1. Ans. (A)

$\det((3AC)(2B)^{-1}) = |3AC|.|(2B)^{-1}|$

$= 9|A|.|C|.\left|\frac{1}{2}.B^{-1}\right| = 9|A|.|C|.\frac{1}{4}.\frac{1}{|B|}$

$= 9.\frac{1}{4}.2.\frac{1}{4}.\frac{1}{9} = \frac{1}{8}$

2. Ans. (C)

$\ell ny = \frac{1}{3}\left\{3\ell nx + \ell n(x^2+1) - \frac{1}{5}\ell n(5-x)\right\}$

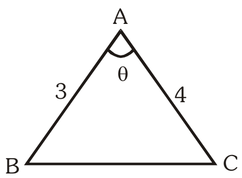
$\frac{1}{y}.y' = \frac{1}{3}\left\{\frac{3}{x} + \frac{2x}{x^2+1} + \frac{1}{5(5-x)}\right\}$

$\frac{y'}{y} = \frac{1}{3}\left\{3+1+\frac{1}{20}\right\}$ = length of sub-tangent

3. Ans. (A)

$\Delta = \frac{1}{3}.12.\sin\theta$

$\Delta = 6\sin\theta$



for maximum area $\theta = \frac{\pi}{2}$

$\Delta = 6$

$a^2 = 3^2 + 4^2$

a = 5

$\Delta = \frac{5+3+4}{2} = s$

4. Ans. (C)

$A^n = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$, $\lim_{n\to\infty}\frac{1}{n}A^n = \lim_{n\to\infty}\begin{bmatrix} \frac{1}{n} & a \\ 0 & \frac{1}{n} \end{bmatrix} = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \quad \lambda = 1$

5. Ans. (C)

$(1+x^2)y = 1$ .... (i)

$(1+x)y = 1$ .... (ii)

solving (i) and (ii)

$1 + x^2 = 1 + x$

x = 0, 1

P = (0, 1), $\quad Q = \left(1, \frac{1}{2}\right)$

Tangent to (i) at these points is y = 1 & x + 2y = 2

6. Ans. (A)

$\because$ LMVT is applicable to given function in [–1, 1]

$\therefore$ $f(x)$ must be continuous in [–1, 1] & derivable in (–1, 1)

$f(0^-) = f(0^+)$

$\frac{\pi}{2} = c$

$f'(0^-) = f'(0^+)$

–1 = m

7. Ans. (C)

The curve cuts x-axis at (1, 0)

$y' = \left(\sqrt{5-x^6}\right).3x^2 - \left(\sqrt{5-x^4}\right)2x$

$y'|_{x=1} = \left(\sqrt{5-1}\right)3 - \left(\sqrt{5-1}\right)2$

$\tan\theta = 6 - 4$

$\theta = \tan^{-1}2 = \cot^{-1}\frac{1}{2}$

8. Ans. (C)

$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

characteristice equation is $|A - \lambda I| = 0$

$\left|\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right| = 0$

$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 1 & 2 & 3-\lambda \end{vmatrix} = 0$

$-\lambda\{\lambda^2 - 3\lambda - 2\} - 1\{-1\} = 0$

$-\lambda^3 + 3\lambda^2 + 2\lambda + 1 = 0$

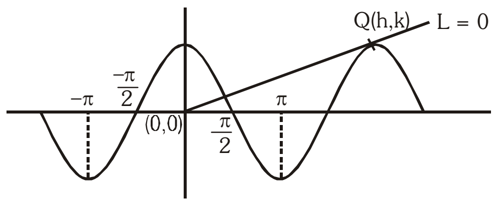
$A^3 - 3A^2 - 2A + I = 0$

$A^3 - I = 2A + 3A^2$

p = 1, q = 2, r = 3

p + q + r = 6

9. Ans. (B)



Let Q(h, k) be point of contact & m be the slope of the tangent

$\therefore \quad \frac{k}{h} = -\sinh$ .... (i)

Also Q(h, k) lies on the curve

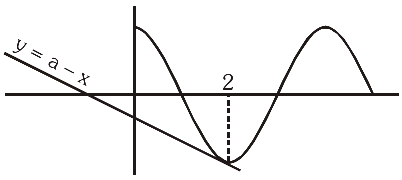
$\therefore \quad k = \cosh$ .... (ii)

$(i)^2 + (ii)^2 \Rightarrow \cos^2 h + \sin^2 h = k^2 + \frac{k^2}{h^2}$

$\Rightarrow \quad \frac{1}{y^2} = 1 + \frac{1}{x^2}$

10. Ans. (A)

For local minima at x = 2



$\cos\pi \ge a - 2$

$a \le 1$

11. Ans. (D)

$\because$ A is an orthogonal matrix

$\Rightarrow \quad \vec{a} = (\hat{i} + 2\hat{j} + x\hat{k})/3, \vec{b} = (2\hat{i} + \hat{j} + y\hat{k})/3$

$\vec{c} = (2\hat{i} - 2\hat{j} + z\hat{k})/3$

are orthogonal triads of unit vectors

$\Rightarrow \quad x^2 + 4 + 1 = 9 \quad \Rightarrow \quad x = \pm 2$

$y^2 + 4 + 1 = 9 \quad \Rightarrow \quad y = \pm 2$

$\vec{a} \cdot \vec{b} = 0$

$\Rightarrow \quad 2 + 2 + xy = 0$

$xy = -4 \quad \Rightarrow$ x & y are of opposite sign.

$\therefore \quad x + y = 0$

12. Ans. (B)

The given equation is of the form,

$ax^2 + bx + c = 0$, where a, b, c are odd integers

Let the equation has rational roots

$\Rightarrow \quad b^2 - 4ac = I^2$, where 'I' is odd integer

$\downarrow \quad \downarrow \quad \downarrow$

odd even odd

$\Rightarrow \quad (b - I)(b + I) = 4ac$

now b = 2n + 1

Let I = 2r + 1, r is an integer

$(2n - 2r)(2n + 2r + 2) = 4ac$

$\underbrace{(n - r)(n + r + 1)}_{\text{even}} = \underbrace{ac}_{\text{odd}}$

$\Rightarrow \quad b^2 - 4ac$ is not a perfect square

hence given equation has irrational roots.

13. Ans. (A)

Let a & b be two real roots of the equation

$f(x) = 0$ in (2, 3)

$\Rightarrow f(a) = f(b)$

$\therefore f'(x) = 0$ must have at least one real root in (a, b) {Rolle's theorem}

$\Rightarrow 3x^2 - 12 = 0$ must have at least one real root in (2, 3)

which is not possible

Hence no such value of 'K' exists.

14. Ans. (D)

$\because \quad \alpha^2 - 2\alpha + 3 = 0 \ \& \ \beta^2 - 2\beta + 3 = 0$

$\alpha^3 - 2\alpha^2 + 3\alpha - 2 = \alpha(\alpha^2 - 2\alpha + 3) - 2 = -2$

$(\beta^3 - 2\beta^2 + 3\beta - 1)^4 = (\beta(\beta^2 - 2\beta + 3) - 1)^4 = 1$

$\therefore$ The quadratic equation whose roots are –2 and 1 is $x^2 - (-2 + 1)x + (-2) = 0$

$\Rightarrow \quad x^2 + x - 2 = 0$

15. Ans. (C)

$$a^3b^3c^3 \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = a^3b^3c^3 \{-1(0-1) + 1(1-0)\}$$

$= 2a^3b^3c^3 \quad \therefore \quad p = q = r = 3$

$p + q + r + 10 = 19$

16. Ans. (A)

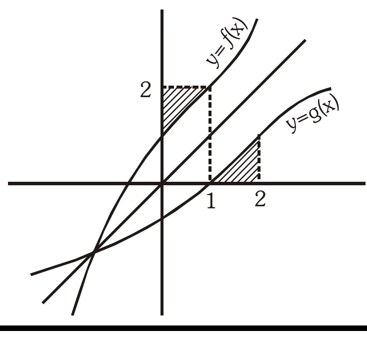
$f(x) = x^3 - 3x^2 + 3x + 1$

$f'(x) = 3x^2 - 6x + 3 = 3(x^2 - 2x + 1)$

$f'(x) = 0 \quad \Rightarrow \quad x = 1$

sign of $f'(x)$ does not change about x = 1, then x = 1 is point of inflection.

Area bounded by y = g(x) and x-axis from x = 1 to x = 2 is same as area bounded by y = f(x) and y-axis from y = 1 to y = 2.



Required area $= 2 - \int_0^1 (x^3 - 3x^2 + 3x + 1)dx$

$= 2 - \left(\frac{1}{4} - \frac{3}{3} + \frac{3}{2} + 1\right) = 2 - \frac{7}{4} = \frac{1}{4}$

17. Ans. (A)

$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r.\frac{dr}{dt}$ $\qquad \frac{dS}{dt} = kt$ (given)

$\Rightarrow \quad kt = 8\pi r.\frac{dr}{dt} \Rightarrow \quad \frac{8\pi r^2}{2} = k.\frac{t^2}{2} + c$

at $t = 0, r = 3$

$\therefore \quad 36\pi = c$

$4\pi r^2 = \frac{kt^2}{2} + 36\pi$

at $t = 2, r = 5$

$\Rightarrow \quad 100\pi = 2k + 36\pi \;\Rightarrow\; k = 32\pi$

$\Rightarrow \quad 4\pi r^2 = 16\pi t^2 + 36\pi$

$r = \sqrt{4t^2 + 9}$

Now, at t = 3 seconds

$r = \sqrt{45} = 3\sqrt{5}$ units

18. Ans. (A)

$y_1y_3 = 3y_2^2 \Rightarrow \quad \frac{y_3}{y_2} = 3.\frac{y_2}{y_1}$

On integrating we get $\ell n y_2 = 3\ell n y_1 + \ell n c$

$\Rightarrow \quad y_2 = cy_1^3 \quad \Rightarrow \quad \frac{y_2}{y_1^2} = cy_1$

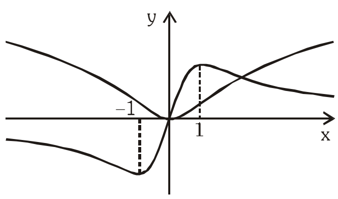
again integrating both sides, we get

$-\frac{1}{y_1} = cy + c' \qquad \Rightarrow \quad -dx = (cy + c')dy$

integrating both sides $-x = \frac{cy^2}{2} + c'y + c''$

$x = A_1y^2 + A_2y + A_3$

19. Ans. (B)



$e^{f(x)} . f'(x) = 2x$

$e^{f(x)} = x^2 + c$

{Integrating both sides with respect to x}

$f(x) = \ell n(x^2 + c)$

$f'(x) = \frac{2x}{x^2 + c}$

$f'(1) = \frac{2}{1+c} = 1 \quad \{\because f'(1) = 1\}$

$\therefore \quad c = 1$

$\therefore \quad f(x) = \ell n(x^2 + 1)$

$f'(x) = \frac{2x}{x^2 + 1}$

20. Ans. (A)

$\ell n(f(x)) > 0 \Rightarrow \quad f(x) > 1$

$\Rightarrow \quad f'(x) + 786f(x) > 0 \ \forall\ x \in R$

$\Rightarrow \quad e^{786x}f'(x) + 786e^{786x} . f(x) > 0$

$\Rightarrow \quad \frac{d}{dx}(f(x) . e^{786x}) > 0 \ \forall\ x \in R$

$\Rightarrow \quad$ g(x) is an increasing function.

