Mathematics

TARGET : JEE 2013

Home Assignment # 01

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STRAIGHT OBJECTIVE TYPE

1. Assume that $f(1) = 0$ and that for all integers m and n, $f(m + n) = f(m) + f(n) + 3(4mn - 1)$, then $f(19) =$

(A) 2049 (B) 2098 (C) 1944 (D) 1998

2. $f(x) = \{x\} + \{x + 1\} + \{x + 2\}........\{x + 99\}$, then $[f(\sqrt{2})]$ where {.} denotes fractional part function & [.] denotes the greatest integer function =

(A) 5050 (B) 4950 (C) 41 (D) 14

3. If $f_0(x) = x/(x + 1)$ and $f_{n+1} = f_0 \text{ o } f_n$ for n = 0, 1, 2,......, then $f_n(x)$ is -

(A) $f_n(x) = \frac{x}{(n+1)x+1}$ (B) $f_0(x)$ (C) $\frac{nx}{nx+1}$ (D) $\frac{x}{nx+1}$

4. $\lim\limits_{x\to 1} \frac{\sqrt{x+8}-\sqrt{8x+1}}{\sqrt{5-x}-\sqrt{7x-3}} =$

(A) $\frac{7}{12}$ (B) 0 (C) –1 (D) does not exist

5. Let $f(x)$ is even and $g(x)$ is an odd function which satisfies $x^2 f(x) - 2f\left(\frac{1}{x}\right) = g(x)$, then $f(1) + f(2) + f(3) + f(4) =$

(A) 10 (B) 0 (C) 24 (D) 4

6. If $f(x)$ be a function such that $f(x + 1) = \frac{f(x)-1}{f(x)+1}$, $\forall$ x $\in$ N and $f(1) = 2$ then $f(999)$ is -

(A) –3 (B) 2 (C) $\frac{1}{3}$ (D) $-\frac{1}{2}$

7. If $f(x) = 3x + 5$ and $h(x) = 3x^2 + 3x + 2$, then function g such that $f$og = h is -

(A) $x + 1$ (B) $x^2 + x - 1$ (C) $9x^2 + 9x + 11$ (D) none of these

8. The principle value of $\cos^{-1}\left(-\sin\frac{7\pi}{6}\right)$ is -

(A) $\frac{5\pi}{3}$ (B) $\frac{7\pi}{6}$ (C) $\frac{\pi}{3}$ (D) none of these

9. $\lim\limits_{x\to 0} \frac{(2^{\sin x}-1)(\ell n(1+\sin 2x))}{x\tan^{-1}x} =$

(A) $\ell n2$ (B) $2\ell n2$ (C) $\ell n^2 2$ (D) 0

FILL THE ANSWER HERE

1. (A) (B) (C) (D) 2. (A) (B) (C) (D) 3. (A) (B) (C) (D) 4. (A) (B) (C) (D)
5. (A) (B) (C) (D) 6. (A) (B) (C) (D) 7. (A) (B) (C) (D) 8. (A) (B) (C) (D)
9. (A) (B) (C) (D)

10. If $f(x) = \begin{cases} Ax+3 & x<1 \\ 2 & x=1 \\ B+x^2 & x>1 \end{cases}$ is continuous for all x ∈ R, then A,B are :

(A) (–1, 0) (B) (0, 1) (C) (–1, 1) (D) (0, 0)

11. $\lim\limits_{x\to\infty}\left(\frac{3x^2+1}{3(x^2+x+1)}\right)^x =$

(A) –1 (B) –2 (C) e^{-2} (D) e^{-1}

12. $\lim\limits_{x\to\infty}\left(3^x+3^{2x}\right)^{1/x} =$

(A) 3 (B) 9 (C) 0 (D) infinity

13. $\cos\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right)+\cos\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right)$ is equal to -

(A) $\pm\sqrt{\frac{a+b}{a}}$ (B) $\pm\sqrt{\frac{b}{a+b}}$ (C) $\sqrt{\frac{a+b}{b}}$ (D) none of these

14. The value of $\tan\left(\lim\limits_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{1+2r}{1+r^2(r+1)^2}\right)\right)$ is -

(A) 1 (B) $\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) none of these

15. The complete solution of the equation 100x – 100[x] = 1, where [.] = the greatest integer less than or equal to x, are -

(A) $x = n+\frac{1}{100}, n \in N$ (B) $x = n-\frac{1}{100}, n \in N$

(C) $x = n+\frac{1}{100}, n \in I$ (D) $n < x < n+1, n \in I$

16. If $f(x)= 5\log_5 x$ then $f^{-1}(\alpha-\beta)$ where α, β ∈ R is equal to -

(A) $f^{-1}(\alpha) - f^{-1}(\beta)$ (B) $\frac{f^{-1}(\alpha)}{f^{-1}(\beta)}$ (C) $\frac{1}{f(\alpha-\beta)}$ (D) $\frac{1}{f(\alpha)-f(\beta)}$

