Mathematics

TARGET : JEE 2013

SCORE

JEE (Advanced)

Home Assignment # 02

ALLEN CAREER INSTITUTE
KOTA (RAJASTHAN)
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HOME ASSIGNMENT # 02

STRAIGHT OBJECTIVE TYPE

1. If A, B & C are matrices of order 2 such that $|A| = \frac{1}{4}$, $|B| = 9$, $|C| = 2$, then $\left|(3AC)(2B)^{-1}\right|$ is equal to -

(A) $\frac{1}{8}$ (B) $\frac{3}{4}$ (C) 2 (D) 6

2. The length of the sub-tangent to the curve $y = \sqrt[3]{\frac{x^3(x^2+1)}{\sqrt[5]{5-x}}}$ at x = 1 is -

(A) $\frac{81}{20}$ (B) $\frac{20}{81}$ (C) $\frac{27}{20}$ (D) $\frac{20}{27}$

3. If a triangle has two sides of length 3 and 4 and has maximum area and Δ and s be respectively area & semi-perimeter of the triangle then -

(A) $\Delta = s$ (B) $2\Delta = s$ (C) $\Delta = 2s$ (D) none of these

4. If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ and $\lim_{n\to\infty} \frac{1}{n} A^n = \begin{bmatrix} 0 & a\lambda \\ 0 & 0 \end{bmatrix}$, then λ is equal to -

(A) –1 (B) 0 (C) 1 (D) none of these

5. The equation of the tangents to the curve $(1 + x^2)y = 1$ at the point of its intersection with the curve $(1 + x)y = 1$ are given by -

(A) x + 2y = 1; y = 1 (B) x + 2y = 2; x = 1 (C) x + 2y = 2; y = 1 (D) x + 2y = 1; x = 1

6. If for the function $f(x) = \begin{cases} \cos^{-1} x, & -1 \le x \le 0 \\ mx + c, & 0 < x \le 1 \end{cases}$, LMVT is applicable in [–1, 1], then (m, c) is -

(A) $\left(-1, \frac{\pi}{2}\right)$ (B) $\left(1, -\frac{\pi}{2}\right)$ (C) $\left(\frac{\pi}{2}, -1\right)$ (D) $\left(-\frac{\pi}{2}, 1\right)$

7. The angle of intersection between the curves $y = \int_{x^2}^{x^3} \left(\sqrt{5 - t^2}\right) dt$ and x-axis is (where $x \ne 0$) -

(A) $\tan^{-1}\frac{1}{2}$ (B) $\cot^{-1} 2$ (C) $\cot^{-1}\frac{1}{2}$ (D) $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

8. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ and $A^3 - pI = qA + rA^2$, then the value of p + q + r is -

(A) 3 (B) 4 (C) 6 (D) none of these

FILL THE ANSWER HERE

1. Ⓐ Ⓑ Ⓒ Ⓓ	2. Ⓐ Ⓑ Ⓒ Ⓓ	3. Ⓐ Ⓑ Ⓒ Ⓓ	4. Ⓐ Ⓑ Ⓒ Ⓓ
5. Ⓐ Ⓑ Ⓒ Ⓓ	6. Ⓐ Ⓑ Ⓒ Ⓓ	7. Ⓐ Ⓑ Ⓒ Ⓓ	8. Ⓐ Ⓑ Ⓒ Ⓓ

9. Tangents are drawn to y = cos x from the point P(0, 0). Points of contacts of these tangents will always lie on -

(A) $\frac{1}{x^2}=\frac{1}{y^2}+1$ (B) $\frac{1}{x^2}=\frac{1}{y^2}-1$ (C) $x^2+y^2=1$ (D) $x^2-y^2=1$

10. Let $f(x)=\begin{cases}\cos\frac{\pi x}{2}, & x\ge 2\\ a-x, & x<2\end{cases}$. If $f(x)$ has local minima at x = 2 then complete set of values of 'a' are -

(A) $(-\infty, 1]$ (B) $(-\infty, -1)$ (C) $(-1, \infty)$ (D) $[1, \infty)$

11. If $A=\frac{1}{3}\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ x & y & z\end{bmatrix}$ is an orthogonal matrix then the value of x + y is equal to -

(A) 3 (B) –3 (C) 1 (D) 0

12. If the quadratic equation $(2m+1)x^2+(2n+1)x+(2p+1)=0$, where m, n, p $\in$ I, has real roots then-

(A) both roots are rational (B) both roots are irrational

(C) both roots are positive (D) both roots are of opposite sign

13. The number of values of K for which, the equation $f(x)=x^3-12x+K=0$ has two real distinct roots in the interval (2, 3) is -

(A) 0 (B) 1 (C) 2 (D) 3

14. If α and β are roots of the equation $x^2-2x+3=0$, then equation whose roots are $\alpha^3-2\alpha^2+3\alpha-2$ and $(\beta^3-2\beta^2+3\beta-1)^4$ is -

(A) $x^2-2x+3=0$ (B) $x^2+2x-3=0$ (C) $x^2-x+2=0$ (D) $x^2+x-2=0$

15. If $\begin{vmatrix}0 & ab^2 & ac^2\\ a^2b & 0 & bc^2\\ a^2c & cb^2 & 0\end{vmatrix}=2a^pb^qc^r$, then 10 + p + q + r is equal to -

(A) 16 (B)13 (C) 19 (D) 10

16. Let $f(x)=x^3-3x^2+3x+1$ and g be the inverse of f, then area bounded by the curve y = g(x) and x-axis from x = 1 to x = 2, is -

(A) $\frac{1}{4}$ sq. units (B) $\frac{7}{4}$ sq. units (C) $\frac{9}{4}$ sq. units (D) can not be determined

17. The surface area of a spherical balloon, being inflated, changes at a rate proportional to time t. If initially its radius is 3 units and after 2 seconds it is 5 units, then radius after 3 seconds is -

(A) $3\sqrt{5}$ units (B) $5\sqrt{3}$ units (C) 9 units (D) $\frac{7}{2}$ units

9.	Ⓐ Ⓑ Ⓒ Ⓓ	10.	Ⓐ Ⓑ Ⓒ Ⓓ	11.	Ⓐ Ⓑ Ⓒ Ⓓ	12.	Ⓐ Ⓑ Ⓒ Ⓓ
13.	Ⓐ Ⓑ Ⓒ Ⓓ	14.	Ⓐ Ⓑ Ⓒ Ⓓ	15.	Ⓐ Ⓑ Ⓒ Ⓓ	16.	Ⓐ Ⓑ Ⓒ Ⓓ
17.	Ⓐ Ⓑ Ⓒ Ⓓ						

18. The solution of the differential equation $y_1y_3 = 3y_2^2$ is -

(A) $x = A_1y^2 + A_2y + A_3$ (B) $x = A_1y + A_2$
(C) $x = A_1y^2 + A_2y$ (D) none of these

19. Let $f(x)$ be a differentiable function satisfying the equation $\frac{f'(x)}{2} = \frac{x}{e^{f(x)}}\ \forall\ x \in R$. If $f'(1) = 1$, then the number of solutions of the equation $f(x) = f'(x)$ is -

(A) 1 (B) 2 (C) 3 (D) none of these

20. Let $f(x)$ be a second degree polynomial function such that $\ell n(f(x)) > 0\ \forall\ x \in R$ & the equation $f'(x) + 786f(x) = 0$, has no real roots. If $g(x) = e^{786x}f(x)$, then -

(A) g(x) is an increasing function (B) g(x) is a decreasing function
(C) g(x) is an even function (D) the graph of g(x) cuts x-axis exactly once.