21. Ans. (A)

$x^3 + 3x^2 + 4x + 5 = 0 \qquad$ .....(i)

$x^3 + 2x^2 + 7x + 3 = 0 \qquad$ .....(ii)

(i) – (ii) $\Rightarrow x^2 - 3x + 2 = 0$

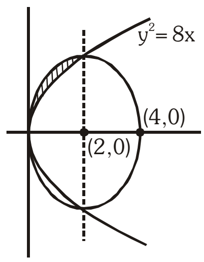
$\Rightarrow \quad x = 1, 2$

If x is the common root, then it has to be either 1 or 2.

But neither 1 nor 2 satisfies equation (i) & (ii)

$\therefore \quad$ Number of common roots = 0

22. Ans. (D)



Area of shaded portion

$= \frac{1}{4}(\pi.2.4) - \frac{1}{2}\left(\frac{8}{3}.2^2\right)$

$= 2\pi - \frac{16}{3}$

23. Ans. (D)

$\frac{dy}{dx} = e^x(\cos x - \sin x) = m$ (let)

Now $\frac{dm}{dx} = -2e^x \sin x$

For m to be max., $\frac{dm}{dx} = 0 \Rightarrow x = n\pi, n \in I$

$\frac{d^2m}{dx^2} = -2e^x(\sin x + \cos x)$

At $x = 0, \frac{d^2m}{dx^2} < 0$

$\therefore \quad x = 0$ is a point of maxima

24. Ans. (A)

$f'(x) = (4a - 3) + (a - 7)\cos x$

For non existence of critical point

$f'(x) \neq 0 \ \forall \ x \in R$

$\Rightarrow \quad \cos x \neq \frac{3-4a}{a-7} \ \forall \ x \in R \quad \Rightarrow \quad \left|\frac{3-4a}{a-7}\right| > 1$

$\Rightarrow \quad |4a-3| - |a-7| > 0$

(i) If $a < \frac{3}{4} \Rightarrow 3 - 4a + a - 7 > 0$

$\Rightarrow a < -\frac{4}{3}$

(ii) If $\frac{3}{4} \leq a < 7 \Rightarrow 4a - 3 + a - 7 > 0$

$\Rightarrow \quad a > 2 \Rightarrow \quad 2 < a < 7$

(iii) $a \geq 7 \Rightarrow \quad 4a - 3 - a + 7 > 0$

$\Rightarrow a > -\frac{4}{3} \Rightarrow \quad a \geq 7$

From (i), (ii) and (iii), $a \in \left(-\infty, -\frac{4}{3}\right) \cup (2, \infty)$

25. $\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$\left.\begin{matrix} \alpha^2 = 1 \ \Rightarrow \ \alpha = \pm 1 \\ \alpha = 4 \end{matrix}\right\} \Rightarrow$ no solution

26. Ans. (A)

$(x^2 + x - 2)(2x^2 - x - 1)(x^2 - 1) = 0$

$x = -2, 1, 1, -\frac{1}{2}, 1, -1 = -\frac{1}{2}, -1, 1, -2$

27. $f(A) = I + A + A^2 + \ldots\ldots\ldots + A^{16}$

$A = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^3 = A^2 . A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

similarly, $A^4 = A^5 = \ldots\ldots = A^{16} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$f(A) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \ldots\ldots\ldots$

$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$

28. If we calculate $A^2 = \begin{bmatrix} 1 & 0 \\ 2\left(\frac{1}{2}\right) & 1 \end{bmatrix}$,

$A^3 = \begin{bmatrix} 1 & 0 \\ 3\left(\frac{1}{2}\right) & 1 \end{bmatrix}, \ldots\ldots\ldots, A^{50} = \begin{bmatrix} 1 & 0 \\ 50\left(\frac{1}{2}\right) & 1 \end{bmatrix}$

29. $f(2x + 1) = 4x^2 + 14x = (2x + 1)^2 + 10x - 1$

$f(2x + 1) = (2x + 1)^2 + 5(2x + 1) - 6$

Let $2x + 1 = t$

$f(t) = t^2 + 5t - 6 \ \Rightarrow \alpha + \beta = -5.$

30. $\frac{a(2x+1)^2}{(x-3)^2} + \frac{b(2x+1)}{(x-3)} + c = 0$

$\Rightarrow \frac{2x+1}{x-3} = \alpha$ or $\frac{2x+1}{x-3} = \beta$

$\Rightarrow 2x + 1 = \alpha x - 3\alpha \ \Rightarrow \ x(\alpha - 2) = 1 + 3\alpha$

$\Rightarrow x = \frac{1+3\alpha}{\alpha-2}, \ \frac{1+3\beta}{\beta-2}.$

31. $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2A \Rightarrow A^3 = 2A^2 = 2^2A$

Similarly $A^4 = 2^3$ A and so on

So $A^n = 2^{n-1}A$

$\Rightarrow A^n = 2^{n-1}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A^n - I = \begin{bmatrix} 2^{n-1} - 1 & 2^{n-1} \\ 2^{n-1} & 2^{n-1} - 1 \end{bmatrix}$

$|A^n - I| = (2^{n-1} - 1)^2 - (2^{n-1})^2 = 1 - 2^n \Rightarrow \ \lambda = 2$

32. $M_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$

$|M_r| = r^2 - (r-1)^2 = 2r - 1$

$\Rightarrow \sum_{r=1}^{n} |M_r| = \sum_{r=1}^{n}(2r-1) = n^2$

33. Let discriminant of $10px^2 - qx + r$, $px^2 - qx - 5r$, $5px^2 - qx - r$ be D_1, D_2 and D_3 respectively.

$\therefore \ D_1 + D_2 + D_3$

$= q^2 - 40pr + q^2 + 20pr + q^2 + 20pr = 3q^2 > 0$

Hence atleast two real roots.

34. $AA^T = I \quad \Rightarrow \quad |A|^2 = 1 \Rightarrow |A| \pm 1$

$\Rightarrow \quad |A| = -1 \qquad \{\because \ |A| \neq 1\}$

$A^{-1} = A^T$ {$\because$ A is orthogonal matrix}

$\therefore \quad \frac{\text{adj}A}{|A|} = A^T$

$\therefore \quad -\begin{bmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$

(where c_{ij} represents cofactor of a_{ij})

$\Rightarrow$ every element is additive inverse of its cofactor.

35. $(I + A)^{10} = I + {}^{10}C_1 A + {}^{10}C_2 A^2 + \ldots\ldots. + {}^{10}C_{10} A^{10}$

$= I + {}^{10}C_1 A + {}^{10}C_2 A + \ldots\ldots. + {}^{10}C_{10} A$

$= I + (2^{10} - 1)A$

$\{\because A^n = A \ \forall\ n \geq 2$ in case of idempotent matrix$\}$

36. $A^{-5} = \begin{bmatrix} \sin^{10}\alpha & 0 & 0 \\ 0 & \sin^{10}\beta & 0 \\ 0 & 0 & \sin^{10}\gamma \end{bmatrix}$

& $\quad A^{-2} = \begin{bmatrix} \sin^4\alpha & 0 & 0 \\ 0 & \sin^4\beta & 0 \\ 0 & 0 & \sin^4\gamma \end{bmatrix}$ & so on

similarly,

$B^{-5} = \begin{bmatrix} \cos^{10}\alpha & 0 & 0 \\ 0 & \cos^{10}\beta & 0 \\ 0 & 0 & \cos^{10}\gamma \end{bmatrix}$

& $\quad B^{-2} = \begin{bmatrix} \cos^4\alpha & 0 & 0 \\ 0 & \cos^4\beta & 0 \\ 0 & 0 & \cos^4\gamma \end{bmatrix}$ & so on

Now $C = A^{-5} + B^{-5} + 5\,A^{-1}B^{-1}(A^{-3} + B^{-3}) + 10A^{-2}B^{-2}(A^{-1} + B^{-1})$

$C_{11} = \cos^{10}\alpha + \sin^{10}\alpha + 5\sin^2\alpha\cos^2\alpha(\sin^6\alpha + \cos^6\alpha) + 10\sin^4\alpha\cos^4\alpha(\sin^2\alpha + \cos^2\alpha)$

$= (\cos^2\alpha)^5 + {}^5C_1(\cos^2\alpha)^4 \,.\, \sin^2\alpha + {}^5C_2(\cos^2\alpha)^3(\sin^2\alpha)^2 + {}^5C_3(\cos^2\alpha)^2(\sin^2\alpha)^3 + {}^5C_4(\cos^2\alpha)(\sin^2\alpha)^4 + {}^5C_5(\sin^2\alpha)^5$

$= (\cos^2\alpha + \sin^2\alpha)^5 = 1$

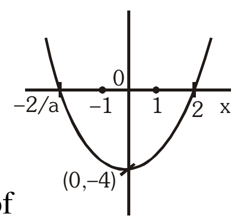
Similarly $C_{22} = C_{33} = 1$

$\therefore \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

37. $f(x) = ax^2 + 2x(1 - a) - 4$

$= x(ax + 2) - 2(ax + 2)$

$= (ax + 2)(x - 2)$



If 'a' is negative then for infinite integral values of x expression will become negative

$\therefore$ a must be positive.

From the graph it is clear that to satisfy the given conditions.

$f(-1) < 0$ and $f(-2) \geq 0$

solving these two we get $a \in [1, 2)$.

38. $\int_0^a f(x)dx = \tan^{-1} a$

$f(a) = \dfrac{1}{1+a^2} \Rightarrow \dfrac{1}{1+x^2} - 1 = \tan^2 x$

$\Rightarrow \sec^2 x = \dfrac{1}{1+x^2 \leq 1} \Rightarrow x = 0$

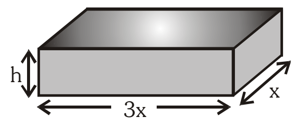
39. **Ans. (B)**

$f'(a) = f'(b)$

$3a^2 + a = 3b^2 + a \Rightarrow a = -b$

so $\quad f(1) = 1 + a + b = 1$

40. **Ans. (B)**



$3x^2h = 1$

$S = 6x^2 + 8hx$

$S(x) = 6x^2 + 8x\dfrac{1}{3x^2}$

$S'(x) = 12x - \dfrac{8}{3x^2}$

$S'(x) = 0 \Rightarrow 36x^3 = 8$

$\Rightarrow x^3 = \dfrac{2}{9}$

also $S''(x) > 0$

so $x = \left(\dfrac{2}{9}\right)^3$ is point of minima so $a^3 = \dfrac{2}{9}$

41. **Ans. (C)**

(A) false as $f(x)$ is not differentiable at $x = 0$

(B) false as $f(x)$ is continuous at and differentiable in $[-5, -1]$

(C) LMVT is applicable and

$2c + 2 = \dfrac{1 - (16 - 8 + 2)}{3} = -3$

$c = -\dfrac{5}{2}$

(D) is incorrect as (C) is correct

42. **Ans. (C)**

$(K + 1)^2 e^{(K+1)x} - 4(K + 1)e^{(K+1)x} + 4e^{(K+1)x} = 0$

$(K + 1)^2 - 4(K + 1) + 4 = 0$

$K + 1 = 2 \quad \Rightarrow K = 1$

43. Ans. (D)

as $\dfrac{f(5)-f(2)}{5-2}=\dfrac{15-0}{3}=5$

So average change is 5 and $f'(x) \le 5$

$\Rightarrow \quad f'(x)=5 \quad \forall\, x \in [2,5] \quad \Rightarrow f(x)=5x+c$

