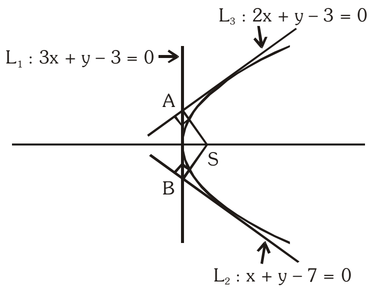
SCORE JEE (Advanced)
HOME ASSIGNMENT # 03

SOLUTION **MATHEMATICS**

1. Ans. (D)



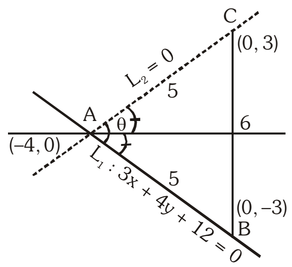
Point of intersection of $L_1 = 0$ & $L_2 = 0$ is $(-2,9)$
Point of intersection of $L_1 = 0$ & $L_3 = 0$ is $(0,3)$
line perpendicular to L_2 and passing through $(-2, 9)$ is $x - y + 11 = 0$ .... (i)
Line perpendicular to L_3 and passing through $(0, 3)$ is $x - 2y + 6 = 0$ .... (ii)
Point of intersection of (i) & (ii) is the focus of the parabola which is $(-16, -5)$.

2. Ans. (C)

$$\theta = \tan^{-1}\left(\frac{24}{7}\right)$$

$$\frac{24}{7} = \left(\frac{m-(-3/4)}{1+m(-3/4)}\right)$$

$$m = \frac{3}{4}$$



$\therefore$ equation of the line $L_2 = 0$ is

$L_2 : 3x - 4y + 12 = 0$

Now, incentre of the ΔABC is

$$\left(\frac{5\times 0+6\times(-4)+5\times 0}{16}, \frac{5\times 3+6\times 0+5\times(-3)}{16}\right)$$

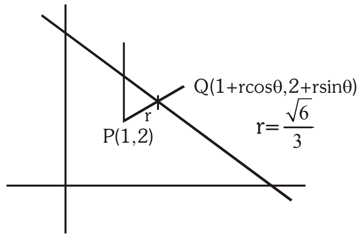
$$\equiv\left(-\frac{3}{2}, 0\right) \equiv (4\lambda, 3k)$$

$$\therefore \quad \lambda = -\frac{3}{8}, \quad k = 0$$

$$10k - 24\lambda = 9$$

4. Ans. (A)

$$1+\frac{\sqrt{6}}{3}\cos\theta + 2 + \frac{\sqrt{6}}{3}\sin\theta = 4$$



$$\cos\theta + \sin\theta = \frac{3}{\sqrt{6}}$$

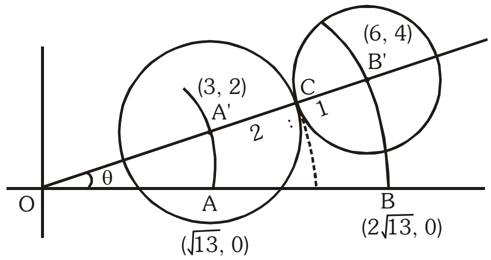
$$\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{\sqrt{3}}{2}$$

$$\sin(\theta + 45^\circ) = \sin 60^\circ$$

$\theta + 45^\circ = 60^\circ$ or $\theta + 45^\circ = 120^\circ$

$\theta = 15^\circ$ or $\theta = 75^\circ$

6. Ans. (A)



$$\theta = \tan^{-1}\frac{2}{3} \quad \Rightarrow \quad \tan\theta = \frac{2}{3}$$

$$\sin\theta = \frac{2}{\sqrt{13}} \quad \& \quad \cos\theta = \frac{3}{\sqrt{13}}$$

$A' \equiv (OA\cos\theta, OA\sin\theta) \equiv (3, 2)$

Similarly

$B' \equiv (OB\cos\theta, OB\sin\theta) \equiv (6, 4)$

It can be checked that C_1 & C_2 touches each other. Let the point of contact be C.

$$C \equiv \left(\frac{3\times 1+2\times 6}{3}, \frac{2\times 1+2\times 4}{3}\right)$$

$$C \equiv \left(5, \frac{10}{3}\right)$$

Required Radical axis is a line perpendicular to A'B' and passing through point C

$$\left(y - \frac{10}{3}\right) = -\frac{3}{2}(x-5)$$

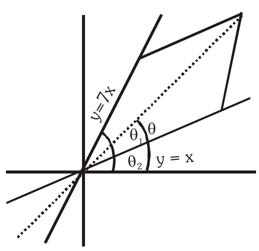
7. Ans. (B)

$$\theta_1 - \theta = \theta - \theta_2$$

$$2\theta = \theta_1 + \theta_2$$

$$\tan 2\theta = \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1\tan\theta_2}$$

$$= \frac{7+1}{1-7.1} = -4/3$$



$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{-4}{3} \Rightarrow 2\tan^2\theta - 3\tan\theta - 2 = 0$

$\Rightarrow \quad (2\tan\theta + 1)(\tan\theta - 2) = 0$

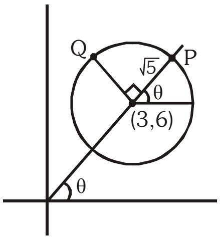
$\Rightarrow \quad \tan\theta = 2$ or $-1/2$

$\therefore$ Slope of longer diagonal is 2.

8. Ans. (A)

$x^2 + y^2 - 6x - 12y + 40 = 0$

$(x-3)^2 + (y-6)^2 = 5$



co-ordinates of P are

$(3 + \sqrt{5}\cos\theta, 6 + \sqrt{5}\sin\theta)$

where $\tan\theta = \frac{6}{3} = 2$

$\therefore \quad P\left(3 + \sqrt{5}.\frac{1}{\sqrt{5}}, 6 + \sqrt{5}.\frac{2}{\sqrt{5}}\right)$

$\Rightarrow \quad P(4, 8)$

After rotation of 90° in anti clockwise sense co-ordinates of Q are

$(3 + \sqrt{5}\cos(90 + \theta), 6 + \sqrt{5}\sin(90 + \theta))$

$\Rightarrow \quad \left(3 - \sqrt{5}\sin\theta,\ 6 + \sqrt{5}\cos\theta\right)$

$\Rightarrow \quad \left(3 - \sqrt{5} \times \frac{2}{\sqrt{5}}, 6 + \sqrt{5}.\frac{1}{\sqrt{5}}\right) \Rightarrow (1, 7)$

13. Ans. (C)

Case-I : when lines are concurrent

$$\therefore \begin{vmatrix} 2 & 1 & -5 \\ 1 & 1 & -2 \\ b & 3 & -1 \end{vmatrix} = 0$$

$\Rightarrow \quad 2(-1 + 6) - 1(-1 + 2b) - 5(3 - b) = 0$

$\Rightarrow \quad 10 + 1 - 2b - 15 + 5b = 0$

$\Rightarrow \quad 3b = 4 \quad \Rightarrow \quad b = \frac{4}{3}$

Case-II : when $L_1 = 0$ & $L_3 = 0$ are parallel

$\frac{b}{2} = \frac{3}{1} \Rightarrow b = 6$

Case-III : when $L_2 = 0$ & $L_3 = 0$ are parallel

$\frac{b}{1} = \frac{3}{1} \Rightarrow b = 3$

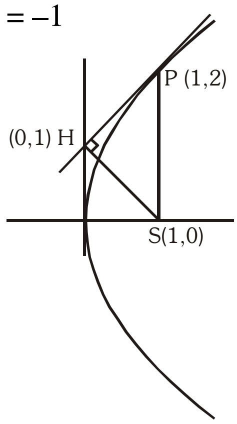
$\therefore$ Sum of values of b is $\frac{4}{3} + 6 + 3 = \frac{31}{3}$

14. Ans. (C)

Tangent at P is $2y = 2(x + 1)$

Putting $x = 0$, we get $y = 1$

Slope of HP × slope of SH = –1



$\angle SHP = \frac{\pi}{2}$

$\Rightarrow$ SP is diameter

so SP = 2

r = 1

$A = \pi$

[A] = 3

15. Ans. (C)

$\frac{x^2}{1} - \frac{y^2}{2} = 1$

The equation of the normal to this curve at $(\sec\theta, \sqrt{2}\tan\theta)$ is

$x\cos\theta + \sqrt{2}y\cot\theta = 3$ ....(i)

Let the foot of perpendicular from centre upon the normal be (h, k).

