} = 0 \Rightarrow b = -d$

from (i) a = c

$\therefore\ z_1 = a + ib$

$z_2 = a - ib$

$\therefore\ z_1 = \bar{z}_2 \Rightarrow |z_1 - \bar{z}_2| = 0$

127. Ans. (A)→(P); (B)→(R); (C)→(S,T); (D)→(Q,T)

(A) Let angle between plane and line is θ.

Also $|\sin\theta| = \left|\frac{(\hat{i} - 3\hat{j} + 2\hat{k}).(2\hat{i} + \hat{j} - 3\hat{k})}{14}\right|$

$= \left|\frac{2-3-6}{14}\right| = \frac{1}{2}$

$\therefore\ |\text{cosec}\ \theta| = 2$

(B) $[\vec{a}\ \vec{b}\ \vec{c}] = \frac{4}{7}$

$[2\vec{a} - \vec{b}\ \ 2\vec{b} - \vec{c}\ \ 2\vec{c} - \vec{a}]$

$= \{(2\vec{a} - \vec{b}) \times (2\vec{b} - \vec{c})\}.(2\vec{c} - \vec{a})$

$= \{4(\vec{a} \times \vec{b}) - 2\vec{a} \times \vec{c} + \vec{b} \times \vec{c}\}.(2\vec{c} - \vec{a})$

$= 8[\vec{a}\ \vec{b}\ \vec{c}] - [\vec{b}\ \vec{c}\ \vec{a}] = 7[\vec{a}\,\vec{b}\,\vec{c}]$

$= 7 \times \frac{4}{7} = 4$

(C) $\vec{r} = (\vec{a} \times \vec{b})\sin x + (\vec{b} \times \vec{c})\cos y + 2(\vec{c} \times \vec{a})$

$\vec{r}.(\vec{a} + \vec{b} + \vec{c}) = 0$

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}](\sin x + \cos y + 2) = 0$

$\Rightarrow$ but $[\vec{a}\ \vec{b}\ \vec{c}] \neq 0$

$\Rightarrow \sin x + \cos y + 2 = 0$

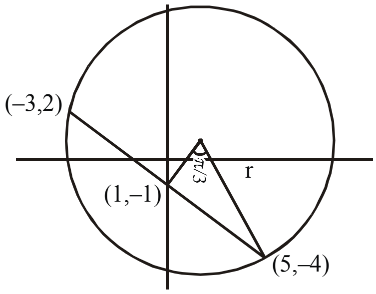
$\Rightarrow \sin x = -1, \cos y = -1.$

$\therefore$ For $x^2 + y^2$ to be minimum,

$x = -\frac{\pi}{2}, y = \pi$

$\therefore\ x^2 + y^2 = \frac{\pi^2}{4} + \pi^2 = \frac{5\pi^2}{4}.$

(D)



$\sin\frac{\pi}{3} = \frac{5}{r} \Rightarrow r = \frac{10\sqrt{3}}{3}$ $\therefore$ p = 3

128. Ans. 712

Suppose that the first term of the sequence ΔA is d then

ΔA = {d, d + 1, d + 2, ... (d + (n – 1)}

hence A = $(a_1, a_1 + d, a_1 + d + (d + 1),$

$a_1 + d + (d + 1) + (d + 2)$..........)

$a_n = a_1 + (n-1)d + \frac{1}{2}(n-1)(n-2)$

so a_n is a quadratic polynomial in n.

so $a_n = \frac{(n-\alpha)(n-\beta)}{2}$

since $a_{19} = a_{92} = 0$ we must have

$a_n = \frac{1}{2}(n-19)(n-92)$

so $a_3 = \frac{1}{2}(3-19)(3-92) = 712$

129. Ans. 45

$36\{x\}^2 = 2[x]\,.\,3x$

$6\{x\}^2 = (x - \{x\})x$

$x^2 - x\{x\} - 6\{x\}^2 = 0$

$(x - 3\{x\})(x + 2\{x\}) = 0$

$\{x\} = \frac{x}{3}$ or $-\frac{x}{2}$

$0 \le \frac{x}{3} < 1$ $\qquad 0 \le -\frac{x}{2} < 1$

$0 \le x < 3$ $\qquad 0 \le -x < 2$

$-2 < x \le 0$ (Rejected)

Case-I : $0 \le x < 1$

$x = \frac{x}{3} \Rightarrow x = 0$ (rejected)

Case-II : $1 \le x < 2$

$x - 1 = \frac{x}{3}$

$\frac{2}{3}x = 1 \Rightarrow x = \frac{3}{2}$

Case-III : $2 \le x < 3$

$x - 2 = \frac{x}{3}$

$\frac{2}{3}x = 2 \Rightarrow x = 3$ (rejected)

$\therefore \quad x = \frac{3}{2}$

Now, given G.P. is 2, 3, $\frac{9}{2}$, ...