21. The number of real roots common between the two equations $x^3 + 3x^2 + 4x + 5 = 0$ and $x^3+2x^2+7x+3=0$ is -

(A) 0 (B) 1 (C) 2 (D) 3

22. The area of the smaller portion above x-axis bounded by the curves $y^2 = 8x$ and $\frac{(x-2)^2}{4} + \frac{y^2}{16} = 1$, is -

(A) $4\pi - \frac{32}{3}$ (B) $8\pi - \frac{16}{3}$ (C) $8\pi - \frac{64}{3}$ (D) $2\pi - \frac{16}{3}$

23. If the slope of tangent to curve $y = e^x \cos x$ possess local maxima at x = a, then 'a' equals -

(A) $\frac{3\pi}{2}$ (B) π (C) $\frac{\pi}{2}$ (D) 0

24. The set of values of 'a' for which the function $f(x) = (4a - 3)(x + \ell n5) + (a - 7)\sin x$ does not posses critical points is -

(A) $\left(-\infty, -\frac{4}{3}\right) \cup (2, \infty)$ (B) $(-\infty, 2)$
(C) $[1, \infty)$ (D) $(1, \infty)$

25. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, and $A^2 = B$, then the number of value(s) of α is -

(A) 1 (B) 2 (C) 3 (D) no value

26. Number of different values of x satisfying the equation $(x^2 + x - 2)^3 + (2x^2 - x - 1)^3 = 27(x^2 - 1)^3$ are-

(A) 4 (B) 6 (C) 2 (D) infinite

18.	Ⓐ Ⓑ Ⓒ Ⓓ	**19.**	Ⓐ Ⓑ Ⓒ Ⓓ	**20.**	Ⓐ Ⓑ Ⓒ Ⓓ	**21.**	Ⓐ Ⓑ Ⓒ Ⓓ
22.	Ⓐ Ⓑ Ⓒ Ⓓ	**23.**	Ⓐ Ⓑ Ⓒ Ⓓ	**24.**	Ⓐ Ⓑ Ⓒ Ⓓ	**25.**	Ⓐ Ⓑ Ⓒ Ⓓ
26.	Ⓐ Ⓑ Ⓒ Ⓓ						

27. If $A = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}$ and $f(x) = 1 + x + x^2 + ... + x^{16}$, then $f(A) =$

(A) 0 (B) $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 5 \\ 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 5 \\ 1 & 1 \end{bmatrix}$

28. If $A = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$, then A^{50} is -

(A) $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 \\ 0 & 50 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$

29. If $f(2x + 1) = 4x^2 + 14x$ then sum of the roots of $f(x) = 0$ is :

(A) $\frac{9}{4}$ (B) 5 (C) $\frac{9}{4}$ (D) –5

30. If α and β are roots of the equation $ax^2 + bx + c = 0$ then roots of the equation $a(2x + 1)^2 - b(2x + 1)(3 - x) + c(3 - x)^2 = 0$ are -

(A) $\frac{2\alpha+1}{\alpha-3}, \frac{2\beta+1}{\beta-3}$ (B) $\frac{3\alpha+1}{\alpha-2}, \frac{3\beta+1}{\beta-2}$ (C) $\frac{2\alpha-1}{\alpha-2}, \frac{2\beta+1}{\beta-2}$ (D) none of these

31. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $\det(A^n - I) = 1 - \lambda^n$, $n \in N$ then λ is -

(A) 1 (B) 2 (C) 3 (D) 4

32. If the matrix M_r is given by $M_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$, r = 1, 2, 3,..., then the value of

$\det(M_1) + \det(M_2) + ... + \det(M_{2008})$ is -

(A) 2007 (B) 2008 (C) $(2008)^2$ (D) $(2007)^2$

33. The equation $(10px^2 - qx + r)(px^2 - qx - 5r)(5px^2 - qx - r) = 0$, $(qpr \neq 0)$ has atleast -

(A) 2 real roots (B) 4 real roots (C) 6 real roots (D) data insufficient

34. If $AA^T = I$ and $\det(A) \neq 1$, then -

(A) every element is equal to its cofactor

(B) every element is equal to additive inverse of its cofactor

(C) every element and its cofactor are multiplicative inverse of each other.

(D) none of these

35. If A is an idempotent matrix then $(I + A)^{10}$ is equal to -

(A) I + A (B) I + 10A (C) I + 1023A (D) I + 1024A

27.	Ⓐ Ⓑ Ⓒ Ⓓ	28.	Ⓐ Ⓑ Ⓒ Ⓓ	29.	Ⓐ Ⓑ Ⓒ Ⓓ	30.	Ⓐ Ⓑ Ⓒ Ⓓ
31.	Ⓐ Ⓑ Ⓒ Ⓓ	32.	Ⓐ Ⓑ Ⓒ Ⓓ	33.	Ⓐ Ⓑ Ⓒ Ⓓ	34.	Ⓐ Ⓑ Ⓒ Ⓓ
35.	Ⓐ Ⓑ Ⓒ Ⓓ						

36. If $A^{-1} = \begin{bmatrix} \sin^2\alpha & 0 & 0 \\ 0 & \sin^2\beta & 0 \\ 0 & 0 & \sin^2\gamma \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} \cos^2\alpha & 0 & 0 \\ 0 & \cos^2\beta & 0 \\ 0 & 0 & \cos^2\gamma \end{bmatrix}$ where $\alpha, \beta, \gamma \in R$ and

$C = A^{-5} + B^{-5} + 5\,A^{-1}\,B^{-1}\,(A^{-3} + B^{-3}) + 10A^{-2}\,B^{-2}\,(A^{-1} + B^{-1})$ then C =

(A) I (B) 5I (C) 32I (D) $2(A^{-1} + B^{-1})^2$

37. The set of values of 'a' for which the quadratic $ax^2 + 2x(1 - a) - 4$ is negative for exactly three integral values of x, is -

(A) (0, 2) (B) (0, 1] (C) [1, 2) (D) $[2, \infty)$

38. If $f(x) \geq 0\ \forall\ x \in R$ and area bounded by the curve $y = f(x)$, $x = 0$, $x = a$ and x-axis is $\tan^{-1}a$, then the number of solutions of the equation $f(x) - 1 = \tan^2 x$ is -

(A) 0 (B) 1 (C) 2 (D) infinitely many

39. Let $f(x) = x^3 + ax + b$ with $a \neq b$ and suppose the tangent lines to the graph of f at $x = a$ and $x = b$ have the same gradient. Then the value of $f(1)$ is equal to -

(A) 0 (B) 1 (C) $-\frac{1}{3}$ (D) $\frac{2}{3}$

40. A solid rectangular brick is to be made from 1 cu feet of clay. The brick must be 3 times as long as it is wide. The width of brick for which it will have minimum surface area is a. Then a^3 is -

(A) $\left(\frac{2}{9}\right)^{1/3}$ (B) $\frac{2}{9}$ (C) $\frac{8}{729}$ (D) $\frac{3}{2}$

41. Consider the function $f(x) = \begin{cases} x^2 + 2x + 2 & x < 0 \\ x + 2 & x \geq 0 \end{cases}$, and identify the correct statement -

(A) Mean value theorem is applicable in [–2, 2]

(B) Mean value theorem is not applicable in [–5, –1]

(C) Mean value theorem is applicable in [–4, –1] and its c is $\frac{-5}{2}$

(D) Mean value theorem is applicable in [–4, –1] and its c is –1

42. If $y = e^{(K+1)x}$ is a solution of differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$, then k =

(A) –1 (B) 0 (C) 1 (D) 2

43. If f is a differentiable function for all real x and $f'(x) \leq 5$, $\forall\ x \in R$. If $f(2) = 0$ and $f(5) = 15$, then $f(3)$ -

(A) 0 (B) 15 (C) 1 (D) 5

36.	Ⓐ Ⓑ Ⓒ Ⓓ	37.	Ⓐ Ⓑ Ⓒ Ⓓ	38.	Ⓐ Ⓑ Ⓒ Ⓓ	39.	Ⓐ Ⓑ Ⓒ Ⓓ
40.	Ⓐ Ⓑ Ⓒ Ⓓ	41.	Ⓐ Ⓑ Ⓒ Ⓓ	42.	Ⓐ Ⓑ Ⓒ Ⓓ	43.	Ⓐ Ⓑ Ⓒ Ⓓ

44. The angle of intersection of $x = \sqrt{y}$ and $x^3 + 6y = 7$ at (1, 1) is -

(A) $\frac{\pi}{5}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

45. Which of the following statements is true for the general cubic function $f(x) = ax^3 + bx^2 + cx + d\ (a \neq 0)$

I. If the derivative $f\,'(x)$ has two distinct real roots then cubic has one local maxima and one local minima.

II. If the derivative $f\,'(x)$ has exactly one real root then the cubic has exactly one relative extremum.

III. If the derivative $f\,'(x)$ has no real roots, then the cubic has no relative extrema

(A) only I & II (B) only II and III (C) only I and III (D) all I, II, III are correct.