equation of normal can also be written as

$hx + ky = h^2 + k^2$ ....(ii)

comparing equation (i) & (ii)

$\frac{\cos\theta}{h} = \frac{\sqrt{2}\cot\theta}{k} = \frac{3}{h^2 + k^2}$

$\Rightarrow \quad \sec\theta = \frac{h^2 + k^2}{3h}, \ \tan\theta = \frac{\sqrt{2}(h^2 + k^2)}{3k}$

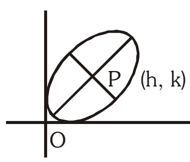
$\Rightarrow \quad \frac{(h^2 + k^2)^2}{9h^2} - \frac{2(h^2 + k^2)^2}{9k^2} = 1$

$(x^2 + y^2)^2(y^2 - 2x^2) = 9x^2y^2$

$\Rightarrow \quad k_1 = 1, k_2 = 2, k_3 = 3$

$k_1 + k_2 + k_3 = 6$

22. Ans. (B)



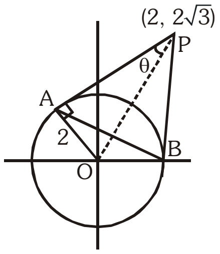
Let the perpendicular lines are xy axis and centre is (h, k) then O lies on its director circle it means OP is constant

$\sqrt{h^2 + k^2} = c \Rightarrow x^2 + y^2 = c^2$ is circle.

24. Ans. (A)

OP = 4

$\Rightarrow \sin\theta = \frac{1}{2}, \theta = \frac{\pi}{6}$



$\Rightarrow$ ΔAPB is equilateral triangle

$\Rightarrow AP = \sqrt{4+12-4} = 2\sqrt{3}$

Area (ΔAPB) = $\frac{\sqrt{3}}{4} \times 12 = 3\sqrt{3}$

29. Ans. (A)

Let $yt = x + 2t^2$ be the tangent passing through (–2, 3)

$\therefore 3t = -2 + 2t^2 \Rightarrow 2t^2 - 3t - 2 = 0$

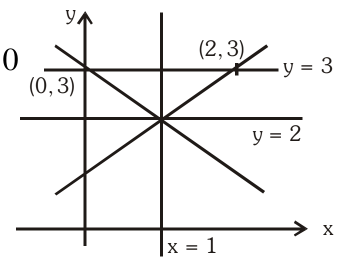
$t_1 + t_2 = \frac{3}{2}$

$\therefore 4t_1 + 4t_2 = 6$

30. Ans. (D)

$x(y-2) - (y-2) = 0$

$(x-1)(y-2) = 0$



Bisectors

$(y-2) = (x-1)$

$\Rightarrow x - y = -1$

$(y-2) = -(x-1)$

$\Rightarrow x + y = 3$

Required circle is $x(x-2) + (y-3)(y-3) = 0$

$(x-1)^2 + (y-3)^2 = 1$

31. Ans. (D)

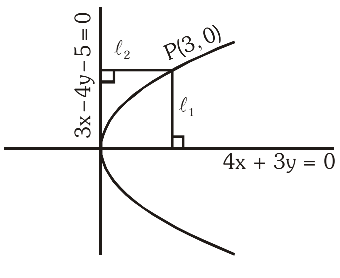
$3\ell = n \Rightarrow (3, 0)$ lies on the given line

$\Rightarrow$ focus lies on directrix

$\Rightarrow$ conic as a pair of straight lines.

32. Ans. (B)

$4a = \text{L. R.} = \frac{\ell_1^2}{\ell_2}$



$= \frac{\left(\frac{12}{5}\right)^2}{\frac{4}{5}} = \frac{36}{5}$

$\therefore$ Distance between focus & directrix $= 2a = 3.6$

34. Ans. (A)

$xy - x - y + 1 = 0$ .... (i)

$(x-1)(y-1) = 0$

$\Rightarrow$ The given pair of lines of which (i) is bisector is equally inclined to co-ordinate axis.

$\Rightarrow h = 0.$

35. Ans. (C)

Put y = 0, we get $x^2 + 4x + c^2 = 0$, which gives equal roots if $16 - 4c^2 = 0 \Rightarrow c^2 = 4$.

$\therefore$ Equation becomes

$x^2 + 4xy - 2y^2 + 4x + 2fy + 4 = 0$

which represents a pair of straight lines if

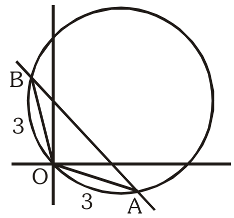
$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow f = 4$

36. Ans. (B)

$S : (x-3)^2 + (y-4)^2 = 25$

$S : x^2 + y^2 - 6x - 8y = 0$



$\because$ Origin lies on the circle. A & B are points on the circle at a distance 3 from origin. Equation of another circle whose centre is (0, 0) and radius is 3, is $S' : x^2 + y^2 = 9$

$\therefore$ chord AB is, $S - S' = 0 \Rightarrow 6x + 8y = 9$

41. Ans. (B)

$\sqrt{a} + \sqrt{d} = \sqrt{c} + \sqrt{b}$

$\Rightarrow a + d + 2\sqrt{ad} = c + b + 2\sqrt{bc}$

$\Rightarrow a + d = c + b$

$\Rightarrow a^2 + d^2 + 2ad = b^2 + c^2 + 2bc$

$\Rightarrow a^2 + d^2 = b^2 + c^2$

Now the given line is

$(a^2 + d^2)x + b^2y + c^2 = 0$

$(b^2 + c^2)x + b^2y + c^2 = 0$

$b^2(x + y) + c^2(x + 1) = 0$

hence given family of lines will pass through the point of intersection of the lines x + y = 0 and x + 1 = 0 i.e. (–1, 1)

43. Ans. (C)

Coordinates of any points lying on the line $y = \sqrt{3}x$ will be $\left(\frac{r}{2}, \frac{r\sqrt{3}}{2}\right)$

If the given line intersects the curve $x^4 + ax^2y + bxy + cx + dy + 6 = 0$, then

$\frac{r^4}{16} + a\frac{r^3\sqrt{3}}{8} + \frac{br^2\sqrt{3}}{4} + \frac{cr}{2} + \frac{dr\sqrt{3}}{2} + 6 = 0$

$r^4 + r^3 2a\sqrt{3} + r^2 4b\sqrt{3} + r8(c + d\sqrt{3}) + 96 = 0$

$\therefore \quad r_1 r_2 r_3 r_4 = 96$

hence OA . OB . OC . OD = 96

45. Ans. (D)

Family of lines

$x + y - 1 = 0$ and $2x + 3y - 2 = 0$

will passes through the point of intersection of

$x + y - 1 = 0$ and $2x + 3y - 2 = 0$

i.e. (1, 0)

According to given condition the second family of line will also pass through the point (1, 0)

$a - 2 = 0 \quad \Rightarrow \quad a = 2$

$b - 5 = 0 \quad \Rightarrow \quad b = 5$

centroid of the given triangle is (3, 7) hence centroid of the triangle is (b – a, a + b)

47. Ans. (A)

Let the equation y = mx + c be the common tangent to the curve $y^2 = 8x$ and $x^2 + y^2 = 2$ then c = 2/m and $c^2 = 2(1 + m^2)$

$\Rightarrow \frac{4}{m^2} = 2(1 + m^2) \Rightarrow m^4 + m^2 - 2 = 0$

$\Rightarrow (m^2 + 2)(m^2 - 1) = 0 \Rightarrow m^2 = 1 \Rightarrow m = \pm 1$

hence there will be two common tangents which are perpendicular to each other.

49. Ans (B)

Let the curve $y = x^2 + px + q$ cuts the x-axis at $A(\alpha_1, 0)$ and $B(\alpha_2, 0)$ then

$\alpha_1 + \alpha_2 = -p$, $\alpha_1\alpha_2 = q$ also

$y = x^2 + px + q$ cuts y-axis at C (0, q)

Now equation of family of circles passing through A and B is

$(x - \alpha_1)(x - \alpha_2) + y^2 + \lambda y = 0$

this circle also passes through point C(0, q)

so $\lambda = -(1 + q)$

so the circle is

$x^2 + y^2 + px + q - (1 + q)y = 0$

which always passes through (0, 1)

54. Ans. (D)

Given hyperbola is $\frac{x^2}{16} - \frac{y^2}{48} = 1$

Equation of its director circle will be

$x^2 + y^2 = -32$

which is not possible, so no point exist from which two perpendicular tangents can be drawn to the hyperbola.