$f(2)=0 \quad \Rightarrow \quad 10+c=0 \quad \Rightarrow \quad c=-10$

$\Rightarrow \quad f(3)=15-10=5$

44. Ans. (D)

$y' = 2\sqrt{y} \quad \Rightarrow \quad m_1 = 2$

$y' = -\dfrac{3x^2}{6} \quad \Rightarrow \quad m_2 = -\dfrac{1}{2}$

as $\quad m_1m_2 = -1 \quad \Rightarrow \quad \theta = \dfrac{\pi}{2}$

45. Ans. (C)

Refer theory

46. Ans. (C)

47. Ans. (B)

$y' = b - ce^{-x}$

$y'' = ce^{-x} \quad \Rightarrow \quad y''' = -ce^{-x} \quad \Rightarrow \quad y''' = -y''$

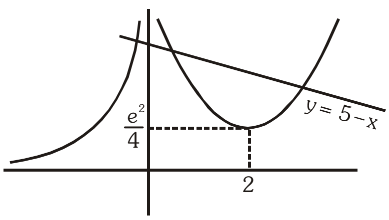
48. Ans. (A)

$$\int \frac{dy}{(y-2)(y+1)} = \int \frac{dx}{(x+3)(x-1)}$$

$$\Rightarrow \quad \int \frac{1}{3}\left(\frac{1}{y-2}-\frac{1}{y+1}\right)dy = \frac{1}{4}\int\left(\frac{1}{x-1}-\frac{1}{x+3}\right)dx$$

$$\frac{1}{3}\ell n\left|\frac{y-2}{y+1}\right| = \frac{1}{4}\ell n\left|\frac{x-1}{x+3}\right| + c$$

49. Ans. (C)



$$f(x) = \frac{e^x}{x^2}$$

$$f'(x) = \frac{x^2 . e^x - 2xe^x}{x^4}$$

$$= \frac{e^x(x-2)}{x^3}$$

$\Rightarrow \quad f(x) = 5 - x$ has 3 distinct roots.

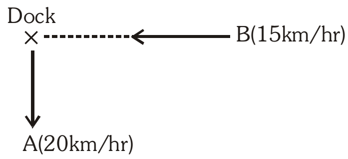
50. Ans. (B)

$$A = \int_{\pi/6a}^{\pi/2a} \cos(ax)dx - \int_{\pi/2a}^{5\pi/6a} \cos(ax)dx$$

$$= \left(\frac{\sin ax}{a}\right)_{\pi/6a}^{\pi/2a} - \left(\frac{\sin ax}{a}\right)_{\frac{\pi}{2a}}^{\frac{5\pi}{6a}}$$

$$= \left(1-\frac{1}{2}\right)\frac{1}{a} - \left(\frac{1}{2}-1\right)\frac{1}{a} = \frac{1}{a} > 3 \quad \Rightarrow \quad 0 < a < \frac{1}{3}$$

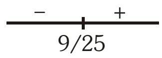
51. Ans. (A)



At 2 PM boat B is 15 km away from Dock. At time t hr boat A is 20t km way & boat B is 15 – 15t km away from dock.

$(AB)^2 = f(t) = 400t^2 + 15^2(1-t)^2$

$f'(t) = 800t + 15^2 . 2(t-1) = 1250t - 450$



AB is minimum at $t = \dfrac{9}{25}$ hrs. = 21 min 36 sec.

$\therefore \quad$ time = 2:21:36 PM

52. Ans. (A)

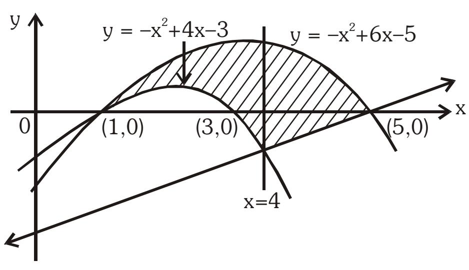
$$\frac{2(xdx-ydy)}{(x^2-y^2)} = -\left(\frac{ydx-xdy}{x^2+y^2}\right)$$

$$\Rightarrow \frac{d(x^2-y^2)}{x^2-y^2} = -d\tan^{-1}\left(\frac{y}{x}\right)$$

on integrating, both side

$\log|x^2 - y^2| = -\tan^{-1}\left(\dfrac{y}{x}\right) + k$

53. Ans. (C)



$$A = \int_1^4 \left\{(-x^2+6x-5)-(-x^2+4x-3)\right\}dx$$

$$+\int_4^5 (-x^2+6x-5)-(3x-15)dx$$

$$A = \frac{73}{6}$$

54. Ans. (B)

$\frac{1}{1+y^2}.\frac{dy}{dx}+2x(\tan^{-1}y)=x^3$

Put $\tan^{-1}y = z$

$\therefore \quad \frac{1}{1+y^2}.\frac{dy}{dx}=\frac{dz}{dx}$

$\frac{dz}{dx}+(2x)z=x^3 \quad \Rightarrow \quad z.e^{x^2}=\frac{1}{2}\int 2e^{x^2}.x^3dx+c$

Put $x^2 = t$

$\Rightarrow \quad (\tan^{-1}y)\ e^{x^2}=\frac{1}{2}\int e^t.t\,dt+c \quad \Rightarrow$

$e^{x^2}(\tan^{-1}y)=\frac{1}{2}\left\{e^t.t-e^t\right\}+c$

$\Rightarrow \quad 2e^{x^2}\tan^{-1}y=x^2e^{x^2}-e^{x^2}+2c$

$\Rightarrow \quad 2\tan^{-1}y=x^2-1+2ce^{-x^2}$

55. Ans. (A)

For point of intersection $x^2y = xy$

Since $y \neq 0$ on only of two curves so we must have $x = 0, 1$. Thus the given curves intersects at $(0, 1)$ and $\left(1, \frac{1}{2}\right)$

Now differentiating $x^2y = 1 - y$ are have

$\frac{dy}{dx}=\frac{-2xy}{1+x^2}$

$\left(\frac{dy}{dx}\right)_{(0,1)}=0$ and $\left(\frac{dy}{dx}\right)_{\left(1,\frac{1}{2}\right)}=-\frac{1}{2}$

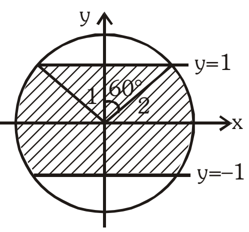
So angle between the tangents is $\tan^{-1}\frac{1}{2}$

56. Ans. (A)

$(x^2 + y^2 - 4)(y^2 - 1) = 0$

Shaded area = Total area – non shaded area

Non shaded area



$=\left(\frac{1}{2}\times4\times\frac{2\pi}{3}-\frac{1}{2}\times2\sqrt{3}\right)\times2$

$=\left(\frac{4\pi}{3}-\sqrt{3}\right)\times2$

$\therefore$ Shaded area $= 4\pi - \frac{8\pi}{3}+2\sqrt{3}=\frac{4\pi}{3}+2\sqrt{3}$

57. Ans.(C)

$\frac{1}{y}\frac{dy}{dx}=\frac{x^2e^x+\ell ny}{x}$

put $\ell ny = t \Rightarrow \frac{1}{y}\frac{dy}{dx}=\frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx}=\frac{x^2e^x+t}{x} \quad \Rightarrow \frac{dt}{dx}-\frac{t}{x}=xe^x$

I.F $= e^{\int -\frac{1}{x}dx}=\frac{1}{x}$

solution is $\frac{t}{x}=\int e^xdx$

$\Rightarrow \quad \frac{t}{x}=e^x+c \Rightarrow \ln y = xe^x + cx$

58. Ans.(C)

We know that $\frac{a}{b}=\frac{c}{d}=\frac{a+c}{b+d}$

$\frac{d(x+y)}{\tan(x+y)+\cot(x+y)}=\frac{dz}{1}$

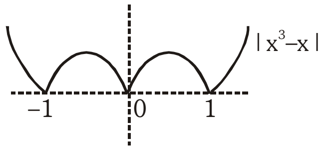
$(x + y) = t$

$\int \sin t\cos t.dt=\int dz \Rightarrow -\frac{1}{4}\cos 2t + c = z.$

$z=-\frac{\cos(2x+2y)}{4}+c$

59. Ans.(B)

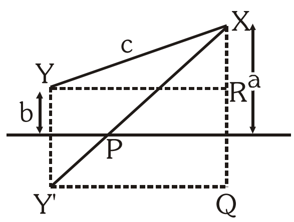
$y = |x^3 - x| = |x|\,|x^2 -1|$ is an even function



Area bounded $=2\int_0^1(x-x^3)dx$

$=2\left|\frac{x^2}{2}-\frac{x^4}{4}\right|_0^1=\frac{2}{4}=\frac{1}{2}$

60. Ans. (D)



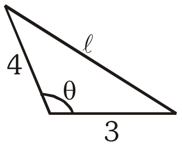
To find the point where pumpset is to be installed reflect point 'Y' w.r.t. river say Y' join with X intersection point of Y'X with river is the point 'P'.

$XR = (a - b), \qquad YR = \sqrt{c^2-(a-b)^2}$

$XY' = \sqrt{c^2-(a-b)^2+(a+b)^2}$

$XY' = \sqrt{c^2 + 4ab}$.

61. Ans.(B)



Let angle between the hands be θ and distance between tips be ℓ

$\ell = \sqrt{25 - 24\cos\theta}$

we have to find ℓ when $\frac{d\ell}{d\theta}$ is maximum

$\frac{d\ell}{d\theta} = \frac{12\sin\theta}{\sqrt{25 - 24\cos\theta}}$ to maximum $\frac{d\ell}{d\theta}$

$$\frac{d^2\ell}{d\theta^2} = 12\left\{\frac{\sqrt{25-24\cos\theta}.\cos\theta - \frac{24\sin^2\theta}{2\sqrt{25-24\cos\theta}}}{(25-24\cos\theta)}\right\} = 0$$

$(25 - 24\cos\theta)\cos\theta - 12\sin^2\theta = 0$

$12\cos^2\theta - 25\cos\theta + 12 = 0$

$\cos\theta = \frac{3}{4}$ or $\cos\theta = \frac{4}{3}$ (Not possible)

hence $\ell = \sqrt{25 - 24.\frac{3}{4}} = \sqrt{7}$

62. Ans.(C)

$f(x) = \cos|x| - 2ax + b$

$f(x) = \cos x - 2ax + b$

$f'(x) = -\sin x - 2a$

$f(x)$ be increasing along entire number line

$\Rightarrow f'(x) \geq 0$

$-\sin x - 2a \geq 0$

$a \leq -\frac{\sin x}{2}$

$-\frac{1}{2} \leq \frac{-\sin x}{2} \leq \frac{1}{2} \Rightarrow a \leq -\frac{1}{2}$

63. Ans. (B)

$\left.\begin{array}{l}\alpha + \beta = -6 \\ 3\alpha + 2\beta = -20\end{array}\right\}$ solving $\alpha = -8, \beta = 2$

$\lambda = \alpha\beta = -16$

64. Ans. (D)

$D \geq 0 \Rightarrow 4p^2 - 20p(p-3) \geq 0$

$\Rightarrow p^2 - 5p(p-3) \geq 0$

$\Rightarrow p(p - 5p + 15) \geq 0$

$\Rightarrow p(4p - 15) \geq 0 \quad \Rightarrow \quad 0 \leq p \leq \frac{15}{4}$

Sum of the roots > 0

Product the roots > 0

$\frac{2p}{p-3} > 0 \quad \& \quad \frac{5p}{p-3} > 0$

combining $p \in \left(3, \frac{15}{4}\right]$

65. Ans. (C)

$D > 0 \quad 4m^2 - 4\ell n > 0 \Rightarrow m^2 - \ell n > 0$

$D' = (\ell^2 + \ell n - 2\ell n + 2m^2)x^2 + (2\ell m + 2mn)x$

$+ (\ell n + n^2 - 2\ell n + 2m^2) = 0$

$D' = 4m^2(\ell + n)^2 - 4(\ell^2 + 2m^2 - \ell n)(n^2 - \ell n + 2m^2)$

$< 4\{m^2(\ell + n)^2 - (\ell^2 + m^2)(n^2 + m^2)\}$

$= 4\{m^2\ell^2 + m^2n^2 + 2\ell nm^2 - \ell^2n^2 - \ell^2m^2 - m^2n^2 - m^4\}$

$= -4(m^2 + \ell n)^2 < 0$

$\Rightarrow$ Roots are imaginary.