46. The order and degree of the differential equation $\sqrt[3]{\frac{dy}{dx}} - 4\frac{d^2y}{dx^2} - 7x = 0$ are a and b, then a + b is -

(A) 3 (B) 4 (C) 5 (D) 6

47. The differential equation of the family of curves represented by $y = a + bx + ce^{-x}$ (where a, b, c are arbitrary constants) is -

(A) $y''' = y'$ (B) $y''' + y'' = 0$ (C) $y''' - y'' + y' = 0$ (D) $y''' + y'' - y' = 0$

48. The solution of differential equation $(x^2 + 2x - 3)dy = (y^2 - y - 2)dx$ is -

(A) $4\ell n\left|\frac{y-2}{y+1}\right| = 3\ell n\left|\frac{x-1}{x+3}\right| + c$

(B) $\frac{1}{4}\ell n\left|\frac{y-2}{y+1}\right| = \frac{1}{3}\ell n\left|\frac{x-1}{x+3}\right| + c$

(C) $4\ell n\left|\frac{y-2}{y+1}\right| = 3\ell n\left|\frac{x+1}{x-3}\right| + c$

(D) $4\ell n\left|\frac{y-1}{y-2}\right| = 3\ell n\left|\frac{x+1}{x-3}\right| + c$

49. Number of real roots of the equation $\frac{e^x}{x^2} = 5 - x$ is -

(A) 1 (B) 2 (C) 3 (D) can not be determined

50. The positive value of the parameter 'a' for which the area of the figure bounded by the curve y = cosax, y = 0, $x = \frac{\pi}{6a}$ and $x = \frac{5\pi}{6a}$ is greater than 3 are -

(A) ϕ (B) $\left(0, \frac{1}{3}\right)$ (C) $(3, \infty)$ (D) $\left(\frac{1}{3}, 3\right)$

51. A boat leaves the dock at 2 PM and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/hr and reaches the same dock at 3 PM. At what time the two boats were closest to each other -

(A) 2:21:36 PM (B) 2:15:00 PM (C) 2:15:15 PM (D) 2:21:40 PM

44.	(A) (B) (C) (D)	**45.**	(A) (B) (C) (D)	**46.**	(A) (B) (C) (D)	**47.**	(A) (B) (C) (D)
48.	(A) (B) (C) (D)	**49.**	(A) (B) (C) (D)	**50.**	(A) (B) (C) (D)	**51.**	(A) (B) (C) (D)

52. The solution of the differential equation $2\left(x - y.\frac{dy}{dx}\right).(x^2 + y^2) = (x^2 - y^2).\left(y - x\frac{dy}{dx}\right)$ is -

(A) $\log|x^2 - y^2| + \tan^{-1}\left(\frac{y}{x}\right) = k$
(B) $\log|x^2 - y^2| - \tan^{-1}\left(\frac{y}{x}\right) = k$
(C) $\log\left|\frac{y}{x}\right| - \tan^{-1}(x^2 - y^2) = k$
(D) $\log\left|\frac{y}{x}\right| + \tan^{-1}(x^2 - y^2) = k$

53. Area bounded by $y = -x^2 + 6x - 5$, $y = -x^2 + 4x - 3$ and $y = 3x - 15$, for $x > 1$, is -

(A) 73 (B) 13/6 (C) 73/6 (D) none of these

54. The general solution of the differential equation $\frac{dy}{dx} = \left(x^3 - 2x\tan^{-1} y\right)\left(1 + y^2\right)$ is -

(A) $2\tan^{-1}x = y^2 - 1 + 2ce^{-x^2}$
(B) $2\tan^{-1}y = x^2 - 1 + 2ce^{-x^2}$
(C) $2\tan^{-1}y = y^2 - 1 + 2ce^{-x^2}$
(D) $2\tan^{-1}x = x^2 - 1 + 2ce^{-y^2}$

55. The angle between the tangents drawn to the curve $x^2y = 1 - y$ at the points where it meets with $xy = 1 - y$ is -

(A) $\tan^{-1}\frac{1}{2}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\tan^{-1} 2$

56. Area bounded by $x^2y^2 + y^4 - x^2 - 5y^2 + 4 = 0$ is equal to -

(A) $\frac{4\pi}{3} + 2\sqrt{3}$ (B) $\frac{4\pi}{3} - 2\sqrt{3}$ (C) $\frac{4\pi}{3} + \sqrt{3}$ (D) $\frac{4\pi}{3} - \sqrt{3}$

57. The solution of differential equation $x\frac{dy}{dx} = x^2y.e^x + y\ell ny$ is -

(A) $\ell nx = e^y + cy$ (B) $e^x + \ell ny = c$ (C) $\ell ny = xe^x + cx$ (D) $\ell nx = e^y + cx$

58. If $\frac{dx}{\tan(x+y)} = \frac{dy}{\cot(x+y)} = \frac{dz}{1}$, then z in terms of x & y can be expressed as -

(A) $z = \frac{\sin(2x+2y)}{4} + C$
(B) $z = -\frac{\sin(2x+2y)}{4} + C$
(C) $z = -\frac{\cos(2x+2y)}{4} + C$
(D) None of these

59. Area bounded by curve $y = |x^3 - x|$, x-axis & the lines $x = -1$ and $x = 1$ is -

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{9}{4}$

52.	Ⓐ Ⓑ Ⓒ Ⓓ	53.	Ⓐ Ⓑ Ⓒ Ⓓ	54.	Ⓐ Ⓑ Ⓒ Ⓓ	55.	Ⓐ Ⓑ Ⓒ Ⓓ
56.	Ⓐ Ⓑ Ⓒ Ⓓ	57.	Ⓐ Ⓑ Ⓒ Ⓓ	58.	Ⓐ Ⓑ Ⓒ Ⓓ	59.	Ⓐ Ⓑ Ⓒ Ⓓ

60. Two villages X & Y are on the same side of a straight river at distances 'a' & 'b' respectively from river. A pumpset is to be installed by the river side at a point P. If the villages are situated at a distance 'c', then -

(A) minimum value of XP + PY is $\sqrt{c^2+2ab}$ (B) minimum value of XP + PY is $\sqrt{c^2-4ab}$

(C) minimum value of XP + PY is $\sqrt{c^2-2ab}$ (D) minimum value of XP + PY is $\sqrt{c^2+4ab}$

61. The hands of an accurate clock have lengths 3 and 4, then distance between the tips of the hands when the distance is increasing most rapidly is -

(A) 2 (B) $\sqrt{7}$ (C) $\sqrt{3}$ (D) $\sqrt{5}$

62. If the function $f(x) = \cos|x| - 2ax + b$ be increases along the entire number line, the range of values of a is given by -

(A) $a \le b$ (B) $a = \frac{b}{2}$ (C) $a \le -\frac{1}{2}$ (D) $a \ge \frac{3}{2}$

63. If α & β are the roots of the equation $x^2 + 6x + \lambda = 0$ and $3\alpha + 2\beta = -20$, then λ is equal to -

(A) –8 (B) –16 (C) 16 (D) 8

64. If the roots of $(p - 3)x^2 - 2px + 5p = 0$ are real and positive and $p \in R$, then p belongs to -

(A) (3, 5) (B) (0, 3) (C) $\left(0, \frac{15}{4}\right]$ (D) $\left(3, \frac{15}{4}\right]$