55. Ans. (D)

The normal at $(a\cos\alpha, b\sin\alpha)$ will be

$$\frac{ax}{\cos\alpha} - \frac{by}{\sin\alpha} = a^2 - b^2$$

If passes through $(a\cos\beta, b\sin\beta)$

$$\frac{a^2\cos\beta}{\cos\alpha} - \frac{b^2\sin\beta}{\sin\alpha} = a^2 - b^2$$

$$\frac{\cos\beta}{\cos\alpha} - 1 = \frac{b^2}{a^2}\left(\frac{\sin\beta}{\sin\alpha} - 1\right)$$

$$\Rightarrow \frac{\cos\beta - \cos\alpha}{\cos\alpha} = \frac{b^2}{a^2}\left(\frac{\sin\beta - \sin\alpha}{\sin\alpha}\right)$$

$$\Rightarrow \frac{2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}}{\cos\alpha} = \frac{b^2}{a^2}\frac{2\cos\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}}{\sin\alpha}$$

$$\Rightarrow \tan\alpha \tan\left(\frac{\alpha+\beta}{2}\right) = -\frac{b^2}{a^2}$$

58. Ans. (C)

Any tangent to $y = x^2$ can be written as

$y = mx - \frac{1}{4}m^2$ if it touches 2^{nd}, then

$$mx - \frac{m^2}{4} = x^2 - 2x + 2$$

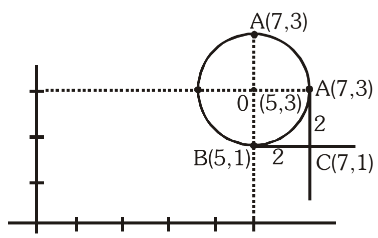
$$\Rightarrow x^2 - (2+m)x + \left(2 + \frac{m^2}{4}\right) = 0$$

$$D = 0 \quad \Rightarrow \quad (m+2)^2 - 4\left(2 + \frac{m^2}{4}\right)$$

$4m - 4 = 0 \Rightarrow m = 1 \Rightarrow 4x - 4y - 1 = 0$

59. Ans. (A,C,D)

$(x - 5)^2 + (y - 3)^2 = 4$



Option (A) : $A(\square OACB) = 4$

Option (B) :

$$S \equiv (x^2 + y^2 - 10x - 6y + 30) + \lambda(2 - y - 4) = 0$$

Obviously radical axis is x–y=4

Option (B) is incorrect

Option (C) : $(x-5)(x-7)+(y-1)(y-3)=0$

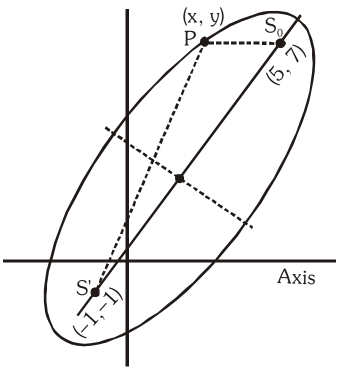
{The required circles diameter is AB}

$x^2+y^2-12x-4y+38=0$

Option (C) is correct

Option (D) is obviously correct

61. Ans. (A,C)



$\sqrt{(x-5)^2+(y-7)^2}+\sqrt{(x+1)^2+(y+1)^2}=12$

PS + PS' = 2a $\Rightarrow$ ellipse

$2ae=\sqrt{36+64}$

$2a=12 \Rightarrow e=\frac{5}{6}$

axis $(y-7)=\frac{7+1}{5+1}(x-5) \Rightarrow 3(y-7)=4(x-5)$

$\Rightarrow 4x-3y+1=0$

63. Ans. (A,D)

$S:(x-2)^2+(y+3)^2=0$

$\therefore$ S is a point circle which represents point (2, –3) and this point (2, –3) also lies on the line

$L:2x+5y+11=0$

Equation of tangents from (2, –3) to the circle

$x^2+y^2=\frac{121}{29}$ is $y+3=m(x-2)$

$\Rightarrow mx-y=3+2m$

Applying p = r

$$\left|\frac{3+2m}{\sqrt{1+m^2}}\right|=\sqrt{\frac{121}{29}}$$

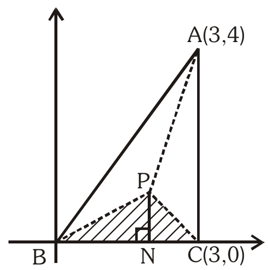
$\Rightarrow 29(9+12m+4m^2)=121(1+m^2)$

$\Rightarrow 5m^2-348m-140=0$

$\therefore m_1+m_2=\frac{348}{5}$

& $m_1m_2=-\frac{140}{5}=-28$

64. Ans. (A,C)



$d(P,BC)\le \min.\{d(P,AB),d(P,AC)\}$

The region represented is shown by shaded region,

where BP and CP are angle bisectors

Clearly maximum of d(P, BC) occurs, when P is incentre of ΔABC

$\therefore$ Maximum of d(P, BC) = PN = ordinate of incentre P = 1

67. Ans. (B,C)

For concurrency

$$\begin{vmatrix}1 & 1 & -1\\ m-1 & m^2-7 & -5\\ m-2 & 2m-5 & 0\end{vmatrix}=0$$

$\Rightarrow 5(2m-5)-5(m-2)-[(m-1)(2m-5)-(m-2)(m^2-7)]=0$

$\Rightarrow 10m-25-5m+10-2m^2+7m-5+m^3-2m^2-7m+14=0$

$\Rightarrow m^3-4m^2+5m-6=0$

$\Rightarrow (m-3)(m^2-m+2)=0$

m = 3 is the only real root

But for m = 3 the given line becomes parallel. Thus the given lines can only be parallel for m = 3.

68. Ans. (A,B)

As the given triangle is isosceles, so the angle bisector of two equal sides will be perpendicular to third side.

Now for angle bisector

$$\frac{7x+3y-20}{\sqrt{58}}=\pm\frac{3x+7y-20}{\sqrt{58}}$$

$(7x+3y-20)=\pm(3x+7y-20)$

so the equation of angle bisectors are $x-y=0$, $x+y-4=0$, so the slopes of the lines perpendicular to these lines will be –1, 1 so equation of third line will be

$y - 3 = (x + 3)$ or $y - 3 = -(x + 3)$

$x - y + 6 = 0$ or $x + y = 0$

69. Ans.(A,B,C,D)

Let $f(x) = x^2 - 6x - a^2$

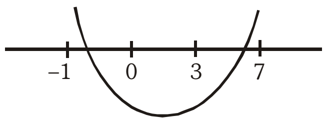
and as (a, 0) lies on the diameter of the circle so maximum value of a^2 can be 4

Now $f(-1) = 1 + 6 - a^2 > 0$

$f(0) = -a^2 < 0$

$f(3) = 9 - 18 - a^2 < 0$

$f(7) = 49 - 42 - a^2 > 0$



so the equation $x^2 - 6x - a^2 = 0$ has one root between –1 and 0 & one root betwen 3 and 7.

70. Ans.(A,B)

Any point on the line x + y = 1 can be taken as $(\lambda, 1 - \lambda)$ so equation of the chord taking this point as mid point will be

$$\lambda x - 2(y + 1 - \lambda) = \lambda^2 - 4a(1 - \lambda)$$

It passes through (2a, a)

$2a\lambda - 2a(a + 1 - \lambda) = \lambda^2 - 4a(1 - \lambda)$

$\Rightarrow \lambda^2 - 2a + 2a^2 = 0$

It has two distinct root then

$-(-2a + 2a^2) > 0$

$a^2 - a < 0 \Rightarrow 0 < a < 1.$

length of latus rectum can be between 0 and 4.

76. Ans. (A,C)

Intersection point of circle $x^2 + y^2 = 9$ and line $y = 2x$ will be $\left(\frac{3}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right)$ and $\left(\frac{3}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right)$

Coordinates of the point of intersection of perpendicular drawn from these points to the major

axis of ellipse and ellipse will be $\left(\frac{3}{\sqrt{5}}, -\frac{4}{\sqrt{5}}\right)$,

$\left(\frac{3}{\sqrt{5}}, -\frac{4}{\sqrt{5}}\right)$, $\left(-\frac{3}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$, $\left(-\frac{3}{\sqrt{5}}, -\frac{4}{\sqrt{5}}\right)$

Equation of tangents at these points will be

$x + 3y - 3\sqrt{5} = 0$, $x - 3y - 3\sqrt{5} = 0$

$x - 3y + 3\sqrt{5} = 0$ and $x + 3y + 3\sqrt{5} = 0$

77. Ans. (B,C,D)

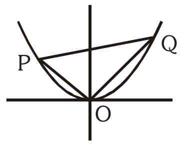
It is locus of P(x, y) moving such that (PS – PS') = 4 < SS', where S(4, 2) and S'(–4, –2). So the locus is hyperbola with S, S' as foci. Centre is mid point of SS'.