66. Ans. (D)

$kx^2 + x(2k-1) + k - 2 = 0$

$D = (2k-1)^2 - 4k(k-2)$

$= 4k^2 + 1 - 4k - 4k^2 + 8k$

$D = 4k + 1$

D must be a perfect seven

$\Rightarrow$ k = 12 (from the options)

67. Ans. (A)

Roots of $ax^2 + bx + c = 0$ & $cx^2 + bx + a = 0$ are reciprocals of each other

$\therefore \quad \alpha < 1 \;\&\; \beta > -\frac{1}{3}$

68. Ans. (A,D)

69. Ans. (A,B,D)

$\frac{dy}{dx} = \frac{e^x}{\sqrt{1 - e^{2x}}}$

$\therefore \quad \int dy = \int \frac{d(e^x)}{\sqrt{1 - (e^x)^2}}$

$y = \sin^{-1}(e^x) + c = \cos^{-1}\sqrt{1-e^{2x}} + c$

$y = \frac{\pi}{2} + c - \cos^{-1}(e^x)$

70. Ans. (A,B,C)

$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 2 & 3 \\ 1 & 1 & 2 \end{vmatrix} = 1.(4-3) - 2(4-3) + 0 = 1 - 2 = -1$

$\therefore \quad A^{-1} = -\text{adj}(A)$

$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 & 1 \\ 2 & 2-\lambda & 3 \\ 1 & 1 & 2-\lambda \end{vmatrix} = (1-\lambda)\{\lambda^2 - 4\lambda + 4 - 3\} - 2(4 - 2\lambda - 3) + (2 - 2 + \lambda)$

$= (1-\lambda)(\lambda^2 - 4\lambda + 1) - 2(1 - 2\lambda) + \lambda = \lambda^2 - 4\lambda + 1 - \lambda^3 + 4\lambda^2 - \lambda - 2 + 4\lambda + \lambda = -\lambda^3 + 5\lambda^2 - 1 = 0$

$\therefore$ characteristic equation is

$A^3 - 5A^2 + I = 0$

$A^2(A - 5I) = -I$

$\therefore \quad A^2(5I - A) = I$

$\therefore \quad A^{-2} = 5I - A$

71. Ans. (A,D)

$6y^2 . y' = 2tx + 3x^2$

$y' = \frac{2t^2 + 3t^2}{6t^2} = \frac{5}{6} \Rightarrow \quad (y - t) = \frac{5}{6}(x - t) \Rightarrow$

$6y - 6t = 5x - 5t$

$\Rightarrow \quad 5x - 6y = -t \quad \Rightarrow \quad p = -t/5, q = t/6$

$p^2 + q^2 = 61$

$\Rightarrow \frac{t^2}{25} + \frac{t^2}{36} = 61 \Rightarrow t^2 = 25.36 \Rightarrow t = \pm 30$

72. Ans. (A,B,C)

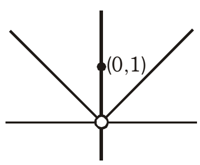
(A) Consider $f(x) = \tan x$

$\frac{3\pi}{4} > \frac{\pi}{4}$

$\not\Rightarrow \quad \tan\left(\frac{3\pi}{4}\right) > \tan\left(\frac{\pi}{4}\right)$

false

(B) $f(x) = \begin{cases} |x|, & x \neq 0 \\ 1, & x = 0 \end{cases}$



false

(C) $f(x) = x^3$ is monotonic function but $f'(x) = 0$ has real roots.

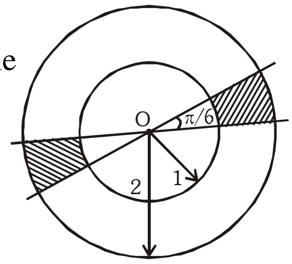
73. Ans. (B,D)

$f(x) = \frac{5x}{x^4 + a}$

for the function to be bounded, denominator must not be equal to zero at any value of x.

$\therefore$ 'a' can be any positive real numbers.

74. Ans. (A)



The angle θ between the lines represented by

$\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$

is given by

$\theta = \tan^{-1}\left(\frac{2\sqrt{4-3}}{\sqrt{3}+\sqrt{3}}\right) = \frac{\pi}{6}$

Hence shaded area $= \frac{\pi}{6}.(2^2 - 1^2) = \frac{\pi}{2}$

75. Ans. (B,C,D)

Clearly $g(x) = f^{-1}(x) = \log(x + \sqrt{1+x^2})$

$\therefore \quad g'(x) = \frac{1}{\sqrt{1+x^2}} > 0 \ \forall x \in R$

$\because \quad g'(x) \neq 0 \ \forall x \in R$

So no tangent is parallel to x axis.

$\Rightarrow$ g(x) has no extremum.

At (0, 0) $\quad g'(0) = 1$

So equation of tangent is $y = x$

76. Ans. (A,B,D)

Let $y = f(x)$ be the curve.

Equation of tangent at (x, y) will be

$Y - y = f'(x)(X - x)$

Intercept cut by the tangent on y-axis will be $y - xf'(x)$

so $y - xf'(x) = y\frac{dy}{dx} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{y}{x+y}$

This is homogeneous equation of first degree & first order.

Rewriting this equation $\frac{dx}{dy} - \frac{x}{y} = 1$ which is a linear equation.

77. $(a + 2)x^2 + bx + c = 0 \quad$ ...(i)

$2x^2 + 3x + 4 = 0 \quad$ ...(ii)

for (2) $D < 0$

$\Rightarrow$ (i) & (ii) have both roots common.

$\therefore \quad \frac{a+2}{2} = \frac{b}{3} = \frac{c}{4}$

$\therefore$ for equation (1) $D < 0$

$\Rightarrow \; b^2 - 4c(a+2) < 0$

$b^2 < 4ac + 8c.$

$\frac{a+2}{2} = \frac{b}{3} = \frac{c}{4} = k(\text{say}) \Rightarrow \{k \in N, k \geq 2\}$

$\frac{a+b+c+2}{9} = k$

$a + b + c = 9k - 2$

$(a + b + c)_{min} = 16.$

78. $\Delta = \begin{vmatrix} 2a & 2b & b-c \\ 2b & 2a & a+c \\ a+b & a+b & b \end{vmatrix}$

$R_2 \to R_2 + R_1$

$\Delta = \begin{vmatrix} 2a & 2b & b-c \\ 2(a+b) & 2(a+b) & a+b \\ a+b & a+b & b \end{vmatrix}$

$\Delta = (a+b)\begin{vmatrix} 2a & 2b & b-c \\ 2 & 2 & 1 \\ a+b & a+b & b \end{vmatrix}$

$C_1 \to C_1 - C_2$

$= (a+b)\begin{vmatrix} 2(a-b) & 2b & b-c \\ 0 & 2 & 1 \\ 0 & a+b & b \end{vmatrix}$

$= (a + b) \,.\, 2(a - b)(2b - a - b)$

$= 2(a + b)(a - b)(b - a) = -2(a + b)(a - b)^2.$

79. $A(\text{adj } A) = |A|\, I = (\text{adj } A)\, A$

$(\text{adj } A)^{-1} A^{-1} = \frac{I}{|A|} = A^{-1}(\text{adj } A)^{-1}$ .....(i)

{taking Inverse}

Also

$A^{-1}(\text{adj } A^{-1}) = |A^{-1}|.\, I$

$\Rightarrow A^{-1}(\text{adj } A^{-1}) = \frac{1}{|A|} I$ .....(ii)

$A^{-1}(\text{adj } A^{-1}) = A^{-1}(\text{adj } A)^{-1}$ {from (i) & (ii)}

$(\text{adj } A^{-1}) = (\text{adj } A)^{-1}$ option (B) is correct.

$A(\text{adj } A) = |A|\, I$

$\Rightarrow \frac{A}{|A|}(\text{adj } A) = I \quad \Rightarrow \quad (\text{adj } A)^{-1} = \frac{A}{|A|}$

option (A) is correct.

80. $-(A - I_3)^T = -(A - AA^T)^T = -(A^T - A^TA) = -A^T(I - A) = A^T(A - I)$

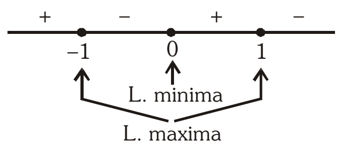
Also. $AA^T = I \Rightarrow |A|^2 = I \Rightarrow |A| = \pm 1.$

81. **Ans. (B,C,D)**

$g'(x) = 14xe^{-x^2} - 14x^3e^{-x^2}$

$= 14xe^{-x^2}(1 - x^2) = -14x(x+1)(x-1)e^{-x^2}$

sign of g'(x)



82. **Ans. (B,D)**

Let the common tangent is

$y = mx \pm \sqrt{12m^2 - 12}$ solving it with

$xy = 8 \quad \Rightarrow \quad mx^2 \pm \sqrt{12(m^2-1)}x - 8 = 0$

$D = 0 \quad \Rightarrow \quad 12(m^2 - 1) + 32m = 0$

$3m^2 + 8m - 3 = 0 \Rightarrow \quad m = -3, \frac{1}{3}$

$y = -3x \pm 4\sqrt{6} \quad m = \frac{1}{3}$ rejected

83. **Ans. (A,D)**

For $x \geq 1$ $f(x)$ is an increasing function. For minimum at $x = 1$, so

$f(1^-) \geq f(1) \quad \Rightarrow \quad -1 + \cos^{-1}a \geq 1$

$\Rightarrow \quad a \leq \cos 2$

84. **Ans. (A,B,C,D)**

From the first equation $\frac{dy}{dx} = b$

and from the second equation $\frac{dy}{dx} = -\frac{x}{y}$

$\Rightarrow \quad b\left(-\frac{x}{y}\right) = -1 \Rightarrow \quad y = bx$

So the curves are perpendicular for every value of a & b

85. **Ans.(A,C)**

$y = A\cos 2x + B\sin^2 x + C$

$y = A(1 - 2\sin^2 x) + B\sin^2 x + C$

$\Rightarrow y = k_1 + k_2\sin^2 x \quad \Rightarrow y' = k_2\sin 2x$

$\Rightarrow y'' = k_2 2\cos 2x \quad \Rightarrow \quad y'' = \frac{2y'\cos 2x}{\sin 2x}$

$\Rightarrow \sin 2x \frac{d^2y}{dx^2} = 2\frac{dy}{dx}(\cos 2x)$

$\Rightarrow \lambda = 0$ & $f(x) = \sin 2x$

Solution of sin2x = 0 in (0,4) are $x = \frac{\pi}{2}$ & π

86. Ans. (A,B,C)

$x\frac{dy}{dx} - y = xy^4$

Dividing both sides by y^4 & putting $\frac{1}{y^3} = t$

$\Rightarrow \quad -\frac{x}{3}\frac{dt}{dx} - t = x \quad$ or $\quad \frac{dt}{dx} + \frac{3t}{x} = -3$

which on solving gives $\frac{x^3}{y^3} = \frac{-3x^4}{4} + c$

$\because \quad y(1) = 2 \quad \Rightarrow \quad c = \frac{7}{8}$

Hence $y^3 = \frac{8x^3}{7-6x^4} \quad \Rightarrow \quad f(x) = \sqrt[3]{\frac{8x^3}{7-6x^4}}$

which is odd and discontinuous at point where $7 - 6x^4 = 0$.