65. If the roots of $\ell x^2 + 2mx + n = 0$ are real & distinct, then the roots of $(\ell + n)(\ell x^2 + 2mx + n) = 2(\ell n - m^2)(x^2 + 1)$ will be -

(A) real & equal (B) real & different

(C) imaginary (D) may be real or imaginary

66. The integral values of k for which the roots of $kx(x + 2) + k - (x + 2) = 0$ are rational is given by -

(A) 5 (B) –3 (C) –6 (D) 12

67. If one of the roots of $ax^2 + bx + c = 0$ is greater than 1 and the other is less than –3 and if the roots of $cx^2 + bx + a = 0$ are α & β, then -

(A) $\alpha < 1$ & $\beta > -\frac{1}{3}$ (B) $\alpha < 1$ & $\beta < -\frac{1}{3}$ (C) $\alpha > 1$ & $\beta > 1$ (D) $\alpha > 1$ & $\beta < -\frac{1}{3}$

60.	Ⓐ Ⓑ Ⓒ Ⓓ	**61.**	Ⓐ Ⓑ Ⓒ Ⓓ	**62.**	Ⓐ Ⓑ Ⓒ Ⓓ	**63.**	Ⓐ Ⓑ Ⓒ Ⓓ
64.	Ⓐ Ⓑ Ⓒ Ⓓ	**65.**	Ⓐ Ⓑ Ⓒ Ⓓ	**66.**	Ⓐ Ⓑ Ⓒ Ⓓ	**67.**	Ⓐ Ⓑ Ⓒ Ⓓ

MULTIPLE OBJECTIVE TYPE

68. Which of the following is true for the differential equation $x = \frac{d^2y}{dx^2} + \sqrt{\left(\frac{d^2y}{dx^2}\right)^2 + b^2}$, where $b \in R$ -

(A) order of differential equation is 2
(B) degree of differential equation is 2
(C) degree of the differential equation is not defined
(D) degree of differential equation is 1

69. The general solution of the differential equation $e^x dx = \sqrt{1-e^{2x}}\,dy$ is -

(A) $y = K - \cos^{-1}(e^x)$
(B) $y = \sin^{-1}(e^x) + C$
(C) $y = \sin^{-1}\sqrt{1-e^{2x}} + C$
(D) $y = \cos^{-1}\sqrt{1-e^{2x}} + C$

70. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$, then -

(A) $A^3 - 5A^2 + I = 0$
(B) $A^{-2} = 5I - A$
(C) $-A^{-1} = \text{adj } A$
(D) A^3 is non-invertible matrix

71. If the tangent to the curve $2y^3 = tx^2 + x^3$ at the point (t, t) cuts of the intercepts p, q on the x & y-axes respectively, where A (p, q) is a point on the circle centred at origin and radius $\sqrt{61}$, then the value of 't' can be -

(A) –30 (B) 10 (C) 20 (D) 30

72. Which of the following statements are false -

(A) If f (x) is increasing function in its domain and x_1, x_2 belongs to the domain of the function, then $x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$
(B) If the function has local maxima at x = a, then it implies function is increasing for $a - \delta < x < a$ and function is decreasing in $a < x < a + \delta$, where 'δ' can be sufficiently small.
(C) If $f(x) = ax^3 + bx^2 + cx + d$ is monotonic function, then $f'(x) = 0$ has no real roots.
(D) The order of the differential equation, representing family of cubic polynomials, having exactly one critical point is three.

73. If $f(x) = \frac{5x}{x^4 + a}$ is a bounded function, then 'a' can be -

(A) –1 (B) 1 (C) –16 (D) 4

74. The area bounded by the circles $x^2 + y^2 = 1$, $x^2 + y^2 = 4$ and the pair of lines $\sqrt{3}(x^2 + y^2) = 4xy$ is equal to -

(A) $\frac{\pi}{2}$ (B) $\frac{5\pi}{2}$ (C) 3π (D) $\frac{\pi}{4}$

68. Ⓐ Ⓑ Ⓒ Ⓓ	69. Ⓐ Ⓑ Ⓒ Ⓓ	70. Ⓐ Ⓑ Ⓒ Ⓓ	71. Ⓐ Ⓑ Ⓒ Ⓓ
72. Ⓐ Ⓑ Ⓒ Ⓓ	73. Ⓐ Ⓑ Ⓒ Ⓓ	74. Ⓐ Ⓑ Ⓒ Ⓓ	

75. If y = g(x) is a curve which is obtained by the reflection of $y = f(x) = \frac{e^x - e^{-x}}{2}$ about the line y = x, then which of the following is/are true -

(A) y = g(x) has exactly one tangent parallel to x-axis

(B) y = g(x) has no tangent parallel to x-axis

(C) The tangent to y = g(x) at (0, 0) is y = x

(D) g(x) has no extremum

76. The differential equation of the curve for which intercept cut by any tangent on y-axis is equal to the length of the sub normal -

(A) is linear
(B) is homogeneous of first degree
(C) has separable variables
(D) is of first order

77. If eqautions $(a + 2)x^2 + bx + c = 0$ and $2x^2 + 3x + 4 = 0$ have a common root where a, b, c ∈ N then -

(A) $b^2 - 4ac < 0$
(B) minimum value of a + b + c is 16
(C) $b^2 < 4ac + 8c$
(D) minimum value of a + b + c = 7

78. $\begin{vmatrix} 2a & 2b & b-c \\ 2b & 2a & a+c \\ a+b & a+b & b \end{vmatrix}$ is divisible by -

(A) (a – b)
(B) $(a - b)^2$
(C) a + b
(D) (a + b + c)

79. If A is an invertible matrix then $(\text{adj}A)^{-1}$ =

(A) $\text{adj}(A^{-1})$
(B) $\frac{A}{\det A}$
(C) A
(D) (detA)A

80. If $AA^T = I$ then identify the correct statement -

(A) $A^T(A - I_3) = -(A - I_3)^T$
(B) A is always invertible
(C) $\det(A - I_3) = 0$
(D) A is singular

81. If $g(x) = 7x^2e^{-x^2}\ \forall\ x \in R$, then g(x) has -

(A) local maximum at x = 0
(B) local minima at x = 0
(C) local maximum at x = –1
(D) two local maxima and one local minima

82. Equation of common tangent(s) of $x^2 - y^2 = 12$ and xy = 8 is (are) -

(A) $y = 3x + 4\sqrt{6}$
(B) $y = -3x + 4\sqrt{6}$
(C) $3y = x + 4\sqrt{6}$
(D) $y = -3x - 4\sqrt{6}$

83. The value of 'a' for which the function $f(x) = \begin{cases} -x^3 + \cos^{-1} a, & 0 < x < 1 \\ x^2, & x \ge 1 \end{cases}$ has a local minimum at x = 1, is -

(A) –1
(B) 1
(C) 0
(D) –1/2

84. The families of curves defined by the equations y = bx and $x^2 + y^2 = a^2$ are orthogonal if -

(A) a = 2, b = 4
(B) a = –3, b = 5
(C) a = –2, b = 3
(D) a = 5, b = 2

75.	Ⓐ Ⓑ Ⓒ Ⓓ	**76.**	Ⓐ Ⓑ Ⓒ Ⓓ	**77.**	Ⓐ Ⓑ Ⓒ Ⓓ	**78.**	Ⓐ Ⓑ Ⓒ Ⓓ
79.	Ⓐ Ⓑ Ⓒ Ⓓ	**80.**	Ⓐ Ⓑ Ⓒ Ⓓ	**81.**	Ⓐ Ⓑ Ⓒ Ⓓ	**82.**	Ⓐ Ⓑ Ⓒ Ⓓ
83.	Ⓐ Ⓑ Ⓒ Ⓓ	**84.**	Ⓐ Ⓑ Ⓒ Ⓓ				

85. If differential equation corresponding to family of curves y = A cos2x + B $\sin^2$ x + C is given by $\lambda \frac{d^3y}{dx^3} + f(x)\frac{d^2y}{dx^2} = 2\frac{dy}{dx}(\cos 2x)$, where λ is real constant and f (x) is some function in x, then -

(A) $\lambda = 0$

(B) $\lambda = 1$

(C) Number of solution of equation $f(x) = \lambda$ in (0,4) are 2

(D) Number of solution of equation $f(x) = \lambda$ in (0,4) are 3

86. If y satisfies the differential equation $xdy = y(xy^3 + 1)dx$ & y(1) = 2, then -

(A) $\lim_{x\to\infty} xy^3 = -\frac{4}{3}$
(B) $\lim_{x\to\infty} y = 0$
(C) curve y = f (x) is symmetric w.r.t. origin
(D) curve y = f (x) is continuous ∀ x ∈ R.