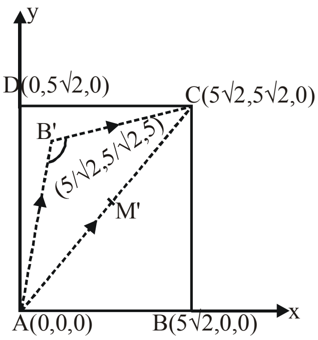
Another G.P. $\frac{1}{4}, \frac{1}{9}, \frac{4}{81}, \ldots\ldots$

$S = \frac{1/4}{1-4/9} = \frac{9}{20}$

$\therefore \quad 100S = 100 \times \frac{9}{20} = 45$

131. Ans. 3

Let $x\hat{i} + y\hat{j} + z\hat{k} = (x, y, z)$



Let M be the mid point of AC

B'M = 5

$\Rightarrow$ coordinates of B' are $\left(\frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}}, 5\right)$

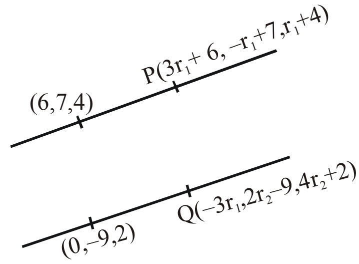
Equation of AB' = $t\left(\frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}}, 5\right)$

equation of CD = $(0, 5\sqrt{2}, 0) + \lambda(5\sqrt{2}, 0, 0)$

Shortest distance between AB and CD

$$\frac{\left|(0, 5\sqrt{2}, 0).\left(\left(\frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}}, 0\right) \times (5\sqrt{2}, 0, 0)\right)\right|}{\left|\left(\left(\frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}}, 0\right) \times (5\sqrt{2}, 0, 0)\right)\right|} = \frac{10}{\sqrt{3}}$$

132 Ans. 4



direction ratio of

$PQ \equiv (-3r_2 - 3r_1 - 6, 2r_2 + r_1 - 16, 4r_2 - r_1 - 2)$

Now, $-3(-3r_2 - 3r_1 - 6) + 2(2r_2, r_1 - 16)$

$+ 4(4r_2 - r_1 - 2) = 0$

$7r_1 + 29r_2 = 22$

$3(-3r_2-3r_1-6) - (2r_2 + r_1 - 16) + (4r_2-r_1-2) = 0$

$7r_1 + 29r_2 = 22$ ...(1)

$3(-3r_2-3r_1-6) - (2r_2 + r_1 - 16) + (4r_2-r_1-2) = 0$

$11r_1 + 7r_2 = -4$ ...(2)

On solving (1) & (2)

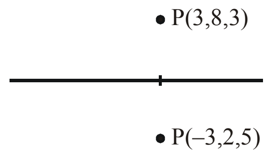
$r_1 = -1, \ r_2 = 1$

so P(3,8,3)

Image of P(3,8,3) w.r.t Plane $3x + 3y - z = 11$

R (–3,2,5)

a + b + c = 4



134 Ans. 1

Clearly, a = 1, b = 2, c = 3.

The second line can be written as

$\frac{x-a}{a} = \frac{y}{0} = \frac{z}{c}$ ....(1)

Now equation of plane through the line

$\frac{y}{b} + \frac{z}{c} = 1$, x = 0 is $\left(\frac{y}{b} + \frac{z}{c} - 1\right) + \lambda x = 0$

or $\lambda x + \frac{y}{b} + \frac{z}{c} = 1$ ....(2)

$\because$ plane (2) is parallel to the line (1)

$\Rightarrow \lambda.a + \frac{1}{6}.0 + \frac{1}{c}.c = 0 \Rightarrow \lambda = -\frac{1}{a}$

$\therefore$ equation of plane Π is

$-\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ or $-\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = 1$

$\therefore$ A(–1, 0, 0), B(0, 2, 0), C(0, 0, 3)

$\therefore$ volume of tetrahedron

$$OABC = \left|\frac{1}{6}[-\hat{i}\ \ 2\hat{j}\ \ 3\hat{k}]\right| = 6.\frac{1}{6}[\hat{i}\ \hat{j}\ \hat{k}] = 1$$