87. Ans.(A,B)

$$f(x) = \begin{cases} 2 - x + k, & x \le 2 \\ 3x + 2, & x > 2 \end{cases}$$

$\because$ y = 2 – x + k is a decreasing function & y = 3x +2 is an increasing function, f (x) will have a local minimum at x= 2 if $f(2) \le f(x)$ for x > 2

$\Rightarrow \quad k \le 3x + 2$ for x > 2

$\Rightarrow \quad k \le 8$

88. Ans. (A,C,D)

$D_1 = (AB + AC) - BC$; $D_3 = (AC + BC) - AB$

$D_2 = (BC + AB) - AC$; for Δ all are +ve (Distinct & real)

If collinear then let AB + BC = AC

$D_3 = 0$ & all others will be +ve

$\left.\begin{matrix}\alpha + \beta = -ve \\ \alpha\beta = +ve\end{matrix}\right\}$ in each case $\Rightarrow$ both roots are negative

89. Ans. (A,B,C,D)

Let m, n $\in$ I satisfy $ax^2 + bx + c = p$

$\Rightarrow \quad m + n = -\frac{b}{a} \quad mn = \frac{c-p}{a}$

Let $\exists\ k \in I$ such that

$ak^2 + bk + c = 2p$

$\Rightarrow \quad ak^2 - a(m+n)k + amn = p$

$\Rightarrow \quad a(k - m)(k - n) = p$

non-prime = prime

$\Rightarrow$ a contradiction

Hence no integral value of x

90. Ans. (A)

Statement-2 : $(A - B)^2 = (A - B)(A - B)$

$= A^2 - AB - BA + B^2 = A^2 - A - B + B^2$

$= A - A - B + B = 0 \qquad \{\because \quad AB = A$

$\Rightarrow \quad ABA = A^2$

$AB = A^2 \quad \Rightarrow \quad A = A^2$

$BA = B \quad \Rightarrow \quad BAB = B^2$

$BA = B^2 \quad \Rightarrow \quad B = B^2\}$

Statement-1 : $(A - B)^8 = (A - B)^2 . (A - B)^6 = 0$

$\therefore \quad k = 0$

91. Ans. (A)

As $A(I_3 + B + B^2) = (I - B)(I + B + B^2)$

$= I^3 - B^3 = I \quad \Rightarrow \quad A^{-1} = I + B + B^2$

Also $Ax = b \quad \Rightarrow \quad X = A^{-1}b = (I + B + B^2)b$

$= b + Bb + B^2b$ is the solution

92. Ans. (A)

Given

$a + d = 14$ .... (i)

$b + c = 12$ .... (ii)

$b^2 = ac$ .... (iii)

$2c = b + d$ .... (iv)

(iv) in (i)

$a + 2c - b = 14$

Let r is common ratio of G.P.

$a - ar + 2ar^2 = 14$ .... (v)

$ra + ar^2 = 12$ .... (vi)

(v) $\div$ (vi)

$\frac{1 - r + 2r^2}{r + r^2} = \frac{7}{6}$

$\Rightarrow \quad 5r^2 - 13r + 6 = 0$

$\Rightarrow \quad 5r^2 - 10r - 3r + 6 = 0$

$r = \frac{3}{5}, 2 \qquad r \ne \frac{3}{5} \qquad$ so r = 2

as increasing G.P.

$a = \frac{12}{2+4} = 2$

a = 2, b = 4, c = 8, d = 12

for these values of a, b, c lines are concurrent and we know if there exist three scalar $\lambda_1, \lambda_2, \lambda_3$ such that $\lambda_1 L_1 + \lambda_2 L_2 + \lambda_3 L_3 = 0$, then lines are concurrent therefore II is correct explanation.

93. **Ans. (A)**

$\det(\text{adj}A) = (\det(A))^{n-1}$

$\because$ A is odd ordered matrix.

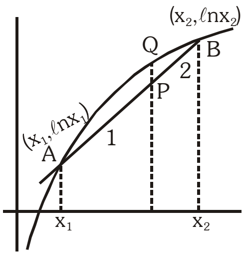
$\Rightarrow$ $|\det(\text{adj}A)|$ is a perfect square

$\Rightarrow$ Statement-1 & Statement-2 are true & Statement-2 explains Statement-1.

94. **Ans. (A)**

Consider $f(x) = \ell nx$

Let P divides AB in the ratio 1 : 2



$$P\left(\frac{2x_1 + x_2}{3}, \frac{2\ell nx_1 + \ell nx_2}{3}\right)$$

From the graph it is clear that

$$f\left(\frac{2x_1 + x_2}{3}\right) > \frac{2\ell nx_1 + \ell nx_2}{3}$$

$$\ell n\left(\frac{2x_1 + x_2}{3}\right) > \frac{2\ell nx_1 + \ell nx_2}{3}$$

95. Statement-1 : Δ_1 can be obtained by replacing each element of Δ_2 by their respective cofactors

$\therefore$ $\Delta_1 = \Delta_2^2 \Rightarrow \Delta_1\Delta_2 = \Delta_2^3$ (True)

Statement-2 : Obviously True

Statement-2 is a correct explanation for Statement-1

Aliter :

$\Delta_1 = y^2z^3 \,.\, xz^3$

$$.xy^2\begin{vmatrix} y^3z^3(z^3-y^3) & -x^3z^3(z^3-x^3) & x^3y^3(y^3-x^3) \\ -(z^6-y^6) & z^6-x^6 & -(y^6-x^6) \\ z^3-y^3 & -(z^3-x^3) & (y^3-x^3) \end{vmatrix}$$

$$\Delta_1 = x^2y^4z^6(z^3-y^3)(z^3-x^3)(y^3-x^3)\begin{vmatrix} y^3z^3 & x^3z^3 & x^3y^3 \\ z^3+y^3 & z^3+x^3 & y^3+x^3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= x^2y^4z^6(z^3-y^3)(z^3-x^3)(y^3-x^3)\begin{vmatrix} y^3z^3 & z^3(x^3-y^3) & y^3(x^3-z^3) \\ z^3+y^3 & x^3-y^3 & x^3-z^3 \\ 1 & 0 & 0 \end{vmatrix}$$

$(C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1)$

$$= x^2y^4z^6(y^3-x^3)^2(z^3-y^3)^2(z^3-x^3)^2$$

$$\Delta_2 = xy^2z^3\begin{vmatrix} 1 & 1 & 1 \\ x^3 & y^3 & z^3 \\ x^6 & y^6 & z^6 \end{vmatrix} = xy^2z^3\begin{vmatrix} x^3-y^3 & y^3-z^3 \\ x^6-y^6 & y^6-z^6 \end{vmatrix}$$

$$= xy^2z^3(x^3-y^3)(y^3-z^3)(z^3-x^3)$$

$$\Delta_1\Delta_2 = x^3y^6z^9(x^3-y^3)^3.(y^3-z^3)^3(z^3-x^3)^3 = \Delta_2^3$$

96. Statement 2 :

$$\begin{bmatrix} 1 & x_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & x_2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & x_1+x_2 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & x_1+x_2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & x_3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & x_1+x_2+x_3 \\ 0 & 1 \end{bmatrix}$$

and so on

$$\Rightarrow \begin{bmatrix} 1 & x_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & x_2 \\ 0 & 1 \end{bmatrix}\ldots\ldots\ldots\begin{bmatrix} 1 & x_n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \sum x_i \\ 0 & 1 \end{bmatrix}$$

using it in st. 1

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\ldots\begin{bmatrix} 1 & 27 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1+2+\ldots.27 \\ 0 & 1 \end{bmatrix}$$

$= \begin{bmatrix} 1 & 378 \\ 0 & 1 \end{bmatrix}$ so 2^{nd} is false.

97. **Ans. (C)**

Statement-1 :

Consider $f(x) = x^4$

Applying LMVT in [a, b], we get $4c^3 = \dfrac{b^4 - a^4}{b-a}$

$$\frac{c}{a+b} = \frac{b^2+a^2}{4c^2}$$

Statement-1 is true

Statement-2 is converse of Rolles theorem which is not true.

98. **Ans. (D)**

Statement-1 :

$$y = \frac{x+1-3}{x+1} = 1 - \frac{3}{x+1}$$

$$y' = \frac{3}{(x+1)^2}$$

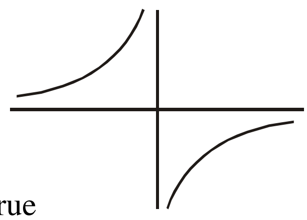
$\Rightarrow$ θ is acute angle $\Rightarrow \cos\theta > 0 \Rightarrow e^{\cos\theta} > 1$

Statement-1 is false.

Statement-2 :

For $xy = -c^2$

$$\frac{dy}{dx} > 0$$



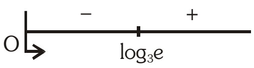
$\Rightarrow$ Statement-2 is true

99. **Ans. (D)**

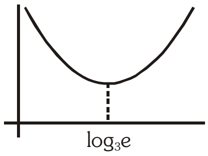
Statement-2 :

$$f(x) = \frac{3^x}{x}$$

$f'(x) = \dfrac{x.3^x \ell n3 - 3^x}{x^2} = \dfrac{3^x(x\ell n3 - 1)}{x^2}$



$f(\log_3 e)$ is the least value of the function



$f(\log_3 e) = \dfrac{e}{\log_3 e}$ is the least value

Hence statement-1 is false

Statement-2 is obviously true.

Paragraph for Question 100 to 102

100. Ans. (D)

$$\Delta = \begin{vmatrix} a^3 - 3a + 1 & 6a & 1 \\ b^3 - 3b + 1 & 6b & 1 \\ c^3 - 3c + 1 & 6c & 1 \end{vmatrix}$$

$$C_1 \to C_1 + \frac{1}{2}C_2 - C_3$$

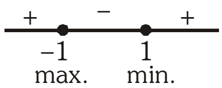
$$\Delta = 6\begin{vmatrix} a^3 & a & 1 \\ b^3 & b & 1 \\ c^3 & c & 1 \end{vmatrix}$$

$= 6(a + b + c)(a - b)(b - c)(c - a)$

$= 0$ $\{as\ (a + b + c) = 0\}$

101. Ans. (D)

$f' = 3(x^2 - 1) = 0$



$\Rightarrow \quad x = 1, -1$

$\alpha = 1, \beta = -1$

$f'' = 0$

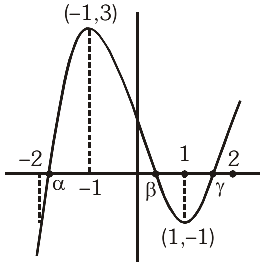
$\Rightarrow \quad x = 0$ is point of inflection

so P(1, –1) Q(–1, 3) R(0, 1)

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ -1 & 3 & 1 \\ 0 & 1 & 1 \end{vmatrix} = \frac{1}{2}[2 - 1 - 1] = 0$$

so collinear

102. Ans. (C)



$f'(x) = 3(x^2 - 1)$

$f(0) = 1$

$f(1) = -1$

$f(-1) = 3$

$f(f(x)) = 0$

$\Rightarrow f^3(x) - 3f(x) + 1 = 0 \Rightarrow f(x) = \alpha, \beta, \gamma$

$f(x) = \alpha$

$\Rightarrow$ exactly one solution as $\alpha \in (-1, -2)$

$f(x) = \beta$

$\Rightarrow$ exactly three distinct solutions as $\beta \in (0, 1)$

$f(x) = \gamma$

$\Rightarrow$ exactly three distinct solutions as $\gamma \in (1, 2)$

so total seven solutions.