87. Let $f(x) = \begin{cases} |x-2|+k, & x \le 2 \\ 3x+2, & x > 2 \end{cases}$. If f(x) has a local minimum at x= 2, then k can be equal to -

(A) 5 (B) 7 (C) 9 (D) 11

88. If three quadratic equations

$x^2 + (\sqrt{AB+AC})x + \frac{BC}{4} = 0$, $x^2 + (\sqrt{BC+AB})x + \frac{AC}{4} = 0$ and $x^2 + (\sqrt{AC+BC})x + \frac{AB}{4} = 0$ are given, where A, B, C are three points in a plane then -

(A) if A, B, C are making a triangle then all equations have real & distinct roots.

(B) if A, B, C are collinear, then all equations have real & equal roots.

(C) if A, B, C are collinear, then two equations have real & distinct, one equation has real & equal roots.

(D) for every position of A, B, C all six roots are negative.

89. If $ax^2 + bx + c$ (a > 1 & a, b, c are integers) is equal to p for two distinct integral values of x, p ∈ prime, then $ax^2 + bx + c$ cannot be equal to 2p for -

(A) x = 0 (B) x = –1 (C) x = 1 (D) x = 3

REASONING TYPE

90. Consider A and B are square matrices of order 3 × 3 such that AB = A and BA = B.

Statement-1 : If $(A – B)^8 = k(A + B)$, then k = 0.

and

Statement-2 : $(A – B)^2$ is a null matrix.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

85. Ⓐ Ⓑ Ⓒ Ⓓ **86.** Ⓐ Ⓑ Ⓒ Ⓓ **87.** Ⓐ Ⓑ Ⓒ Ⓓ **88.** Ⓐ Ⓑ Ⓒ Ⓓ
89. Ⓐ Ⓑ Ⓒ Ⓓ **90.** Ⓐ Ⓑ Ⓒ Ⓓ

91. Let B is square matrix of order 3 such that $B^3 = 0$ and $A = I_3 - B$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ & $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

Statement-1 : System of linear equations $AX = b$ has the solution $X = b + Bb + B^2b$.

and

Statement-2 : $A^{-1} = I_3 + B + B^2$.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

92. Four numbers a, b, c, d are such that the first three form an increasing geometric progression and last three form an arithmetic progression also $a + d = 14$, $b + c = 12$ and three lines are defined as :

$L_1 = ax - 3y + 5 = 0$
$L_2 = 3x + by - 7 = 0$
$L_3 = 9x - 5y + c = 0$

Statement-1 : L_1, L_2, L_3 are concurrent.

and

Statement-2 : $L_3 = 3L_1 + L_2$.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

93. Consider matrix A, such that $A = [a_{ij}]_{3 \times 3}$, $\forall\ a_{ij} \in I$.

Statement-1 : $\text{sgn}(\det(\text{adj}A) - 8) \neq 0$, for any set of value of a_{ij}.

and

Statement-2 : $\left\{\sqrt{|\det(\text{adj}A)|}\right\} = 0$ for every set of values of a_{ij}. {.} represents fractional part function.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

94. **Statement-1 :** $\dfrac{2\ell n x_1 + \ell n x_2}{3} < \ell n\left(\dfrac{2x_1 + x_2}{3}\right)$, where $x_1, x_2 > 0$.

and

Statement-2 : If $f'(x) > 0$ & $f''(x) < 0\ \ \forall\ x \in D_f$, then $f(x)$ increases with concavity downward and any chord of the curve lies below the curve.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

91. (A) (B) (C) (D)	**92.** (A) (B) (C) (D)	**93.** (A) (B) (C) (D)	**94.** (A) (B) (C) (D)

95. **Statement-1 :** If $\Delta_1 = \begin{vmatrix} y^5z^6(z^3-y^3) & x^4z^6(x^3-z^3) & x^4y^5(y^3-x^3) \\ y^2z^3(y^6-z^6) & xz^3(z^6-x^6) & xy^2(x^6-y^6) \\ y^2z^3(z^3-y^3) & xz^3(x^3-z^3) & xy^2(y^3-x^3) \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & y^2 & z^3 \\ x^4 & y^5 & z^6 \\ x^7 & y^8 & z^9 \end{vmatrix}$ then $\Delta_1\Delta_2$ is equal to Δ_2^3.

and

Statement-2 : If elements of any determinant Δ of order 3×3 are replaced by their respective cofactors then the value of determinant thus formed is equal to Δ^2.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

96. **Statement-1 :** $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots \begin{bmatrix} 1 & 27 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 756 \\ 0 & 1 \end{bmatrix}$

and

Statement-2 : $\prod_{i=1}^{n} \begin{bmatrix} 1 & x_i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \sum_{i=1}^{n} x_i \\ 0 & 1 \end{bmatrix}$

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

97. **Statement-1 :** If $a, b \in R$ and $a < b$, then there is atleast one real number $c \in (a, b)$ such that $\frac{c}{a+b} = \frac{b^2+a^2}{4c^2}$

and

Statement-2 : If $f(x)$ is continuous in $[a, b]$ and derivable in (a, b) & $f'(c) = 0$ for atleast one $c \in (a, b)$, then it necessarily implies that $f(a) = f(b)$.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

95. Ⓐ Ⓑ Ⓒ Ⓓ 96. Ⓐ Ⓑ Ⓒ Ⓓ 97. Ⓐ Ⓑ Ⓒ Ⓓ

98. **Statement-1 :** If the tangent to the curve $y = \dfrac{x-2}{x+1}$ at point $P\left(e, \dfrac{e-2}{e+1}\right)$ makes an angle θ with positive x-axis, then $e^{\cos\theta} < 1$.

and

Statement-2 : For the curve $xy = -c^2$, $\left.\dfrac{dy}{dx}\right|_{x=t} > 0 \;\forall\; t \in R$

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

99. **Statement-1 :** $\dfrac{2}{\log_3 2} < \dfrac{e}{\log_3 e} < \dfrac{5}{\log_3 5}$.

and

Statement-2 : The function $f(x) = \dfrac{3^x}{x}$ decreases in $\left(0, \dfrac{1}{\ell n3}\right)$ & increases in $\left(\dfrac{1}{\ell n3}, \infty\right)$.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

COMPREHENSION

Paragraph for Question 100 to 102

Let $f(x) = x^3 - 3x + 1$, $x \in R$ and a, b, c are roots of $f(x) = 0$, $P(\alpha, f(\alpha))$ is point of local minima $Q(\beta, f(\beta))$ is point of local maxima and $R(\gamma, f(\gamma))$ is point of inflection in the graph of $y = f(x)$.