$2a = 4,\ 2ae = 4\sqrt{5} \Rightarrow e = \sqrt{5}$

auxiliary circle is concentric circle with radius 'a' so $x^2 + y^2 = 4$.

79. Ans.(A,B,C)

Let $P(2t_1, t_1^2)$, $Q(2t_2, t_2^2)$



$m_{OP}m_{OQ} = -1 \Rightarrow \frac{t_1}{2}\cdot\frac{t_2}{2} = -1$

$\Rightarrow t_1t_2 = -4$

Let centroid G(h, k)

so $h = \frac{2t_1 + 2t_2}{3}$, $k = \frac{t_1^2 + t_2^2 + 1}{3}$

$3k - 1 = (t_1 + t_2)^2 - 2t_1t_2$

$\Rightarrow 3k - 1 = \left(\frac{3h}{2}\right)^2 + 8$

$\Rightarrow 3(k - 3) \cdot 4 = 9h^2 \Rightarrow 3x^2 = 4(y - 3)$

so vertex (0, 3), LR = $\frac{4}{3}$,

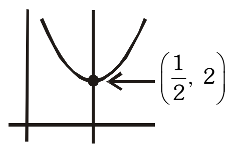
axis x = 0, tangent at vertex y = 3

Paragraph for Question 80 to 82

$4x^2 - 4x + 1 = 24y - 48$

$\Rightarrow (2x - 1)^2 = 24(y - 2)$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 = 6(y - 2)$



80. Directrix $y - 2 = -\frac{3}{2} \Rightarrow y = \frac{1}{2}$

Focus $\left(\frac{1}{2}, 2 + \frac{3}{2}\right)$ LR = 6

81. Let mid point is (h, k) so chord is $T = S_1$

$\Rightarrow 4hx - 2(x + h) - 12(y + k) + 49$

$= 4h^2 - 4h - 24k + 49$

as it passes through origin it will satisfy (0, 0) so $-2h - 12k = 4h^2 - 4h - 24k$

$\Rightarrow \quad 4x^2 - 2x - 12y = 0$

or $\quad 6y = 2x^2 - x$ which is a parabola.

82. L_1 L_2 must be in the form of $y = mx$

solving it with $S = 0$

$4x^2 - 4x - 24mx + 49 = 0$

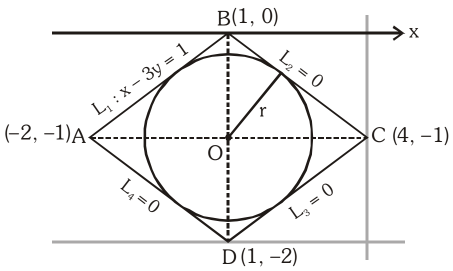
$\Rightarrow \quad 4x^2 - 4x(1 + 6m) + 49 = 0$

by $D = 0 \Rightarrow 16(6m + 1)^2 - 16.49 = 0$

$6m + 1 = \pm 7 \quad \Rightarrow \quad m = 1 \text{ or } -\frac{4}{3}$

$m_1 + m_2 = -\frac{1}{3}$

Paragraph for Question 83 to 85



83. After I[st] reflection, equation of $L_2 = 0$ is

Slope $= -1/3$ and point $(1, 0)$

$y - 0 = -\frac{1}{3}(x - 1) \qquad \Rightarrow \quad x + 3y = 1$

point of incidence on the mirror $x - 4 = 0$ is $(4, -1)$. Equation of $L_3 = 0$ is $y + 1 = \frac{1}{3}(x - 4)$

$x - 3y = 7$

again point of incidence on the line mirror $y + 2 = 0$ is $(1, -2)$

equation of $L_4 = 0$ is

$$y + 2 = -\frac{1}{3}(x - 1)$$

$$x + 3y + 5 = 0$$

84. Closed figure is rhombus

Area $= 4 \times \frac{1}{2} \times 3 \times 1 = 6$ sq. units

85. Centre of the circle is $(1, -1)$ and

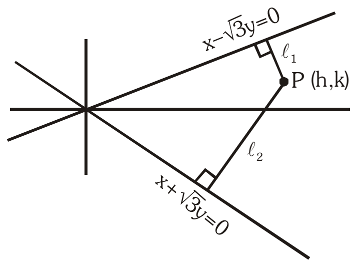
$$r = \left|\frac{1 - 3 - 1}{\sqrt{1 + 9}}\right| = \frac{3}{\sqrt{10}}$$

$\therefore$ equation of circle is $(x - 1)^2 + (y + 1)^2 = \frac{9}{10}$

Paragraph for Question 86 to 88

86. **Ans. (C)**

Equation of line is $(x - \sqrt{3}y)(x + \sqrt{3}y) = 0$



Let point P is (h, k)

$$\therefore \quad \left(\frac{h - \sqrt{3}k}{2}\right)^2 + \left(\frac{h + \sqrt{3}k}{2}\right)^2 = 12$$

$2h^2 + 6k^2 = 48$

$\frac{x^2}{24} + \frac{y^2}{8} = 1 = \frac{x^2}{a^2} + \frac{y^2}{b^2} \qquad \Rightarrow \quad a^2 = 24, \ b^2 = 8$

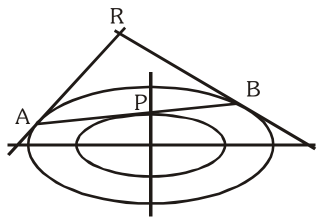
$e^2 = 1 - \frac{8}{24} \Rightarrow e = \sqrt{\frac{2}{3}}$

87. **Ans. (A)**

$C_2 : bx^2 + ay^2$

$= a^2b + ab^2$

$$\frac{x^2}{a(a + b)} + \frac{y^2}{b(a + b)} = 1$$



Equation of AB is $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$ ......(i)

let $R \equiv (h, k)$

AB is chord of contact to curve C_2.

equation of AB is $\frac{xh}{a(a + b)} + \frac{yk}{b(a + b)} = 1$ ......(ii)

from (i) & (ii)

$$\frac{h}{a(a + b)} = \frac{\cos\theta}{a}, \ \frac{k}{b(a + b)} = \frac{\sin\theta}{b}$$

$\Rightarrow \quad h^2 + k^2 = (a + b)^2$

$\Rightarrow \quad OR = (a + b) = 2\sqrt{6} + 2\sqrt{2}$

88. **Ans. (D)**

Standard property of ellipse.

Paragraph for Question 89 to 91

Let the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

If passes through (1, 0)

$\therefore \quad 1 + 2g + c = 0$ .... (i)

Let $(t^2, 2t)$ be the point of intersection of two curves

$\therefore$ equation of tangent is

$yt = x + t^2$ .... (ii)

$\because$ Circle & parabola cuts orthogonaly

$\therefore$ (ii) passes through the centre of the circle

$\therefore$ $-ft = t^2 - g$

$ft = g - t^2$ .... (iii)

$(t^2, 2t)$ also satisfies equation of circle

$\therefore$ $t^4 + 4t^2 + 2gt^2 + 4ft + c = 0$

Putting the value of c & ft in above equation

$t^4 + 4t^2 + 2gt^2 + 4g - 4t^2 - 1 - 2g = 0$

$t^4 + 2gt^2 + 2g - 1 = 0$

$(t^4 - 1) + 2g(t^2 + 1) = 0$

$t^2 = 1 - 2g$

squaring equation (iii) we get

$f^2t^2 = (g - t^2)^2$

$\Rightarrow f^2(1 - 2g) = (3g - 1)^2$ {Putting the value of t^2}

replacing $-g$ by x & $-f$ by y we get,

$y^2(1 + 2x) = (1 + 3x)^2$

89. **Ans. (D)**

$\Rightarrow$ $a = 1, b = 2, c = 3$

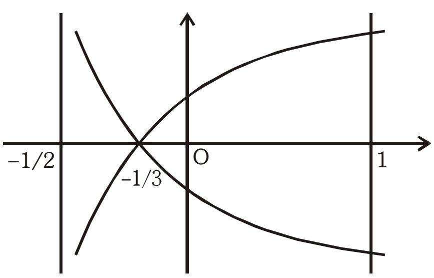
$\Rightarrow$ $a + b + c = 6$

90. **Ans. (B)**

$y^2 = \frac{(1+3x)^2}{(1+2x)} \Rightarrow y = \pm\frac{(1+3x)}{\sqrt{1+2x}}$

Consider $y = \frac{1+3x}{\sqrt{1+2x}}$

$y' = \frac{3x+2}{(1+2x)^{3/2}} > 0 \ \forall \ x > -\frac{1}{2}$



$A = 2\int_{-1/3}^{1} \frac{1+3x}{\sqrt{1+2x}}dx$

Put $1 + 2x = t^2$

$dx = tdt$

Solving we get $A = \frac{20\sqrt{3}}{9}$

91. **Ans. (C)**

Paragraph for Question 92 to 94

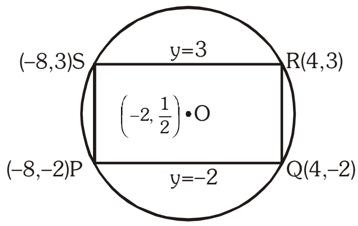
Let O(h, k) be the centre of the circle C