Paragraph for Question 103 to 105

103. Consider $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= \dfrac{(a + b + c)}{2}\left((a - b)^2 + (b - c)^2 + (c - a)^2\right)$

if $a \neq b \neq c$ & $a + b + c \neq 0$, then $\Delta \neq 0$

lines are not concurrent & no solution.

104. a = b = c makes all the lines coincident

$a(x + y + 1) = 0$

$a \neq 0$ because $a + b + c \neq 0$

$x + y + 1 = 0 \Rightarrow$ infinite solutions, all lying on the line $x + y + 1 = 0$.

105. $a + b + c = 0$ & $a = b = c$

simultaneously given $a = b = c = 0$

All equations are $(0)x + (0)y + (0) = 0$, become identities which satisfied by all point in xy plane.

Paragraph for Question 106 to 108

$x - 2y + bz = 3$

$ax + 2z = 2$

$5x + 2y = 1$

$$\Delta = \begin{vmatrix} 1 & -2 & b \\ a & 0 & 2 \\ 5 & 2 & 0 \end{vmatrix} = -24 + 2ab$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 & 3 \\ a & 0 & 2 \\ 5 & 2 & 1 \end{vmatrix}$$

$\Delta_1 = 8a - 24$

$$\Delta_2 = \begin{vmatrix} 1 & 3 & b \\ a & 2 & 2 \\ 5 & 1 & 0 \end{vmatrix}$$

$\Delta_2 = 28 + ab - 10b.$

$$\Delta_3 = \begin{vmatrix} 3 & -2 & b \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{vmatrix}$$

$\Delta_3 = 16 + 4b.$

106. $ab = 12 \Rightarrow \Delta = 0$

$a \neq 3 \Rightarrow \Delta_1 \neq 0$

Hence no solution

107. $ab \neq 12 \Rightarrow \Delta \neq 0$

$\Rightarrow$ system has unique solution.

108. $a = 3$ & $b = 4$

$\Rightarrow \Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$\Rightarrow$ system has infinite solution.

Paragraph for Question 109 to 111

$AB = -\dfrac{D}{4a} = +2$

$\because \quad D = b^2 - 4ac = -4$

$\therefore \quad \dfrac{4}{4a} = 2 \Rightarrow \boxed{a = \dfrac{1}{2}}$

$AC = 3 \Rightarrow -\dfrac{b}{2a} = -3 \Rightarrow b = 6a$

$b = 6 \cdot \dfrac{1}{2} \Rightarrow \boxed{b = 3}$

$b^2 - 4ac = -4$

$9 - 4\left(\dfrac{1}{2}\right) c = -4 \Rightarrow \boxed{c = \dfrac{13}{2}}$

109. $a + b + c = \dfrac{1}{2} + 3 + \dfrac{13}{2} = 10.$

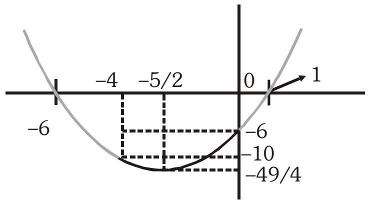
110. $\alpha = b + \sqrt{a+c} = 3 + \sqrt{7}$ & $\beta = 3 - \sqrt{7}$

$\alpha + \beta = 6, \alpha\beta = 2$

$\therefore$ Equation is $x^2 - 6x + 2 = 0$

111. $g(x) = x^2 + 5x - 6 = (x + 6)(x - 1)$

$g(-4) = -10$ & $g(0) = -6$



$g(x)_{min} = -\dfrac{49}{4}$ at $x = -\dfrac{5}{2}$

$\therefore$ Range of g(x) is $\left[-\dfrac{49}{4}, -6\right]$

Paragraph for Question 112 to 114

112. $\dfrac{2}{3} \leq x \leq \dfrac{4}{5} \Rightarrow \dfrac{3}{2} \geq \dfrac{1}{x} \geq \dfrac{5}{4} \Rightarrow 6 \geq \dfrac{4}{x} \geq 5$

$\Rightarrow 3 \geq \dfrac{4}{x} - 3 \geq 2 \Rightarrow 2 \leq \dfrac{4-3x}{x} \leq 3$

113. $-3 < \dfrac{2x-7}{x} < -1 \Rightarrow -5 < -\dfrac{7}{x} < -3$

$\Rightarrow 5 > \dfrac{7}{x} > 3 \Rightarrow \dfrac{5}{7} > \dfrac{1}{x} > \dfrac{3}{7}$

$\Rightarrow \dfrac{7}{5} < x < \dfrac{7}{3}$

114. $-2 < x \leq 3 \Rightarrow \dfrac{1}{x} < -\dfrac{1}{2}$ or $\dfrac{1}{x} \geq \dfrac{1}{3}$

$\Rightarrow \dfrac{6}{x} < -3 \Rightarrow \dfrac{6}{x} \geq 2$

$\Rightarrow 5 + \dfrac{6}{x} < 2 \Rightarrow 5 + \dfrac{6}{x} \geq 7$

$\therefore \quad 5 + \dfrac{6}{x} \in (-\infty, 2) \cup [7, \infty)$

Paragraph for Question 115 to 117

$$C = \begin{bmatrix} 3x^2 & 0 & 0 \\ 0 & -3x & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$D^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

$$Y = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} + \begin{bmatrix} \frac{1}{2^2} & 0 & 0 \\ 0 & \frac{1}{2^2} & 0 \\ 0 & 0 & \frac{1}{3^2} \end{bmatrix} + \ldots.$$

(infinitely many terms)

$$Y = \begin{bmatrix} \dfrac{\frac{1}{2}}{1-\frac{1}{2}} & 0 & 0 \\ 0 & \dfrac{\frac{1}{2}}{1-\frac{1}{2}} & 0 \\ 0 & 0 & \dfrac{\frac{1}{3}}{1-\frac{1}{3}} \end{bmatrix}$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$$

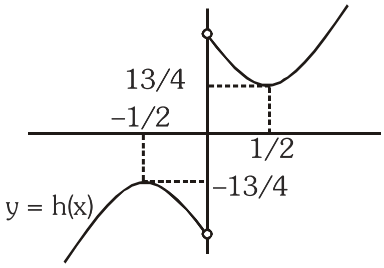
$$Z = -\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \{\because Y + Z = I\}$$

$$Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$$

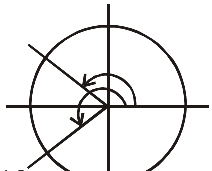
$f(x) = 3x^2 - 3x + 4$ {Trace of matrix C}

$$g(x) = \begin{cases} 3x^2 - 3x + 4, & x > 0 \\ -3x^2 - 3x - 4, & x < 0 \end{cases}$$

115.



116. Range of $f(x)$ is $\left[\frac{13}{4}, \infty\right)$



$\frac{13}{4}$

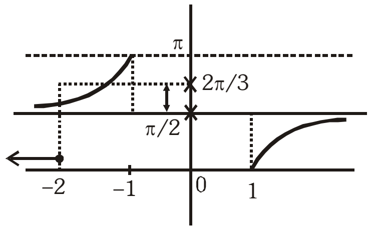
$\therefore \cos^{-1}\cos(a) = \cos^{-1}\cos\frac{13}{4} = \pi - \left(\frac{13}{4} - \pi\right)$

$= 2\pi - \frac{13}{4}$.

117. Let $t(x) = \sec^{-1}\left(h(x) + \frac{5}{4}\right)$

$-\infty < h(x) \le -\frac{13}{4}$

$-\infty < h(x) + \frac{5}{4} < -2$



from graph the range of t(x) is $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$.

Paragraph for Question 118 to 120

118. Ans. (C)

$v(x) = \pi(x + 1)^2 \times 1 + \pi h\{(x + 1)^2 - x^2\}$

$= \pi(x + 1)^2 + \pi h(2x + 1)$ ....(i)

also $27\pi = \pi x^2 h$

$h = \frac{27}{x^2}$ ....(ii)

from (i) and (ii)

$v(x) = \pi(x + 1)^2 + \frac{27\pi}{x^2}(2x + 1)$

$= \pi\left\{(x+1)^2 + \frac{54}{x} + \frac{27}{x^2}\right\}$

a = 1, b = 54, c = 27

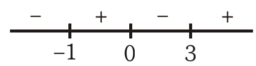
$\frac{a.b}{c} = 2 \quad \Rightarrow \quad$ option (C)

119. Ans. (A)

$v'(x) = \pi\left\{2(x+1) - \frac{54}{x^2} - \frac{54}{x^3}\right\}$

$= 2\pi\left\{\frac{x^4 + x^3 - 27x - 27}{x^3}\right\} = 2\pi\left\{\frac{x^3(x+1) - 27(x+1)}{x^3}\right\}$

$= 2\pi\left\{\frac{(x+1)(x-3)(x^2+3x+9)}{x^3}\right\}$

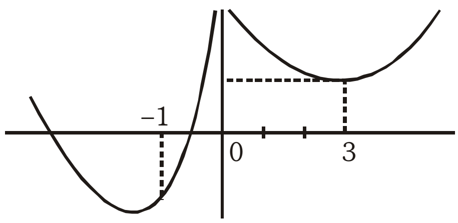


at x = 3 function has global minima

at x = 3, h = 3 from equation (i)

120. Ans. (B)

On the basis of sign scheme of v'(x) in Q.10 graph of v(x) is as shown below.



from figure it is clear that the equation v(x) = 0 has 2 roots.