On the basis of above information answer the following :

100. $\begin{vmatrix} f(a) & f''(a) & 1 \\ f(b) & f''(b) & 1 \\ f(c) & f''(c) & 1 \end{vmatrix}$ is equal to -

(A) (a – b)(b – c)(c – a) (B) 3(a – b)(b – c) (c – a)
(C) $\cos^{-1}(\cos 1)$ (D) $\cos^{-1}1$

101. Triangle PQR is -

(A) equilateral (B) isosceles (C) right angle (D) none of these

102. Number of distinct real solutions of $f(f(x)) = 0$ are -

(A) 3 (B) 5 (C) 7 (D) 9

98. Ⓐ Ⓑ Ⓒ Ⓓ 99. Ⓐ Ⓑ Ⓒ Ⓓ 100. Ⓐ Ⓑ Ⓒ Ⓓ 101. Ⓐ Ⓑ Ⓒ Ⓓ
102. Ⓐ Ⓑ Ⓒ Ⓓ

Paragraph for Question 103 to 105

$ax + by + c = 0$

$bx + cy + a = 0$

$cx + ay + b = 0$ is a system of linear equation then answer the following questions :

103. If $a \neq b \neq c$ and $a + b + c \neq 0$ then the system of linear equation have -

(A) infinite solution all lying on a line
(B) entire xy plane as solution
(C) unique solution
(D) no solution

104. If $a = b = c$ and $a + b + c \neq 0$ then the system of linear equation have -

(A) infinite solution all lying on a line
(B) entire xy plane as solution
(C) unique solution
(D) no solution

105. If $a = b = c$ and $a + b + c = 0$ then the system of linear equation have -

(A) infinite solution all lying on a line
(B) entire xy plane as solution
(C) unique solution
(D) no solution

Paragraph for Question 106 to 108

Consider the system of linear equation

$$x - 2y + bz = 3$$
$$ax + 2z = 2$$
$$5x + 2y = 1$$

On the basis of above informations, answer the following questions.

106. If $ab = 12$ and $a \neq 3$ then system of linear equations has -

(A) no solution
(B) infinite solution
(C) unique solution
(D) finitely many solutions

107. If $ab \neq 12$ then system of linear equations has -

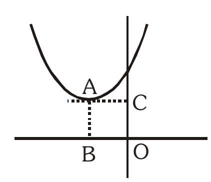
(A) no solution
(B) infinite solution
(C) unique solution
(D) finitely many solutions

108. If $a = 3$ & $b = 4$ then system of equations has -

(A) no solution
(B) infinite solution
(C) unique solution
(D) finitely many solutions

Paragraph for Question 109 to 111

Graph of $f(x) = ax^2 + bx + c$ is shown adjacently, for which $\ell(AB) = 2$, $\ell(AC) = 3$ and $b^2 - 4ac = -4$.



On the basis of above informations, answer the following questions :

109. The value of $a + b + c$ is equal to -

(A) 7
(B) 8
(C) 9
(D) 10

103.	Ⓐ Ⓑ Ⓒ Ⓓ	**104.**	Ⓐ Ⓑ Ⓒ Ⓓ	**105.**	Ⓐ Ⓑ Ⓒ Ⓓ	**106.**	Ⓐ Ⓑ Ⓒ Ⓓ
107.	Ⓐ Ⓑ Ⓒ Ⓓ	**108.**	Ⓐ Ⓑ Ⓒ Ⓓ	**109.**	Ⓐ Ⓑ Ⓒ Ⓓ		

110. The quadratic equation with rational coefficients whose one of the roots is $b+\sqrt{a+c}$, is -

(A) $x^2 - 6x + 2 = 0$ (B) $x^2 - 6x - 1 = 0$ (C) $x^2 + 6x + 2 = 0$ (D) $x^2 + 6x - 1 = 0$

111. Range of $g(x) = (a+\frac{1}{2})\,x^2 + (b + 2)x - (c-\frac{1}{2})$ when $x \in [-4, 0]$ is -

(A) $[-10, -6]$ (B) $\left[-\frac{49}{4},-10\right]$ (C) $\left[-\frac{49}{4},-6\right]$ (D) $\left[-\frac{49}{4},\infty\right)$

Paragraph for Question 112 to 114

If $a \le x \le b$ then $\frac{1}{a} \ge \frac{1}{x} \ge \frac{1}{b}$ provided a & b are of same sign but if a and b are of opposite sign then $\frac{1}{x} \in \left(-\infty,\frac{1}{a}\right] \cup \left[\frac{1}{b},-\infty\right)$.

On the basis of above information, answer the following :

112. If $\frac{2}{3} \le x \le \frac{4}{5}$ then $\frac{4-3x}{x}$ belongs to -

(A) $[-3, -2]$ (B) $[2, 3]$ (C) $\left[\frac{1}{3},\frac{1}{2}\right]$ (D) $\left[-\frac{1}{2},-\frac{1}{3}\right]$

113. If $-3 < \frac{2x-7}{x} < -1$ then x belongs to -

(A) $\left(\frac{7}{5},\frac{7}{3}\right)$ (B) $\left(-\frac{7}{3},-\frac{7}{5}\right)$ (C) $\left(\frac{7}{3},\infty\right)$ (D) $\left(-\infty,\frac{7}{5}\right)$

114. If $x \in (-2, 3]$ then $\frac{5x+6}{x}$ belongs to -

(A) $(2, 7]$ (B) $(-\infty, 7]$ (C) $[2, \infty)$ (D) $(-\infty, 2) \cup [7, \infty)$

Paragraph for Question 115 to 117

Consider matrix $A = \begin{bmatrix} x^2 & 0 & 0 \\ 0 & -x & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

Let $C = AB$,

$D = B - I$

$X = D^{-1} + D^{-2} + D^{-3} + \ldots\ldots + D^{-n}$.

As n approaches to infinity matrix X tends to matrix Y.

Let $Y + Z = I$.

f(x) = trace of matrix C.

$g(x) = \begin{cases} f(x), & \forall\ x > 0 \\ h(x), & \forall\ x < 0 \end{cases}$, where g(x) is an odd function.

On the basis of above information, answer the following :

110. Ⓐ Ⓑ Ⓒ Ⓓ **111.** Ⓐ Ⓑ Ⓒ Ⓓ **112.** Ⓐ Ⓑ Ⓒ Ⓓ **113.** Ⓐ Ⓑ Ⓒ Ⓓ

114. Ⓐ Ⓑ Ⓒ Ⓓ

115. The matrix Z is -

(A) $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$ (D) $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$

116. If the range of f(x) is $[a, \infty)$ then the value of $\cos^{-1}\cos(a)$ is -

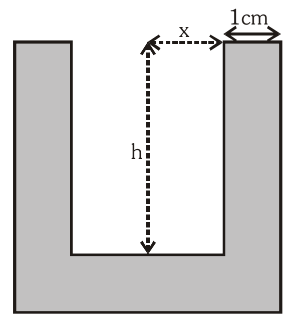
(A) $\frac{13}{4}-\pi$ (B) $2\pi-\frac{13}{4}$ (C) $\frac{13}{4}-2\pi$ (D) none of these

117. The range of the function $\sec^{-1}\left(h(x)+\frac{5}{4}\right)$ is -

(A) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$ (B) $\left[\frac{2\pi}{3}, \pi\right)$ (C) $\left[\frac{\pi}{3}, \frac{\pi}{2}\right)\cup\left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$ (D) none of these

Paragraph for Question 118 to 120

Consider an open cylindrical reservoir whose cross-sectional view is as shown in the figure. The thickness of the wall of cylinder is 1 cm as shown in the figure & the volume of the reservoir is 27π cm^3.



Let x & h be the radius and the height of the cylinder and $v(x) = \pi\left\{(x+a)^2+\frac{b}{x}+\frac{c}{x^2}\right\}$, where v(x) represents the volume of the material required to construct the cylinder expressed as a function of radius 'x' of the cylinder.

On the basis of above informations, answer the following questions :

118. $\frac{a.b}{c}$ is -

(A) prime number but not an even number (B) even number but not a prime number
(C) even prime number (D) irrational number

119. If the material cost to construct the cylinder is minimum, then which of the following relations must hold between x & h -

(A) x = h (B) 2x = h (C) x = 2h (D) none of these

120. If the function v(x) is redefined such that $v : R_0 \to R$, then number of solutions of the equation v(x) = 0 are -

(A) 1 (B) 2 (C) 3 (D) 4

Paragraph for Question 121 to 123

Consider the differential equation $e^x(ydx - dy) = e^{-x}(ydx + dy)$.

Let $y = f(x)$ be a particular solution to this differential equation which passes through the point (0, 2)

Let $C \equiv y = \log_{\frac{1}{4}}\left(x-\frac{1}{4}\right)+\frac{1}{2}\log_4(16x^2-8x+1)$, be another curve.