$\Rightarrow$ $2k - 4h = 9$ .... (i)

Also OP = OQ = Radius of the circle

$\Rightarrow$ $OP^2 = OQ^2$ or $h + 8 = \pm(h - 4)$

$\Rightarrow$ $h + 8 = -h + 4$ $\Rightarrow$ $h = -2$



Putting in (i), $k = \frac{1}{2}$

$\therefore$ Radius = OP = $\frac{13}{2}$

$\therefore$ PR = 13

In ΔPQR, $PR^2 = PQ^2 + QR^2$

$\Rightarrow$ $169 = 144 + QR^2 \Rightarrow QR = 5$

Also co-ordinates of R & S are respectively (4,3) & (–8, 3)

92. **Ans. (C)**

Area of Rectangle PQRS = $12 \times 5 = 60$ sq.units

93. **Ans. (B)**

Now $C_1 : (x - 4)(x + 8) + (y - 3)(y - 3) = 0$

or $x^2 + y^2 + 4x - 6y - 23 = 0$

centre (–2, 3) & Radius = 6

Equation of tangent through P to C_1 is

$y + 2 = m_1 (x + 8)$

$\Rightarrow \left|\frac{6m_1 - 5}{\sqrt{1+m_1^2}}\right| = 6 \Rightarrow m_1 = -\frac{11}{60}$

Similarly, equation of tangent through Q to C_1 is $y + 2 = m_2 (x - 4)$

$\Rightarrow \left|\frac{-6m_2 - 5}{\sqrt{1+m_2^2}}\right| = 6 \Rightarrow m_2 = \frac{11}{60}$

$\therefore |m_1| + |m_2| = \frac{11}{30}$

94. **Ans. (D)**

The line through (7, 5) at maximum distance from centre O of circle C will be the line perpendicular to line joining (7, 5) & point O.

$\Rightarrow$ slope of required line $= \dfrac{-1}{\dfrac{5-1/2}{7+2}} = -2$

Paragraph for Question 98 to 100

98. Ans. (A)

Clearly L_1 will pass through intersection point of B_1 & B_2. So its equation will be

$(3x + 4y - 10) + \lambda(4x - 3y - 5) = 0$

as L_1 passes through origin so $\lambda = -2$

hence equation of L_1

$-5x + 10y = 0$

$x = 2y$

99. Ans. (B)

as B_1 is equally inclined to L_1 & L_2.

Let slope of L_2 is m then

$$\frac{m-\frac{4}{3}}{1+\frac{4m}{3}} = \frac{\frac{4}{3}-\frac{1}{2}}{1+\frac{4}{3}\times\frac{1}{2}} \Rightarrow \frac{3m-4}{3+4m} = \frac{1}{2}$$

$\Rightarrow$ $6m - 8 = 3 + 4m$

$m = \dfrac{11}{2}$

also L_2 will pass through point of intersection of B_1 & B_2. i.e (2, 1) so equation of L_2 is $y - 1 = \dfrac{11}{2}(x-2)$

$2y - 2 = 11x - 22 \Rightarrow 11x - 2y = 20$

100. Ans. (C)

Joint equation of the lines joining A & B to origin is

$$y^2 - 4ax\left(\frac{11x-20y}{20}\right) = 0$$

$20y^2 - 44ax^2 + 80axy = 0$

As AB subtends 90° at origin. So $20 - 44a = 0$

$\Rightarrow$ $a = \dfrac{5}{11}$

Paragraph for Question 101 to 103

Let equation of straight line through O is

$$\frac{x}{\cos\theta} = \frac{y}{\sin\theta} = r$$

Let $OL = r_1$ and $OM = r_2$, then the coordinates of L and M are given by

$L(r_1\cos\theta, r_1\sin\theta)$ and $m(r_2\cos\theta, r_2\sin\theta)$.

Let N(h, k) be the variable point such that $ON = r_3$, then $h = r_3\cos\theta$, $k = r_3\sin\theta$, since L and M lie on $3x + 4y - 5 = 0$ and $x + 2y - 3 = 0$ so

$r_1(3\cos\theta + 4\sin\theta) = 5$ and $r_2(\cos\theta + 2\sin\theta) = 3$

$\Rightarrow$ $\dfrac{r_1}{r_3} = \dfrac{5}{3h+4k}$ and $\dfrac{r_2}{r_3} = \dfrac{3}{h+2k}$

101. Ans. (A)

Now ON is the A.M. of OL and OM

so $r_3 = \dfrac{r_1+r_2}{2} \Rightarrow \dfrac{r_1}{r_3} + \dfrac{r_2}{r_3} = 2$

$$\frac{4}{3h+4k} + \frac{3}{h+2k} = 2$$

$5(h + 2k) + 3(3h + 4k) = 2(3h + 4k)(h + 2k)$

$\Rightarrow$ $6h^2 + 16k^2 + 20hk - 14h - 22k = 0$

hence locus of point N is

$3x^2 + 8y^2 + 10xy - 7x - 11y = 0$

102. Ans. (C)

Now $\dfrac{r_1}{r_2} = \dfrac{5}{3}$

$$\frac{5}{(3\cos\theta+4\sin\theta)} \times \frac{(\cos\theta+2\sin\theta)}{3} = \frac{5}{3}$$

$\Rightarrow$ $\cos\theta + 2\sin\theta = 3\cos\theta + 4\sin\theta$

$\Rightarrow$ $2\cos\theta + 2\sin\theta = 0 \Rightarrow \tan\theta = -1$

$\therefore$ equation of L_1 will be $x + y = 0$

103. Ans. (B)

$$\frac{3x+4y-5}{5} = \pm\frac{x+2y-3}{\sqrt{5}}$$

Now $3(2) + 4(3) - 5 > 0$ and $2 + 2(3) - 3 > 0$ so '+' sign bisector will contain the point (2, 3)

$3x + 4y - 5 = \sqrt{5}x + 2\sqrt{5}y - 3\sqrt{5}$

$(3 - \sqrt{5})x + 2(2 - \sqrt{5})y + \sqrt{5}(3 - \sqrt{5}) = 0$

Paragraph for Question 107 to 109

Coordinates of A, B and C are A(1,1), B(5, –1), C(–1, 5). Now equation of the circles taking AB, AC and BC as diameter respectively will be

$S_1 \equiv (x - 1)(x - 5) + (y - 1)(y + 1) = 0$

$\Rightarrow x^2 + y^2 - 6x + 4 = 0$

$S_2 \equiv (x-1)(x+1)+(y-1)(y-5)=0$

$\Rightarrow x^2+y^2-6y+4=0$

$S_3 \equiv (x-5)(x+1)+(y+1)(y-5)=0$

$\Rightarrow x^2+y^2-4x-4y-10=0$

107. Ans.(C)

Equation of radical axis of S_1 and S_2 will be

$L_{12} \equiv S_1 - S_2 = 0$

$\Rightarrow x - y = 0$

108. Ans. (B)

Vertex of the parabola is (1, 1) and directrix is $x + 1 = 0$

Focus of this parabola will be (3, 1), it will lie on side BC.

109. Ans.(C)

Equation of radical axis of S_2 and S_3 will be

$L_{23} \equiv 4x - 2y + 14 = 0$

$\Rightarrow 2x - y + 7 = 0$

clearly radical centre will be the point of intersection of $L_{12} = 0$ & $L_{23} = 0$

i.e., (–7, –7).