Paragraph for Question 121 to 123

121. Ans. (A)

$ydx(e^x - e^{-x}) = (e^x + e^{-x})dy$

$\Rightarrow \quad \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{dy}{y}$

$\ell n(e^x + e^{-x}) = \ell n(ky)$, where k is a constant

$e^x + e^{-x} = ky$

$\because$ $y = f(x)$ passes through (0, 2)

$\therefore$ $k = 1$

$\therefore$ $f(x) = e^x + e^{-x}$

$g(x) = \log_2(e^x + e^{-x})$

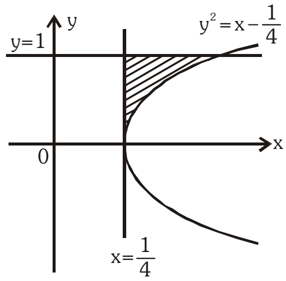
$\therefore$ $1 \le g(x) < \infty$

122. Ans. (D)

$$y = \log_{\frac{1}{4}}\left(x - \frac{1}{4}\right) + \frac{1}{2}\log_4\left\{(16)\left(x - \frac{1}{4}\right)^2\right\}$$

$$= \log_{\frac{1}{4}}\left(x - \frac{1}{4}\right) + 1 - \log_{\frac{1}{4}}\left(x - \frac{1}{4}\right)$$

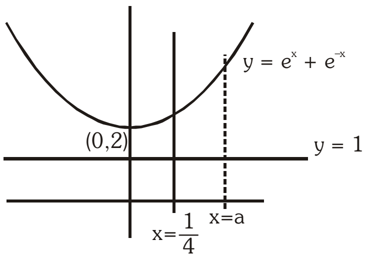
$\therefore$ $y = 1 \ \forall\ x > \frac{1}{4}$



$$A = \int_0^1 \left(y^2 + \frac{1}{4} - \frac{1}{4}\right)dy$$

$$= \frac{1}{3}$$

123. Ans. (B)



$$\int_{1/4}^{a} (e^x + e^{-x} - 1)dx = 4 - \ell n4 + \frac{1}{\sqrt[4]{e}} - \sqrt[4]{e}$$

$\Rightarrow$ $(e^x - e^{-x} - x)_{1/4}^{a} = 4 - \ell n4 + \frac{1}{\sqrt[4]{e}} - \sqrt[4]{e}$

$\Rightarrow$ $e^a - e^{-a} - a - \sqrt[4]{e} + \frac{1}{\sqrt[4]{e}} + \frac{1}{4}$

$$= 4 - \ell n4 + \frac{1}{\sqrt[4]{e}} - \sqrt[4]{e}$$

$\Rightarrow$ $e^a - e^{-a} - a + \frac{1}{4} = 4 - \ell n4$ $\Rightarrow$ $a = \ell n4$

Paragraph for Question 124 to 126

Let points on both the curves be $(x, f(x))$ & $(x, g(x))$ where $g(x) = y_2$

$f(x) - m_1x = g(x) - m_2x$

where $m_1 = f'(x)$ and $m_2 = g'(x) = f(x)$

$f(x) - x.f'(x) = g(x) - x.f(x)$

$f'(x) - xf''(x) - f'(x) = g'(x) - xf'(x) - f(x)$

$\Rightarrow f''(x) = f'(x)$ $\Rightarrow f'(x) = f(x) + c$

$\because f'(0) = 1$ and $f(0) = 0 \Rightarrow c=1$

$\Rightarrow f'(x) = f(x) + 1$ $\Rightarrow f(x) = ke^x - 1$

$\because f(0) = 0 \Rightarrow k=1$ $\Rightarrow f(x) = e^x - 1$

124. Ans. (C)

$\because$ $y_1 = e^x - 1$ and $y_2 = \int_0^x e^t dt = e^x - x - 1$

$$\Rightarrow \text{required area} = \int_0^e |y_2 - y_1|dx = \frac{e^2}{2}$$

125. Ans. (C)

$\because$ $y_2 = e^x - x - 1$

$$\Rightarrow \quad \underset{x\to 0}{\text{Lt}} \frac{e^x - x - 1}{x^2} = \frac{1}{2}$$

127. Ans. (D)

$y_1 - y_2 = k \Rightarrow x = k$

$\Rightarrow$ only one point of intersection

Paragraph for Question 127 to 129

127. Ans. (A)

$\because$ $y^2 + y - x = 2$ $\Rightarrow$ $2y\frac{dy}{dx} + \frac{dy}{dx} = 1$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{2y+1}$$

put $x = 4 \Rightarrow y = 2, -3$

$y \in \left(-\infty, -\frac{1}{2}\right) \Rightarrow y = -3$

$$\Rightarrow \quad \left.\frac{dy}{dx}\right|_{x=4} = \left.\frac{1}{2y+1}\right|_{y=-3} = -\frac{1}{5}$$

Now $(1+2y)\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 0$

$$\Rightarrow \quad \left.\frac{d^2y}{dx^2}\right|_{x=4} = \frac{2}{125} \quad \Rightarrow \quad f''(4) = \frac{2}{125}$$

128. Ans. (B)

$\because$ g(x) is inverse of $f(x)$

$\therefore$ interchanging x and y, we get

$x^2 + x - y = 2$ where $y = g(x)$

$\Rightarrow$ $g'(x) = 2x + 1$ $\Rightarrow$ $g'(-2) = -3$

129. Ans. (D)

$\because$ h(x) is mirror image of $f(x)$ in $y = -\frac{1}{2}$

$\Rightarrow \quad \frac{f(x)+h(x)}{2} = -\frac{1}{2} \quad \Rightarrow \quad f(x)+h(x) = -1$

$\Rightarrow \quad f''(x) + h''(x) = 0 \;\; \forall \; x \in \left(-\frac{9}{4}, \infty\right)$

Paragraph for Question 130 to 132

130. Ans. (A)

The condition for polynomial $Ax^2 + Bx + C$ having both roots on opposite sides of given number 'd' is

$A \,.\, f(d) < 0$

$\Rightarrow \quad 4f(1) < 0$

$\Rightarrow \quad 4(4 - 4a + a^2 - 2a + 2) < 0$

$\Rightarrow \quad a^2 - 6a + 6 < 0$

critical points of a are, $\frac{6 \pm \sqrt{36-24}}{2} = 3 \pm \sqrt{3}$

$\Rightarrow \quad a \in \left(3-\sqrt{3},\; 3+\sqrt{3}\right)$

131. Ans. (C)

$f\left(\frac{1}{4}\right) = 0 \Rightarrow \quad \frac{1}{4} - a + a^2 - 2a + 2 = 0$

$\Rightarrow \quad a^2 - 3a + \frac{9}{4} = 0$

$\Rightarrow \quad \left(a - \frac{3}{2}\right)^2 = 0 \qquad \Rightarrow \quad a = \frac{3}{2}$

Now $1 + \frac{1}{a} + \frac{1}{a^2} + ...\infty - \frac{1}{1-\frac{1}{a}} = \frac{1}{1-\frac{2}{3}} = 3$

132. Ans. (B)

For a = 2, the $f(x) = 4x^2 - 8x + 2$

$= 4x^2 - 8x + 4 - 2$

$= (2x - 2)^2 - 2$

hence minimum value is –2.

Paragraph for Question 133 to 135

133. Ans. (D)

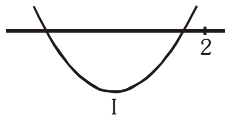
$2\left(x^2 + \frac{1}{x^2}\right) + k\left(x + \frac{1}{x}\right) + 22 = 0$

Let $x + \frac{1}{x} = t$

$2(t^2 - 2) + kt + 22 = 0$

$\Rightarrow \quad 2t^2 + kt + 18 = 0$

$t \geq 2$ or $t \leq -2$



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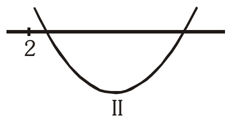

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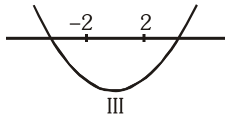
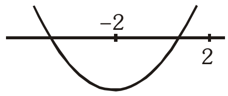


134. Ans. (D)



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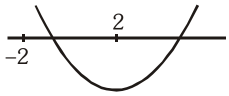


$f(-2) \,.\, f(2) \leq 0$

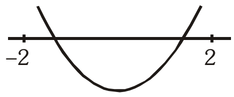
135. Ans. (A)

D of $2t^2 + kt + 18 = 0$ is < 0

$\Rightarrow \quad k^2 - 144 < 0$

$\Rightarrow \quad k \in (-12, 12)$

or



136. Ans. (A) → (R, S); (B) → (P, R); (C) → (Q); (D) → (P, R)

$f(x) = \frac{x^2 - 7x + 12}{2x^2 - 18x + 28}$

$f(x) - \frac{1}{2} = \frac{1}{2}\left\{\frac{x^2 - 7x + 12}{x^2 - 9x + 14} - 1\right\}$

$= \frac{1}{2}\left\{\frac{7x + 12 + 9x - 14}{x^2 - 9x + 14} - 1\right\}$

$\Rightarrow \quad \left\{\frac{2x - 7}{(x-7)(9-2)}\right\} = \frac{x-1}{(x-7)(x-2)}$

$f(x) = \frac{(x-4)(x-3)}{2(x-7)(x-2)}$

(A) If $3 < x < 4$

$f(x) - \frac{1}{2} < 0 \;\Rightarrow\; f(x) < \frac{1}{2}$

& $f(x) > 0$ (r, s)

(B) If $1 < x < 2$

$f(x) - \frac{1}{2} > 0 \;\Rightarrow\; f(x) > \frac{1}{2}$

& $f(x) > 0$ (p, r)

(C) If $4 < x < 7$

$f(x) - \frac{1}{2} < 0 \;\Rightarrow\; f(x) < \frac{1}{2}$

& $f(x) < 0$ (q)

(D) If $x > 7$

$f(x) - \frac{1}{2} > 0 \;\Rightarrow\; f(x) > \frac{1}{2}$

& $f(x) > 0$ (p, r)

137. Ans. (A)→(Q); (B)→(P, R); (C)→(P); (D)→(R)

$\lambda x + y + z = 1$
$x + \lambda y + z = \lambda$
$x + y + \lambda z = \lambda^2$

$$\Delta = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = (\lambda+2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix}$$

$$\Delta = (\lambda+2)\begin{vmatrix} 0 & 1-\lambda & 0 \\ 0 & \lambda-1 & 1-\lambda \\ 1 & 1 & \lambda \end{vmatrix}$$

$$\Delta = (\lambda+2)(\lambda-1)^2$$

$$\Delta_x = \begin{vmatrix} 1 & 1 & 1 \\ \lambda & \lambda & 1 \\ \lambda^2 & 1 & \lambda \end{vmatrix} = (\lambda^2-1)(1-\lambda) \ ; \ \Delta_y = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & \lambda^2 & \lambda \end{vmatrix} = (\lambda-1)^2 \ ;$$

$$\Delta_z = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & \lambda \\ 1 & 1 & \lambda^2 \end{vmatrix} = (\lambda-1)^2(\lambda+1)^2$$

(A) For $\lambda = 1$

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$ & all the three equations become identical so number of solutions is infinite.

(B) For $\lambda \neq 1$

but $\lambda = -2$ then $\Delta = 0$ & $\Delta_x, \Delta_y, \Delta_z \neq 0$

$\Rightarrow$ No solution

and if $\lambda \neq -2$ then $\Delta \neq 0$

$\Rightarrow$ unique solution

(C) for $\lambda \neq 1, \lambda \neq -2$

$\Delta \neq 0 \Rightarrow$ unique solution

(D) $\lambda = -2$

$\Delta = 0$ and $\Delta_x, \Delta_y, \Delta_z \neq 0 \Rightarrow$ No solution

138. Ans. (A)→(R); (B)→(P); (C)→(S); (D)→(Q)

$P^TP = I$

$R_K = P^T(PAP^T)(PAP^T)........(PAP^T)P$

$= (P^TP)A(P^TP)........A(P^TP)$

$(I)A(I)A.............A(I) = \underbrace{AA.......A}_{k\text{ times}} = A^K$

Similarly $T_K = B^K$

$R_K = A^K$,
$T_K = B^K$

$$A^2 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} \frac{1}{2} & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{2} & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2^2} & \frac{9}{2} \\ 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 7 & -12 \\ 3 & -5 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} \frac{1}{2^3} & \frac{21}{4} \\ 0 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 9 & -16 \\ 4 & -7 \end{bmatrix}; A^5 = \begin{bmatrix} 11 & -20 \\ 5 & -9 \end{bmatrix}$$

(A) $\sum_{K=1}^{5} a_K = a_1 + a_2 + ... + a_5$

$= 3 + 5 + 7 + 9 + 11 = 35$

(B) $\sum_{K=1}^{3} b_K = b_1 + b_2 + b_3 = -1 - 3 - 5 = -9$

(C) $\sum_{K=1}^{\infty} x_K = x_1 + x_2 + ... \infty$

$$= \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \infty = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