On the basis of above information, answer the following questions :

115. Ⓐ Ⓑ Ⓒ Ⓓ 116. Ⓐ Ⓑ Ⓒ Ⓓ 117. Ⓐ Ⓑ Ⓒ Ⓓ 118. Ⓐ Ⓑ Ⓒ Ⓓ
119. Ⓐ Ⓑ Ⓒ Ⓓ 120. Ⓐ Ⓑ Ⓒ Ⓓ

121. The range of the function $g(x) = \log_2(f(x))$ is -

(A) $[1, \infty)$ (B) $[2, \infty)$ (C) $[0, \infty)$ (D) none of these

122. The area bounded by the curve C, parabola $x = y^2 + \frac{1}{4}$ and the line $x = \frac{1}{4}$ is -

(A) 1 (B) 3 (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

123. If the area bounded by the curve $y = f(x)$, curve C, ordinate $x = \frac{1}{4}$ & the ordinate $x = a$ is $4 - \ell n4 + \frac{1}{\sqrt[4]{e}} - \sqrt[4]{e}$, then the value of a is -

(A) $\ell n6$ (B) $\ell n4$ (C) 4 (D) $\ell n12$

Paragraph for Question 124 to 126

The slope of the tangent to the curve $y_1 = f(x)$ at (0,0) is 1 and $y_2 = \int_0^x f(t)dt$. If the tangents to both curves at the points having equal abscissae cut the y-axis at same point, then

On the basis of above information answer the following :

124. Area bounded by curves y_1 and y_2 between x=0 and x=e, is -

(A) e+1 (B) e–1 (C) $\frac{e^2}{2}$ (D) 1

125. $\lim_{x\to 0}\left(\frac{y_2}{x^2}\right)$ is -

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) doesn't exist

126. Number of solutions of the equation $y_1 - y_2 = k$ (where k is constant), is -

(A) 0 (B) 2 (C) 3 (D) 1

Paragraph for Question 127 to 129

Consider the function defined implicitly $y^2 + y - x = 2$ on various intervals on the real line. If $y \in \left(-\infty, -\frac{1}{2}\right)$, the equation implicitly defines a unique real valued differentiable function $y = f(x)$.

On the basis of above informations, answer the following questions.

127. The value of $f''(4)$ is -

(A) $\frac{2}{125}$ (B) $\frac{125}{2}$ (C) $\frac{1}{5}$ (D) $\frac{1}{125}$

128. If $y = g(x)$ is inverse of $y = f(x)$, then $g'(-2)$ is -

(A) $-\frac{1}{3}$ (B) –3 (C) –2 (D) 0

121.	Ⓐ Ⓑ Ⓒ Ⓓ	**122.**	Ⓐ Ⓑ Ⓒ Ⓓ	**123.**	Ⓐ Ⓑ Ⓒ Ⓓ	**124.**	Ⓐ Ⓑ Ⓒ Ⓓ
125.	Ⓐ Ⓑ Ⓒ Ⓓ	**126.**	Ⓐ Ⓑ Ⓒ Ⓓ	**127.**	Ⓐ Ⓑ Ⓒ Ⓓ	**128.**	Ⓐ Ⓑ Ⓒ Ⓓ

129. If y = h(x) is mirror image of y = f (x) about the line 2y + 1 = 0, then $f''(x_0) + h''(x_0)$, where $x_0 \in \left(-\frac{9}{4}, \infty\right)$ is -

(A) $2x_0$ (B) $\frac{2}{2f(x_0)+1}$ (C) $\frac{2}{(2f(x_0)+1)^3}$ (D) 0

Paragraph for Question 130 to 132

Let $f(x) = 4x^2 - 4ax + a^2 - 2a + 2$ be a quadratic polynomial in x, a be any real number.
On the basis of above information, answer the following questions :

130. If one root of $f(x) = 0$ is smaller than 1 and other root is greater than 1, then the value of a belongs to -

(A) $\left(3-\sqrt{3}, 3+\sqrt{3}\right)$ (B) $\left(3+\sqrt{3}, \infty\right)$ (C) $\left(-\infty, 3-\sqrt{3}\right)$ (D) none of these

131. If $x = \frac{1}{4}$ is a root of $f(x) = 0$, then the sum of the series $1+\frac{1}{a}+\frac{1}{a^2}+......\infty$ is -

(A) 1 (B) 2 (C) 3 (D) 4

132. For a = 2, the minimum value of $f(x)$ is -

(A) –1 (B) –2 (C) 0 (D) 1

Paragraph for Question 133 to 135

A polynomial equation is given by $2x^4 + kx^3 + 22x^2 + kx + 2 = 0$.
On the basis of above information, answer the following questions :

133. Interval of k for which the equation has no imaginary roots can be -

(A) $k \in (12, 14)$ (B) $k \in (11, 12)$ (C) $[-12, -11]$ (D) $k \in [-13, -12]$

134. Interval of k for which exactly 2 roots of the equation are real can be -

(A) $k \in (-10,-8)$ (B) $k \in (-14,-12)$ (C) $k \in \phi$ (D) $k \in (13, 16)$

135. Interval of k for which there are no real roots of the equation can be -

(A) $k \in [-10,10]$ (B) $k \in [3,13]$ (C) $k \in (-13,-11]$ (D) $k \in [0,13)$

MATCH THE COLUMN

136. $f(x)=\frac{x^2-7x+12}{2x^2-18x+28}$

Column-I	Column-II
(A) If 3 < x < 4 then $f(x)$ satisfies	(P) $f(x) > 1/2$
(B) If 1 < x < 2 then $f(x)$ satisfies	(Q) $f(x) < 0$
(C) If 4 < x < 7 then $f(x)$ satisfies	(R) $f(x) > 0$
(D) If x > 7 then $f(x)$ satisfies	(S) $0 < f(x) < 1/2$

129. Ⓐ Ⓑ Ⓒ Ⓓ 130. Ⓐ Ⓑ Ⓒ Ⓓ 131. Ⓐ Ⓑ Ⓒ Ⓓ 132. Ⓐ Ⓑ Ⓒ Ⓓ
133. Ⓐ Ⓑ Ⓒ Ⓓ 134. Ⓐ Ⓑ Ⓒ Ⓓ 135. Ⓐ Ⓑ Ⓒ Ⓓ

136. (A) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ
(B) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ
(C) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ
(D) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ

137. Match the following for the system of linear equations

$\lambda x + y + z = 1,\ x + \lambda y + z = \lambda,\ x + y + \lambda z = \lambda^2$

Column-I	**Column-II**
(A) $\lambda = 1$	(P) unique solution
(B) $\lambda \neq 1$	(Q) infinite solutions
(C) $\lambda \neq 1, \lambda \neq -2$	(R) no solution
(D) $\lambda = -2$	(S) finite many solutions

138. Consider the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$; $B = \begin{bmatrix} \frac{1}{2} & 3 \\ 0 & 1 \end{bmatrix}$

Let P be an orthogonal matrix and $Q = PAP^T$, $R_K = P^TQ^KP$, $S = PBP^T$ & $T_K = P^TS^KP$. Where $K \in N$.

Column-I	**Column-II**
(A) $\sum_{K=1}^{5} a_K$, where a_K represents the element of first row & first column in matrix R_K.	(P) –9
(B) $\sum_{K=1}^{3} b_K$, where b_K represents the element of second row & second column in matrix R_K.	(Q) 10
(C) $\sum_{K=1}^{\infty} x_K$, where x_K represents the element of first row & first column in matrix T_K.	(R) 35
(D) $\sum_{K=1}^{10} y_K$, where y_K represents the element of second row & second column in matrix T_K.	(S) 1

139.