135 Ans. 288

$$V_1 = (\vec{a}+\vec{b}).((\vec{b}+\vec{c})\times(\vec{c}+\vec{a})) = 2[\vec{a}\ \vec{b}\vec{c}]$$

$$V_2 = \frac{1}{6}(\vec{a}\times\vec{b}).[(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})]$$

$$= \frac{1}{6}(\vec{a}\times\vec{b}).[((\vec{b}\times\vec{c}).\vec{a})\vec{c} - ((\vec{b}\times\vec{c}).\vec{c})\vec{a}]$$

$$= \frac{1}{6}[\vec{a}\ \vec{b}\vec{c}]^2$$

$$[\vec{a}\ \vec{b}\vec{c}] = \begin{vmatrix} -3 & 1 & 1 \\ 4 & 2 & 4 \\ 2 & 2 & 0 \end{vmatrix} = 36$$

$\therefore V_1 = 76$ and $V_2 = 216$

136 Ans. 3

Let $O(\vec{O}), A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$

D is $\dfrac{\vec{a}+\vec{b}}{2}$ and E is $\dfrac{\vec{c}}{2}$

$$\overrightarrow{DE}.\overrightarrow{AC} = \left(\frac{\vec{c}-(\vec{a}+\vec{b})}{2}\right).(\vec{c}-\vec{a})$$

$$= \frac{|\vec{c}|^2}{2} - \frac{\vec{c}.\vec{a}}{2} - \frac{\vec{a}.\vec{c}}{2} - \frac{\vec{b}.\vec{c}}{2} + \frac{|\vec{a}|^2}{2} + \frac{\vec{a}.\vec{b}}{2}$$

$$= \frac{1-\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)-\left(\frac{1}{2}\right)+1+\left(\frac{1}{2}\right)}{2} = \frac{1}{2}$$

$\because$ angle between adjacent sides is $\dfrac{\pi}{3}$

137. Ans. 7

$|(\vec{q}-\vec{p})| = 3 \Rightarrow \vec{q}^2+\vec{p}^2-2\vec{p}.\vec{q} = 9$ .....(1)

$|\vec{r}-\vec{q}| = 4 \Rightarrow \vec{q}^2+\vec{r}^2-2\vec{r}.\vec{q} = 16$ .....(2)

$(2)-(1) \Rightarrow (\vec{r}^2-\vec{p}^2)-2\vec{q}(\vec{r}-\vec{p}) = 7$

$(\vec{r}^2-\vec{p}^2).(\vec{r}+\vec{p}-2\vec{q}) = 7$

$\overrightarrow{PR}.(\overrightarrow{QR}+\overrightarrow{QP}) = 7$

138. Ans. 5

Point of intersection of the lines is (0, 1, 1) which lies on the plane P.

D.r. of L_1 is (2, 2, 1) & D.r. of L_2 is (1, 1, –1)

$\Rightarrow$ a vector parallel to P is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -3\hat{i}+3\hat{j}$$

$\therefore$ Normal to the plane $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 3 & 0 \\ 1 & 1 & -1 \end{vmatrix}$

$= 3\hat{i}-3\hat{j}-6\hat{k}$

$\Rightarrow$ Equation of plane is

1.(x – 0) + (y – 1) + 2(z – 1) = 0

$\Rightarrow$ x + y + 2z = 3

$\because$ (a, –2, 0) lies on P $\Rightarrow$ a = 5

139. Ans. 4

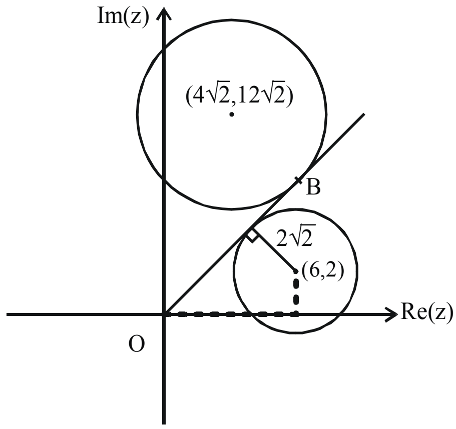
$\hat{a}\times\hat{b}+\hat{c} = \hat{b}-\hat{c}$

which is possible iff

$\hat{b}-\hat{c} = \vec{0} \Rightarrow \hat{b} = \hat{c}$

140. Ans. 8

Let the equation of OAB be y = mx



$$\Rightarrow \left|\frac{6m-2}{\sqrt{1+m^2}}\right| = 2\sqrt{2} \Rightarrow \frac{(3m-1)^2}{(1+m^2)} = 2$$

$$\Rightarrow \frac{9m^2-6m+1}{1+m^2} = 2 \Rightarrow 7m^2-6m-1 = 0$$

$\Rightarrow 7m^2 - 7m + m - 1 = 0$

$7m(m-1) + (m-1) = 0 \Rightarrow m = 1$

$\Rightarrow$ y = x must be tangent to the second circle

$$\Rightarrow \left|\frac{12\sqrt{2}-4\sqrt{2}}{\sqrt{2}}\right| = k \Rightarrow k = 8$$