(D) $\sum_{K=1}^{10} y_K = y_1 + y_2 + ...+ y_{10} = \underbrace{1+1+......+1}_{10\text{ times}}$

$= 10$

139. Ans. (A)→(R), (B)→(Q), (C)→(P), (D)→(S)

(A) LHS of given equation is $f'(x)$ where $f(x) = (x-1)(x-2)(x-3)(x-4)$

$\because$ $f(x)$ is continuous & derivable and $f(1) = f(2) = 0$

$\therefore$ $f'(x) = 0$ has atleast one root in (1, 2) similarly $f'(x) = 0$ has atleast one root in (2, 3) & (3, 4)

$\because$ $f'(x)$ is a cubic function hence it has exactly one real root in (1, 2), (2, 3), (3, 4)

(B) $g(x) = 2f\left(\frac{x^2}{2}\right) + f(6-x^2)$

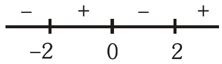
$$g'(x) = 2x\left\{f'\left(\frac{x^2}{2}\right) - f'(6-x^2)\right\}$$

$\frac{x^2}{2} > 6 - x^2 \Rightarrow x < -2$ or $x > 2$

for $\frac{x^2}{2} > 6 - x^2$

$f'\left(\frac{x^2}{2}\right) - f'(6 - x^2)$

$\{\because \; f''(x) > 0 \Rightarrow f'(x)$ is increasing$\}$



$\Rightarrow \quad g'(x) > 0 \;\; \forall\, x \in (2, \infty) \;\Rightarrow\; a = 2$

(C) Applying LMVT in [0, 1]

$\frac{e - 1}{1 - 0} = e^c$ for come $c \in (0, 1)$

$e^c + 1 = e \quad \Rightarrow \quad \ell n(e^c + 1) = 1$

(D) $f(x) = \begin{cases} mx + c, & x < 0 \\ e^x, & x \ge 0 \end{cases}$

for application of LMVT function must be continuous & derivable in [–2, 2]

$f(0^+) = f(0^-) \quad \Rightarrow \quad c = 1$

$f'(0^+) = f'(0^-) \quad \Rightarrow \quad m = 1$

$\Rightarrow \; m + 3c = 4$

140. Ans. (A)→(T); (B)→(P); (C)→(Q); (D)→(T)

(A) Length of subtangent = $\left|\frac{y}{(dy/dx)}\right|$

$\sqrt{x} + \sqrt{y} = 3$

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0 \Rightarrow \left(\frac{dy}{dx}\right)_{(4,1)} = -\frac{1}{2}$

Length of subtangent $= \left|\frac{1}{-1/2}\right| = 2$

(B) $\left(\frac{dy}{dx}\right) = \frac{4t - 2}{2t + 3}$

If $\quad x = 2 \; t = -5 \;\&\; 2$

If $\quad y = -1 \; t = 2 \;\&\; -1$

common value of t is 2

$\left(\frac{dy}{dx}\right)_{t=2} = \frac{6}{7}$

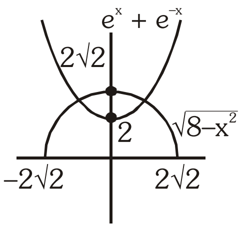
(C) It is given that da = RdA

$a = 2R \sin A \quad \Rightarrow da = 2R \cos A \, dA$

$\Rightarrow \cos A = \frac{1}{2}$

(D) The number of solutions ≡ number of intersection points of the curves

$e^x + e^{-x} = \sqrt{8 - x^2}$



hence 2 number of solutions.

141. Ans. (A)→(P); (B)→(Q); (C)→(R,S); (D)→(Q)

(A) $\{x(x + 1)\}^2 + x^2 = 3(x + 1)^2$

$\Rightarrow \quad x^2 + \left(\frac{x}{x + 1}\right)^2 = 3$

$\left(x - \frac{x}{x + 1}\right) + \frac{2x^2}{x + 1} = 3$

$\left(\frac{x^2}{x + 1}\right)^2 + \frac{2x^2}{x + 1} = 3$

Put $\frac{x^2}{x + 1} = t$

$t^2 + 2t = 3$

(B) $x(x^2 - 1)(x + 2) + 1 = 0$

$x(x + 1)(x^2 + x - 2) + 1 = 0$

$(x^2 + x)(x^2 + x - 2) + 1 = 0$

$x^2 + x = t$

(C) $x^2 = \frac{5 \pm \sqrt{5}}{2}$

(D) $x^3 - x^2 - 3x + 2 = 0$

x = 2 is a factor by hit & trial

& then proceed

142. Ans. (A)→(P,S); (B)→(P); (C)→(T); (D)→(Q)

(A) $|x + 1| + |x - 2| + |x - 3| \ge 6$

take critical points & solve

(B) $||x| - 1| < 1 - x$

$1 - x > 0 \Rightarrow \quad x < 1$

Case-I : $\quad x < -1$

$-x - 1 < 1 - x$

$\Rightarrow \quad 0 < 2 \;\Rightarrow\; \quad x < -1$

Case-II : $-1 < x < 0$

$x + 1 < 1 - x$

$x < 0$

$1 - x < 1 - x$

false

(C) $|x-4| < \sqrt{3-x}$

$x^2 - 8x + 16 < 3 - x$

$x^2 - 7x + 13 < 0$

$\Rightarrow$ No solution

$x \in \phi$

(D) $\sqrt{x^3 - 4x^2 + 4x - 3} + \sqrt{3-x} + x^2 \geq 9$

$\sqrt{(x-3)(x^2-x+1)} + \sqrt{3-x} + x^2 \geq 9$

Domain $x \geq 3$ & $x \leq 3$

$\Rightarrow$ $x = 3$

it satisfy

143. Ans. 9

$$\frac{(A-5)A + 10A - 10}{2A^2 + A - 20} = 14$$

$\Rightarrow$ $A^2 + 5A - 10 = 28A^2 + 14A - 280$

$27A^2 + 9A - 270 = 0$

$\Rightarrow$ $A_1 + A_2 = -\frac{9}{27}, -27(A_1 + A_2) = 9.$

144. Ans. 8

$$f(n) = -\begin{vmatrix} n^2 & (n+1)^2 & (n+2)^2 \\ (n+1)^2 & (n+2)^2 & (n+3)^2 \\ (n+2)^2 & (n+3)^2 & (n+4)^2 \end{vmatrix}$$

$C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$

$$= -\begin{vmatrix} 2n+1 & 2n+3 & (n+2)^2 \\ 2n+3 & 2n+5 & (n+3)^2 \\ 2n+5 & 2n+7 & (n+4)^2 \end{vmatrix}$$

$C_1 \to C_1 - C_2$

$$= 2\begin{vmatrix} 1 & 2n+3 & (n+2)^2 \\ 1 & 2n+5 & (n+3)^2 \\ 1 & 2n+7 & (n+4)^2 \end{vmatrix}$$

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_2$

$$= 2\begin{vmatrix} 1 & 2n+3 & (n+2)^2 \\ 0 & 2 & 2n+5 \\ 0 & 2 & 2n+7 \end{vmatrix}$$

$= 2\{4n + 14 - 4n - 10\} = 8$

145. Ans. 9

Equation the tangent to the curve $y = f(x)$ is

$(Y - y) = f'(x)(X - x)$

equation of the tangent to the curve

$y_1 = g(x) = \int_{-\infty}^{x} f(t)dt$ is

$(Y - y_1) = g'(x)(X - x)$

$\Rightarrow$ $(Y - y_1) = f(x)(X - x)$

Given that tangent with equation abscissa intersects on x-axis

$\therefore$ $x - \frac{y}{f'(x)} = x - \frac{y_1}{f(x)}$

$\frac{f(x)}{f'(x)} = \frac{y_1}{f(x)}$

$\frac{f(x)}{y_1} = \frac{f'(x)}{f(x)}$

$\frac{g'(x)}{g(x)} = \frac{f'(x)}{f(x)}$

Integrating both sides we get, $\ell n g(x) = \ell n f(x) + c$

$\Rightarrow$ $\ell n\left(\frac{g(x)}{f(x)}\right) = c$ $\Rightarrow$ $g(x) = kf(x)$

$\Rightarrow$ $g(0) = kf(0) \Rightarrow$ $k = 1/3$

$g(x) = \int_{-\infty}^{x} f(x)dx$

$kf(x) = \int_{-\infty}^{x} f(x)dx$

$kf'(x) = f(x)$

$\frac{f'(x)}{f(x)} = 3$

$\ell n f(x) = 3x + c$

$f(x) = \lambda e^{(3x)}$

$1 = \lambda$ {$\because$ curve passes through (0,1)}

$\Rightarrow$ $f(x) = e^{3x}$

$\ell n f(3) = 9$

146. Ans. 3

$a = \frac{4 - \pi + \pi}{2} = 2$

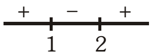
$b = \frac{3(6 - 2\pi + 2\pi)}{2} = 9$

$c = 10 - 3\pi + 3\pi + 2 = 12$

$d = 5\{2\pi - (2\pi - 5)\} = 25$

$f(x) = 2x^3 - 9x^2 + 12x + 25$

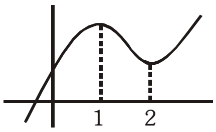
$f'(x) = 6(x-2)(x-1)$



from graph

p = 2

q = 1



147. Ans. 2

$$\int_2^x f(t)dt = \frac{x^2}{2} + \int_x^2 t^2 f(t)dt$$

Differentiating w.r.t. x, we get

$f(x)\,(1+x^2) = x \Rightarrow f(x) = \frac{x}{1+x^2}$, which is an odd function.

Now $\int_{-\pi/4}^{\pi/4} \frac{f(x)+x^9-x^3+x+1}{\cos^2 x}dx$

$$= \int_{-\pi/4}^{\pi/4} \frac{\frac{x}{1+x^2}+x^9-x^3+x}{\cos^2 x}dx + \int_{-\pi/4}^{\pi/4} \sec^2 x dx$$

$= 0 + 2$

148. Ans. 1

$f'(x) = ax^3 + x + \cos^2 x > 0 \ \forall\ x \in [1, 3]$

$\Rightarrow$ $f(x)$ is increasing function

$\Rightarrow$ Difference between maximum & minimum value $= f(3) - f(1)$

$$= \int_0^3 (at^3 + t + \cos^2 t)dt - \int_0^1 (at^3 + t + \cos^2 t)dt$$

$$= \left|\frac{at^4}{4} + \frac{t^2}{2} + \frac{t}{2} + \frac{\sin 2t}{4}\right|_0^3 - \left|\frac{at^4}{4} + \frac{t^2}{2} + \frac{t}{2} + \frac{\sin 2t}{4}\right|_0^1$$

$$= 20a + 4 + 1 + \frac{\sin 6 - \sin 2}{4}$$

$= 20a + 5 + \sin 1 \cos 1 \cos 4 \Rightarrow \quad a = 1$

149. Ans. 4

Here $2a + 2b = \ell$ i.e. $a + b = \frac{\ell}{2}$

Area (A) of rectangle ABCD = AB × BC

$= (AP + PB)(BQ + QC) = (b\sin\theta + a\cos\theta)(a\sin\theta + b\cos\theta)$

$$= ab + \frac{(a^2+b^2)}{2}\sin 2\theta$$

$\therefore \quad A \le ab + \frac{a^2+b^2}{2}$ or $A_{max} = \frac{(a+b)^2}{2} = 32$

$\therefore \quad \frac{\ell^2}{8} = 32$ or $\ell = 16$