Column-I	**Column-II**
(A) The number of positive roots of the equation $(x-1)(x-2)(x-3) + (x-1)(x-2)(x-4) + (x-2)(x-3)(x-4) + (x-1)(x-3)(x-4) = 0$	(P) 1
(B) If the function $g(x) = 2f\left(\frac{x^2}{2}\right) + f(6-x^2)\ \forall x \in R$ increases in the interval (a, ∞), where $f''(x) > 0\ \forall x \in R$, then the value of a is	(Q) 2
(C) If $f(x) = e^x\ \forall x \in [0,1]$ & $f(1) - f(0) = f'(c)$, where $c \in (0, 1)$ then $\ell n(e^c + 1)$ is equal to	(R) 3
(D) If for the function $f(x) = \begin{cases} mx + c, & x < 0 \\ e^x, & x \geq 0 \end{cases}$, Lagrange's mean value theorem (LMVT) is applicable in $[-2, 2]$, then m + 3c is	(S) 4

137.		138.		139.	
(A)	Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ	(A)	Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ	(A)	Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ
(B)	Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ	(B)	Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ	(B)	Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ
(C)	Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ	(C)	Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ	(C)	Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ
(D)	Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ	(D)	Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ	(D)	Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ

140. **Column-I** **Column-II**

(A) The length of the sub-tangent of the curve $\sqrt{x}+\sqrt{y}=3$ at the point (4,1) is — (P) $\frac{6}{7}$

(B) The slope of tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point (2,–1) is — (Q) $\frac{1}{2}$

(C) A variable triangle is inscribed in a circle of radius 'R'. If rate of change of a side is R times the rate of change of the opposite angle, then cosine of that angle is — (R) $\frac{\sqrt{3}}{2}$

(D) The number of solutions of the equation $e^x + e^{-x} = \sqrt{8-x^2}$, is — (S) 0

(T) 2

141. Match the set of values of x in column-II which satisfy the equations in column-I.

Column-I		**Column-II**	
(A)	$\{x(x+1)\}^2 + x^2 = 3(x+1)^2$	(P)	2 cos 36°
(B)	$x(x^2-1)(x+2)+1=0$	(Q)	2 sin 18°
(C)	$x^4 - 5x^2 + 5 = 0$	(R)	2 sin 36°
(D)	$x^3 - x^2 - 3x + 2 = 0$	(S)	2 cos 18°

142. Match the set of values of x in column-II which satisfy the inequations in column-I.

Column-I		**Column-II**	
(A)	$\|x+1\|+\|x-2\|+\|x-3\| \geq 6$	(P)	$x < 0$
(B)	$\|\|x\|-1\| < 1-x$	(Q)	$x = 3$
(C)	$\|x-4\| < \sqrt{3-x}$	(R)	$x \leq 3$
(D)	$\sqrt{x^3-4x^2+4x-3}+\sqrt{3-x}+x^2 \geq 9$	(S)	$x \geq 4$
		(T)	$x \in \phi$

140.		141.		142.	
(A)	P Q R S T	(A)	P Q R S T	(A)	P Q R S T
(B)	P Q R S T	(B)	P Q R S T	(B)	P Q R S T
(C)	P Q R S T	(C)	P Q R S T	(C)	P Q R S T
(D)	P Q R S T	(D)	P Q R S T	(D)	P Q R S T

INTEGER TYPE / SUBJECTIVE TYPE

143. There are two possible values of A (say A_1 & A_2) in the solution of matrix equation

$$\begin{bmatrix} 2A+1 & -5 \\ -4 & A \end{bmatrix}^{-1} \begin{bmatrix} A-5 & B \\ 2A-2 & C \end{bmatrix} = \begin{bmatrix} 14 & D \\ E & F \end{bmatrix}$$ then find $-27(A_1 + A_2)$

144. If $f(n) = -\begin{vmatrix} n^2 & (n+1)^2 & (n+2)^2 \\ (n+1)^2 & (n+2)^2 & (n+3)^2 \\ (n+2)^2 & (n+3)^2 & (n+4)^2 \end{vmatrix}$ then find the value of $f(1) \cdot f(2) \cdot f(3)$.

145. Given two curves : $y = f(x)$ passing through (0, 1) and $y = \int_{-\infty}^{x} f(t)dt$ passing through $\left(0, \frac{1}{3}\right)$. The tangents drawn to both the curves at the points with equal abscissas intersects on x-axis. Find the value of $\ell n f(3)$.

146. Consider $f(x) = ax^3 - bx^2 + cx + d$,

where $a = \dfrac{\cot^{-1}(\cot 4) + \pi}{2}$

$b = \dfrac{3(\text{cosec}^{-1}\text{cosec}6 + 2\pi)}{2}$

$c = (\tan^{-1}\tan 10) + 3\pi + 2$

$d = 5(2\pi - \sec^{-1}\sec 5)$

If p represents the number of extremum of the function $f(x)$ and q represents the number of real roots of the equation $f(x) = 0$, then find the value of p + q.

147. Let $y = f(x)$ be a differentiable curve satisfying $2 + \int_{2}^{x} f(t)dt = \frac{x^2}{2} + \int_{x}^{2} t^2 f(t)dt$,

then $\int_{-\pi/4}^{\pi/4} \frac{f(x) + x^9 - x^3 + x + 1}{\cos^2 x} dx$ equals

148. If difference between greatest & least value of function $f(x) = \int_{0}^{x} (at^3 + t + \cos^2 t)dt,\ a > 0\ \forall\ x \in [1, 3]$ is 25 + sin1 cos1 cos4, then value of a is

149. ABCD and PQRS are two variable rectangles such that P, Q, R and S lie on AB, BC, CD and DA respectively and perimeter 'ℓ' of PQRS is constant. If maximum area of ABCD is 32, then $\sqrt{\ell}$ is equal to

143.		**144.**		**145.**		**146.**	
147.		**148.**		**149.**			

ANSWER KEY

1. A	**2.** C	**3.** A	**4.** C	**5.** C	**6.** A	**7.** C	**8.** C	**9.** B	**10.** A
11. D	**12.** B	**13.** A	**14.** D	**15.** C	**16.** A	**17.** A	**18.** A	**19.** B	**20.** A
21. A	**22.** D	**23.** D	**24.** A	**25.** D	**26.** A	**27.** B	**28.** B	**29.** D	**30.** B
31. B	**32.** C	**33.** A	**34.** B	**35.** C	**36.** A	**37.** C	**38.** B	**39.** B	**40.** B
41. C	**42.** C	**43.** D	**44.** D	**45.** C	**46.** C	**47.** B	**48.** A	**49.** C	**50.** B
51. A	**52.** A	**53.** C	**54.** B	**55.** A	**56.** A	**57.** C	**58.** C	59. B	60. D
61. B	62. C	63. B	64. D	65. C	66. D	67. A	68. A,D		

69. A,B,D 70. A,B,C 71. A,D 72. A,B,C 73. B,D

74. A 75. B,C,D 76. A,B,D 77. B,C 78. A,B,C

79. A,B 80. A,B 81. B,C,D 82. B,D 83. A,D

84. A,B,C,D 85. A,C 86. A,B,C 87. A,B 88. A,C,D

89. A,B,C,D 90. A 91. A 92. A 93. A 94. A 95. D 96. D 97. C

98. D	99. D	100. D	101. D	102. C	103. D	104. A	105. B	106. A	107. C
108. B	109. D	110. A	111. C	112. B	113. A	114. D	115. C	116. B	117. A
118. C	119. A	120. B	121. A	122. D	123. B	124. C	125. C	126. D	127. A
128. B	129. D	130. A	131. C	132. B	133. D	134. D	135. A		

136. (A) → **(R,S); (B) → (P,R); (C) → (Q); (D) → (P,R)**

137. (A) → (Q); (B) → (P,R); (C) → (P); (D) → (R)

138. (A) → (R); (B) → (P); (C) → (S); (D) → (Q)

139. (A) → (R), (B) → (Q), (C) → (P), (D) → (S)

140. (A) → (T); (B) → (P); (C) → (Q); (D) → (T)

141. (A) → (P); (B) → (Q); (C) → (R,S); (D) → (Q)

142. (A) → (P,S); (B) → (P); (C) → (T); (D) → (Q)

143. 9 144. 8 145. 9 146. 3 147. 2 148. 1 149. 4