Paragraph for Question 110 to 112

110. Ans.(B)

Equation of family of circles passing through A & B is given by

$(x-3)(x-6)+(y-7)(y-5)+\lambda(2x+3y-27)=0$

$\Rightarrow x^2+y^2+(2\lambda-9)x+(3\lambda-12)y+53-27\lambda=0$

Equation of the common chord of (i) and C is

$(2\lambda-9+4)x+(3\lambda-12+6)y+53-27\lambda+3=0$

$(5x+6y-56)-\lambda(2x+3y-27)=0$

which passes through the point of intersection of $5x+6y-56=0$ and $2x+3y-27=0$

i.e. (2, 23/3)

111. Ans. (C)

Centre (2,3) will lie on the common chord so $-28+14\lambda=0 \Rightarrow \lambda=2$

so equation of required circle will be

$x^2+y^2-5x-6y-1=0$

112. Ans. (C)

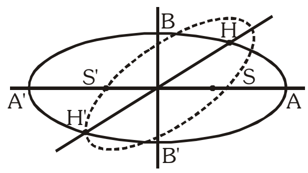
Difference of the square of the length of the tangents from A and B to the circle C is

$|(9+49-12-42-3)-(36+25-24-30-3)| = 3$

$AB^2 = 13,\ OP^2 = 13,\ AP^2 = 17,\ BP^2 = 20$

Paragraph for Question 113 to 115

113. Ans. (B)



Foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and

$S(ae_1, 0)$ and $S'(-ae_1, 0)$ and its centre is at origin. Now there is another ellipse whose axis in inclined at an angle θ with the axis of ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Clearly O is the mid point of SS' and HH'. Thus diagonals of the quadrilateral HSH'S' bisects each other. So its is a parallelogram

$\therefore$ SH = S'H' and HS' = H'S'

Since sum of the focal distances of a point on an ellipse is equal to length of major axis so,

HS + HS' = 2a

$\Rightarrow$ H'S' + HS' = 2a ($\because$ HS = H'S')

114. Ans. (A)

In ΔOHS' are have

$(HS')^2 = (OH)^2 + (OS'^2) - 2OH \cdot OS' \cos(180 - \theta)$

$(HS')^2 = a^2e_2^2 + a^2e_1^2 + 2ae_1\, ae_2 \cos\theta$

$\Rightarrow (HS')^2 = a^2e_2^2 + a^2e_1^2 + 2a^2e_1e_2\cos\theta$

$\because$ HS' will be maximum when $\cos\theta = 1$

$\therefore (HS')_{max} = a(e_1 + e_2)$

115. Ans. (C)

In ΔOHS we have

$HS^2 = OS^2 + OH^2 - 2OS.\ OH\cos\theta$

$\Rightarrow HS^2 = a^2e_1^2 + a^2e_2^2 - 2(ae_1)(ae_2)\cos\theta$

$\Rightarrow HS^2 = a^2e_1^2 + a^2e_2^2 - 2a^2e_1e_2\cos\theta$

HS will be minimum when $\cos\theta = 1$

$\therefore (HS)_{min} = |a(e_1 - e_2)|$

Paragraph for Question 116 to 118

116. Ans. (D)

Equation of hyperbola whose asymptotes are $3x - 4y + 7 = 0$ and $4x + 3y + 1 = 0$ will be

$(3x - 4y + 7)(4x + 3y + 1) + \lambda = 0$

If it passes though (2, 3), then $\lambda = -18$

So hyperbola will be

$(3x - 4y + 7)(4x + 3y + 1) - 18 = 0$

$12x^2 - 12y^2 - 7xy + 31x + 17y - 11 = 0$

117. Ans. (B)

Equation of lines parallel to the lines will be $x + y = \lambda$ and $x - y = \mu$ respectively. If these lines represents asymptotes then these lines must pass through the centre of the hyperbola i.e. (1, 2)

so asymptotes will be

$x + y - 3 = 0$ and $x - y + 1 = 0$

118. Ans. (D)

Asymptotes of the hyperbola $\frac{x^2}{16} - \frac{y^2}{25} = 1$ will be

$$\frac{x^2}{16} - \frac{y^2}{25} = 0 \quad \Rightarrow \quad \left(\frac{x}{4} + \frac{y}{5}\right)\left(\frac{x}{4} - \frac{y}{5}\right) = 0$$

Now any point on the hyperbola will be $(4\sec\theta, 5\tan\theta)$

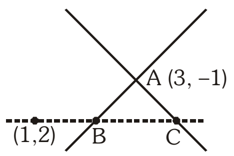
length of the perpendicular from this point to the asymptotes will be

$$p_1 = \frac{(20\sec\theta + 20\tan\theta)}{\sqrt{41}}$$

$$\& \quad p_2 = \frac{(20\sec\theta - 20\tan\theta)}{\sqrt{41}} \quad \therefore \quad p_1 p_2 = \frac{400}{41}$$

Paragraph for Question 122 to 124

Line BC will be parallel to angle bisectors of angle A(3, –1)



$$\text{so} \quad \frac{3x + 4y - 5}{5} = \pm \frac{4x - 3y - 15}{5}$$

$\Rightarrow$ by +ve sign $x - 7y - 10 = 0$ ...... B_1

–ve sign $7x + y - 20 = 0$ ...... B_2

So BC can be $x - 7y = \lambda_1 \Rightarrow \lambda_1 = -13$

or $7x + y = \lambda_2 \Rightarrow \lambda_2 = 9$

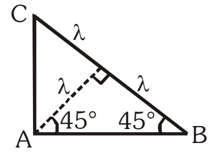
so $x - 7y = -13$ or $7x + y = 9$

Distance of $x - 7y = -13$ from A(3, –1) is $\frac{23}{\sqrt{50}} > 2$

and distance of $7x + y = 9$

from A(3, –1) is $\frac{11}{5\sqrt{2}} < 1$ so BC is $7x + y = 9$

122. Ans. (B)



ΔABC is right isosceles triangle

so area of ΔABC

$$= \frac{1}{2} 2\lambda . \lambda = \lambda^2 = \left(\frac{11}{5\sqrt{2}}\right)^2 = \frac{121}{50}$$

123. Ans. (A)

Now for 'B' $3x + 4y = 5$

$7x + y = 9$

$-25x = -31$

$$\Rightarrow \quad x = \frac{31}{25} \quad \Rightarrow \quad y = \frac{8}{25}$$

$B\left(\frac{31}{25}, \frac{8}{25}\right)$

Now for 'C' $4x - 3y = 15$

$7x + y = 9$

$$25x = 42 \Rightarrow x = \frac{42}{25} \Rightarrow y = -\frac{69}{25}$$

So $C\left(\frac{42}{25}, -\frac{69}{25}\right)$

Circumcentre will be mid point of BC $\left(\frac{73}{25}, -\frac{61}{25}\right)$

124. Ans. (C)

As bisector B_1 is such that point B and C shows opposite sign so it is internal bisector.

Paragraph for Question 125 to 127

$y^2 = 4ax$

Let the centroid is (h, k) and vertices of Δ are $A(at_1^2, 2at_1)$, $B(at_2^2, 2at_2)$, $C(at_3^2, 2at_3)$

So $3h = a(t_1^2 + t_2^2 + t_3^2)$ .... (i)

$3k = 2a(t_1 + t_2 + t_3)$ .... (ii)

Let r is the radius of circle and t_1, t_2, t_3 are roots of equation $(at^2 - h)^2 + (2at - k)^2 = r^2$

$\Rightarrow\ a^2t^4 + t^2(4a^2 - 2ah) - 4akt + h^2 + k^2 - r^2 = 0$

$t_1 + t_2 + t_3 + t_4 = 0,\ \Sigma t_1t_2 = \dfrac{4a^2 - 2ah}{a^2}$

by (ii) $\ t_4 = -\left(\dfrac{3k}{2a}\right)$

by $(t_1 + t_2 + t_3 + t_4)^2 = 0$

$\Rightarrow\ \Sigma t_1^2 = -2\Sigma t_1t_2$

$\dfrac{3h}{a} + \left(\dfrac{9k^2}{4a^2}\right) = -2\left(\dfrac{4a^2 - 2ah}{a^2}\right)$

$\Rightarrow\ 3ah + \dfrac{9k^2}{4} = -8a^2 + 4ah$

$\Rightarrow\ 9k^2 = -32a^2 + 4ah\ \Rightarrow\ 9y^2 = 4a(x - 8a)$

here $a = \dfrac{9}{4}$ so locus is $y^2 = (x - 18)$

125. Ans. (B)

So LR = 1

126. Ans. (A)

Locus of point of intersection of perpendicular tangents is directrix $x - 18 = -\dfrac{1}{4}$

$\Rightarrow\ 4x - 71 = 0$

127. Ans. (D)

Chord of contact is T = 0

$0.y = \dfrac{x + 0}{2} - 18$

x = 36, so (36,0)

Paragraph for Question 128 to 130

128. Ans. (A)

Ellipse $\dfrac{x^2}{4} + y^2 = 1$

Let P(h, k) and equation of any tangent is

$y = mx \pm \sqrt{4m^2 + 1}\ \Rightarrow\ (k - mh)^2 = 4m^2 + 1$

$\Rightarrow\ m^2(h^2 - 4) - 2mkh + (k^2 - 1) = 0$

$\theta_1 + \theta_2 = \alpha$

$\tan(\theta_1 + \theta_2) = \tan\alpha$

$\Rightarrow\ \dfrac{m_1 + m_2}{1 - m_1m_2} = \tan\alpha\ \Rightarrow\ \dfrac{\frac{2hk}{h^2 - 4}}{1 - \frac{k^2 - 1}{h^2 - 4}} = \tan\alpha$

$\Rightarrow\ 2xy = \tan\alpha(x^2 - y^2 - 3)$

$\Rightarrow\ x^2 - 2xy\cot\alpha - y^2 = 3$

$\Delta = 3 - 0 - 0 - 0 + 3(\cot\alpha)^2 \neq 0$

$H^2 > AB$ so hyperbola

129. Ans. (B)

$\tan 45° = \left|\dfrac{m_1 - m_2}{1 + m_1m_2}\right|$

$(1 + m_1m_2)^2 = (m_1 + m_2)^2 - 4m_1m_2$

$\left(1 + \dfrac{k^2 - 1}{h^2 - 4}\right)^2 = \left(\dfrac{2hk}{h^2 - 4}\right)^2 - 4\dfrac{k^2 - 1}{h^2 - 4}$

$(x^2 + y^2 - 5)^2 = 4[x^2y^2 - (y^2 - 1)(x^2 - 4)]$

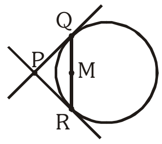
$\Rightarrow\ (x^2 + y^2 - 5)^2 = 4[4y^2 + x^2 - 4]$

130. Ans. (D)

Let mid point is M(h, k), $T = S_1$

$hx + 4ky = h^2 + 4k^2$

also let $P(x_1, y_1)$ which lies on director circle.



so $x_1^2 + y_1^2 = 5$

$xx_1 + 4yy_1 = 4$

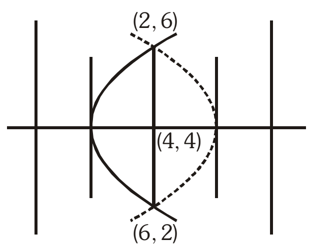
$\dfrac{x_1}{h} = \dfrac{y_1}{k} = \dfrac{4}{h^2 + 4k^2}$

$(4h)^2 + (4k)^2 = 5(h^2 + 4k^2)^2$

$16(x^2 + y^2) = 5(x^2 + 4y^2)^2$

131. Ans. (A)→(R), (B)→(R), (C)→(Q), (D)→(Q)

(A) Length of the LR is $4\sqrt{2} = 4a$



$a = \sqrt{2}$

$\therefore$ equation of directrix $x + y + \lambda = 0$ whose distance from (4, 4) is $2\sqrt{2}$

$\therefore\ \left|\dfrac{4 + 4 + \lambda}{\sqrt{2}}\right| = 2\sqrt{2}\ \Rightarrow\ \lambda = \pm 4 - 8$

$\lambda = -4$ or -12

$x + y = 4$ or $x + y = 12$

$\lambda_1 + \lambda_2 = 4 + 12 = 16$

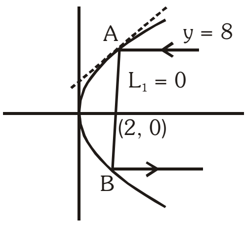
(B) $y = mx - \sqrt{25m^2+16}$ which is tangent to the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose foci are $(\pm 3, 0)$

$\therefore$ Product of the perpendiculars drawn from $(\pm 3, 0)$ upon the given line (tangent) is $= 16$ $(= b^2)$.

(C) co-ordinates of A at $y = 8$, $x = 8$

$\therefore$ A(8, 8)

After reflection ray passes through focus (2, 0)



$L_1 : 4x - 3y - 8 = 0$

point of intersection of $L_1 = 0$ with the parabola

$y^2 = 8x$ is $B\left(\frac{1}{2}, -2\right)$

After 2nd reflection ray moves along the line which is parallel to the axis of the parabola.

$\therefore$ equation of L_2 is $y = -2$

$y + 2 = 0$

$\therefore$ $2y + 4 = 0$

$a = 0, c = 4$

$a + 2c = 0 + 2 \times 4 = 8$

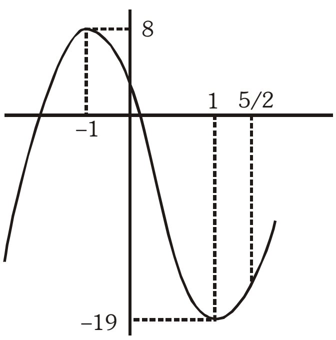
(D) $f'(x) = 6x^2 - 6x - 12 = 0$

$\Rightarrow (x-2)(x+1) = 0 \Rightarrow x = -1, 2$

$f''(x) = 12x - 6$

$f''(-1) = -18 < 0 \Rightarrow x = -1$ is pt. of maxima

$f''(2) = 18 > 0 \Rightarrow x = 2$ is pt. of minima



$f(-1) = -2 - 3 + 12 + 1 = 8$

$f(2) = 16 - 12 - 24 + 1 = -19$

$$f\left(\frac{5}{2}\right) = 2.\frac{125}{8} - 3.\frac{125}{4} - 12.\frac{5}{2} + 1$$

$$= \frac{50}{4} - 29 = -\frac{33}{2}$$

$f(-2) = -16 - 12 + 24 + 1 = -5$

$\therefore$ Maximum value on $\left[-2, \frac{5}{2}\right]$ is 8

132. Ans. (A)→(S); (B)→(P); (C)→(Q); (D)→(R)

(B) Coordinates of point lying at distance r units from P on line passing through P and with slope ℓ will be $\left(\frac{1}{\sqrt{2}}+\frac{r}{\sqrt{2}}, \frac{1}{\sqrt{2}}+\frac{r}{\sqrt{2}}\right)$

Now this line intersects the curve

$2x^2 + xy - 1 = 0$ so

$$2\left(\frac{(r+1)}{\sqrt{2}}\right)^2 + \frac{(r+1)^2}{2} - 1 = 0$$

$3r^2 + 6r + 1 = 0$

$\Rightarrow r_1 + r_2 = -2, r_1r_3 = \frac{1}{3}$

Now $\left|\frac{1}{r_1} - \frac{1}{-r_2}\right| = \left|\frac{-2}{1/3}\right| = 6$

(as point P lies inside the $2x^2 + xy - 1 = 0$)

134. Ans. (A)→(Q), (B)→ (S), (C)→(R), (D)→ (P,Q,R,S)

(A) If x is a perfect square, then Px will be a perfect square only if P is a perfect square, which is not possible as P is a prime number. Hence y cannot be a perfect square. So number of such points will be only one i.e. (0, 0)

(B) H.M. of the lengths of the segments of focal chord is equal to length of semilatus rectum

so $2a = \frac{2\times 2\times 6}{2+6} \Rightarrow a = 3/2$

$\therefore$ length of latus rectum $= 4a = 6$

(C) Equation of the tangent to the parabola $y = x^2 + 7x + 2$ parallel to $y = 3x - 3$ will be $y = 3x + \lambda$

this line touches the parabola so

$x^2 + 7x + 2 = 3x + \lambda$

$\Rightarrow x^2 + 4x + 2 - \lambda = 0$

$B^2 - 4AC = 0$

$\Rightarrow 16 - 8 + 4\lambda = 0 \quad \Rightarrow \lambda = -2$

hence equation of tangent will be

$y = 3x - 2$

the point of contact will be the nearest point to the given straight line i.e. (–2,–8)

$\Rightarrow a = -2, b = -8$

$\therefore 2a - b = -4 + 8 = 4.$

(D) Any tangent to the parabola is

$ty = x + at^2$ at the point $(at^2, 2at)$

If this is normal to the circle then it will pass through the centre of the circle which is (a, 2a)

so $2at = a + at^2$

$\Rightarrow t^2 - 2t + 1 = 0$

$\Rightarrow t = 1$ for any value of a

so the condition satisfies ∀ real value of a.

135. Ans. (A)→(S), (B)→(R), (C)→(P), (D)→(Q)

(A) Equation of common chord

$10x + 4y - b - a = 0$

this chord should pass through the centre of the circle $x^2 + y^2 + 6x + 6y - b = 0$ i.e. (–3, 3)

$-30 - 12 - b - a = 0$

$a + b = -42$

$|a + b| = 42$

(B) Equation of chord of contact drawn from point (a, b) will be $ax + by = 10$

comparing it with given chord

$x + y = 2$

$a = 5, b = 5$

so $a^2 + b^2 = 50$

(C) Equation of the family of circles passing through AB will be

$x^2 + y^2 - 5x + \lambda(2x - y) = 0$

$\Rightarrow x^2 + y^2 + x(2\lambda - 5) - \lambda y = 0$

centre $\left(-\frac{(2\lambda-5)}{2}, \frac{\lambda}{2}\right)$

If AB is a diameter then centre must lie on AB, so

$\frac{\lambda}{2} = -(2\lambda - 5)$

$\lambda = -4\lambda + 10$

$\lambda = 2$

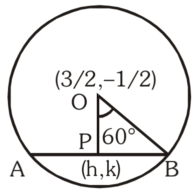
$\therefore$ required circle will be

$x^2 + y^2 - x - 2y = 0$

$a = 1, b = 2$

$\therefore 2a + 9b = 20$

(D)



Let mid point of the chord is (h, k)

In ΔOPB

$\cos 60° = \frac{OP}{OB}$

$\frac{1}{2} = \frac{\sqrt{(h-3/2)^2 + (k+1/2)^2}}{\sqrt{2}}$

$\Rightarrow (h - 3/2)^2 + (k + 1/2)^2 = 1/2$

$\Rightarrow h^2 + k^2 - 3h + k + 2 = 0$

$\therefore$ Required locus

$x^2 + y^2 - 3x + y + 2 = 0$

$\Rightarrow a = 3, b = -1$

So $a^3 + b^3 = 27 - 1 = 26.$

136. Ans. (A)→(Q), (B)→(S), (C)→(P), (D)→(S)

(A) The given asymptotes are perpendicular so hyperbola will be rectangular hyperbola so eccentricity is $\sqrt{2}$

(B) Equation of normal at any point P as a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ will be

$ax\cos\theta + by\cot\theta = a^2 + b^2$

It intersection point with transverse and conjugate axes are L$\left(\frac{a^2+b^2}{a}\sec\theta, 0\right)$ and $\left(0, \frac{a^2+b^2}{a}\tan\theta\right)$

Locus of mid point of L and M will be

$h = \frac{a^2+b^2}{2a}\sec\theta$, $k = \frac{a^2+b^2}{2a}\tan\theta$

$\Rightarrow \quad \sec\theta = \frac{2ah}{a^2+b^2}, \ \tan\theta = \frac{2bk}{a^2+b^2}$

$\Rightarrow \quad \frac{4a^2h^2}{(a^2+b^2)^2} - \frac{4b^2k^2}{(a^2+b^2)^2} = 1$

$\therefore$ Required locus

$$\frac{x^2}{\left(\frac{a^2+b^2}{2a}\right)^2} - \frac{y^2}{\left(\frac{a^2+b^2}{2b}\right)^2} = 1$$

It eccentricity will be $e = \sqrt{\frac{a^2}{b^2}+1}$

Now eccentricity of the given hyperbola is 2

$\therefore \quad \frac{b^2}{a^2} = 3$

$\therefore \quad e = \sqrt{\frac{1}{3}+1} = \frac{2}{\sqrt{3}}$

(C) As the given point lies outside the ellipse so 2 real tangents can be drawn.

(D) Equation of the asymptote of the hyperbola is $\frac{x}{a} - \frac{y}{b} = 0$

$\Rightarrow \quad \frac{b}{a} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \frac{b^2}{a^2} = \frac{1}{3}$

and eccentricity $e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{1}{3}} = \frac{2}{\sqrt{3}}$

137. Ans. (A)→(Q), (B)→(S), (C)→(P), (D)→(R)

(A) Equation of tangent to the ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$ parallel to the line x + y = 7 will $y = -x \pm \sqrt{6+3} \quad \Rightarrow \quad x+y = \pm 3$

Clearly x + y = 3 is at the shortest distance from x + y = 7

point of contact is (2, 1)

so a = 2, b = 1 $\quad \therefore \quad$ a + b = 3

(B) Two perpendicular tangents can be drawn to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ from any point lying on the circle $x^2 + y^2 = 25$ i.e. (5cosθ, 5sinθ)

Equation of chord of contact will be

$$= \frac{1}{\sqrt{\frac{25}{256}\cos^2\theta + \frac{25}{81}\sin^2\theta}}$$

$$= \frac{144}{5\sqrt{81\cos^2\theta + 256\sin^2\theta}} = \frac{144}{5\sqrt{81+175\sin^2\theta}}$$

If will be maximum when $\sin^2\theta = 0$

$\therefore \quad$ maximum distance $= \frac{144}{5\times 9} = \frac{16}{5}$

$\therefore \quad$ a = 16, b = 6

$\therefore \quad$ a – 2b = 16 – 10 = 6

(C) Equation of normal to the hyperbola $\frac{x^2}{4}-\frac{y^2}{1}=1$ will be 2xcosθ + ycotθ = 5

It has equal intercepts on positive x and y

so $-\frac{2\cos\theta}{\cot\theta} = -1$

$\sin\theta = \frac{1}{2} = 1, \ \theta = \frac{\pi}{6}$

hence equation of normal is $x\sqrt{3}+y\sqrt{3}=5$

$x + y = \frac{5}{\sqrt{3}}$

It touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

so $\frac{25}{3} = a^2+b^2$

(D) Equation of the hyperbola can be written as (x – 3)(y – 3) = 2

It is a rectangular hyperbola so a = b

and $\frac{a^2}{2} = 2 \quad \Rightarrow \quad a = 2$

So length of the latus rectum $= \frac{2b^2}{a} = 4$

140. Ans. 8

The curve is $(y + 1)^2 = 4(x - 1)$

equation of the normal to the given curve is

$y + 1 = m(x - 1) - 2m - m^3$

$y = mx - 3m - m^3 - 1$

which passes through (h, k)

$m^3 + m(3 - h) + 1 + k = 0 \quad \begin{cases} m_1 \\ m_2 \\ m_3 \end{cases}$

$m_1m_2m_3 = -(1 + k)$

$m_3 = (1 + k) \quad \because \quad m_1m_2 = -1$

$(1 + k)^3 + (1 + k)(3 - h) + (1 + k) = 0$

$(1 + k)^2 + 3 - h + 1 = 0$

$(y + 1)^2 = x - 4$

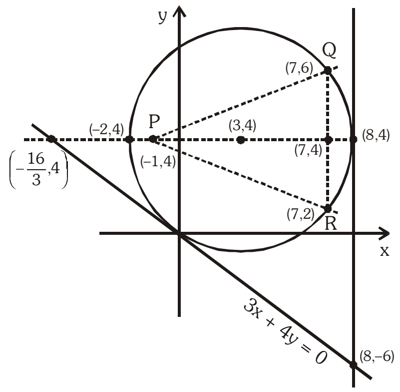
$\therefore$ locus of (h, k) are

$C_1 : (y + 1)^2 = x - 4$

$\frac{k}{2} = -1 \quad \Rightarrow \quad k = -2, \ \lambda = 4$

$\therefore \quad \lambda - 2k = 8$

141. Ans.200



$x^2 + y^2 - 6x - 8y < 0$

$(x - 3)^2 + (y - 4)^2 - 25 < 0$

Point atleast distance from

(–2, 4) is $P(a, b) \equiv P(-1, 4)$

Points which are greatest distance from (–2, 4) are Q(c, d)

& $R(e, f) \equiv Q(7, 6)$ & R(7, 2)

ΔPQR is an isosceles triangle & internal bisector of $\angle P$ is $y = 4$

Equation of tangent at origin is $3x + 4y = 0$

equation of tangent at $\left(\frac{c+e}{2} + 1, b\right) \equiv (8, 4)$ is $x = 8$

Area of the right angled triangle formed by above three lines is $\Delta = \frac{1}{2} \times 10 \times \frac{40}{3} = \frac{200}{3}$

$\therefore \quad 3\Delta = 200$