17. If $f(n) = \lim\limits_{x\to0}\sum_{K=1}^{n^2}\frac{\ell n(1+Kx)}{x}$, $g(n) = \lim\limits_{x\to0}\sum_{K=1}^{n}\frac{e^{K^3x}-1}{x}$, then $\lim\limits_{x\to\infty}\frac{f(x)}{g(x)} =$

(A) does not exist (B) 2 (C) 0 (D) data inadequate

10.	Ⓐ Ⓑ Ⓒ Ⓓ	**11.**	Ⓐ Ⓑ Ⓒ Ⓓ	**12.**	Ⓐ Ⓑ Ⓒ Ⓓ	**13.**	Ⓐ Ⓑ Ⓒ Ⓓ
14.	Ⓐ Ⓑ Ⓒ Ⓓ	**15.**	Ⓐ Ⓑ Ⓒ Ⓓ	**16.**	Ⓐ Ⓑ Ⓒ Ⓓ	**17.**	Ⓐ Ⓑ Ⓒ Ⓓ

18. If $f(x) = \lim_{n\to\infty} n\left(x^{1/n} - 1\right)$, x > 0, then $f(xy)$ is -

(A) $f(x)f(y)$ (B) $f(x) + f(y)$ (C) $f(x) - f(y)$ (D) $\dfrac{f(x)}{f(y)}$

19. The number of points where $f(x) = [\sin x - \cos x]$ is not continuous in $[0, 2\pi]$ are (where $[\cdot]$ denotes the greatest integer function) -

(A) 6 (B) 5 (C) 7 (D) 4

20. Let $f(x) = \begin{cases} \dfrac{1}{|x|}, & |x| \geq 1 \\ ax^2 + b, & |x| < 1 \end{cases}$ be continuous and differentiable everywhere. Then a and b are -

(A) $-\dfrac{1}{2}, \dfrac{3}{2}$ (B) $\dfrac{1}{2}, -\dfrac{3}{2}$ (C) $\dfrac{1}{2}, \dfrac{3}{2}$ (D) $\dfrac{3}{2}, \dfrac{3}{2}$

21. The complete solution set of the inequality $[\tan^{-1}x]^2 - 8[\tan^{-1}x] + 16 \leq 0$, where [.] denotes the greatest integer function is -

(A) $(-\infty, \tan 4]$ (B) $[\tan 4, \tan 3]$ (C) $[\tan 4, \tan 5)$ (D) no solution

22. $\lim_{x\to 0} \sin^{-1} \sin \cot^{-1}\left(\dfrac{1}{x}\right)$ is equal to -

(A) 0 (B) $\dfrac{\pi}{2}$ (C) $-\dfrac{\pi}{2}$ (D) non-existent

23. Let $f(x) = (1 + \cos x)^{\frac{2}{2x-\pi}}$, $x \neq \dfrac{\pi}{2}$. If f(x) is continuous at $x = \dfrac{\pi}{2}$, then the value of $f\left(\dfrac{\pi}{2}\right)$ is -

(A) 1 (B) –1 (C) $\dfrac{1}{e}$ (D) e

24. If $f(x) = \lim_{n\to\infty} (1 + x)(1 + x^2)(1 + x^{2^2})(1 + x^{2^3})......(1 + x^{2^n})$ where $|x| < 1$, then -

(A) $\dfrac{1}{1-x}$ (B) $\dfrac{1}{1+x}$ (C) $\dfrac{1}{1-x^2}$ (D) $\dfrac{1}{1+x^2}$

25. Set of points where $f(x) = \dfrac{4x}{5+6|x|}$ is differentiable, is -

(A) $(-\infty, 0) \cup (0, \infty)$ (B) $(-\infty, -1) \cup (-1, \infty)$ (C) $(-\infty, \infty)$ (D) $(0, \infty)$

26. If $y = x + e^x$, then at x = 1, $\dfrac{d^2x}{dy^2}$ is equal to -

(A) e (B) $\dfrac{-e}{(1+e)^3}$ (C) $\dfrac{-e}{(1+e)}$ (D) $\dfrac{-e}{(1+e)^2}$

18.	Ⓐ Ⓑ Ⓒ Ⓓ	**19.**	Ⓐ Ⓑ Ⓒ Ⓓ	**20.**	Ⓐ Ⓑ Ⓒ Ⓓ	**21.**	Ⓐ Ⓑ Ⓒ Ⓓ
22.	Ⓐ Ⓑ Ⓒ Ⓓ	**23.**	Ⓐ Ⓑ Ⓒ Ⓓ	**24.**	Ⓐ Ⓑ Ⓒ Ⓓ	**25.**	Ⓐ Ⓑ Ⓒ Ⓓ
26.	Ⓐ Ⓑ Ⓒ Ⓓ						

27. Let $f(x)=\begin{cases}(x-1)\sin\left(\dfrac{1}{x-1}\right), & x\neq 1\\ 0, & x=1\end{cases}$, which of the following statements is true ?

(A) f is differentiable at x = 1 but not at x = 0
(B) f is neither differentiable at x = 0 nor at x = 1
(C) f is differentiable at x = 0 and at x = 1
(D) f is differentiable at x = 0 but not at x = 1

28. Let f & g be differentiable functions satisfying $g'(m)=4$ & $g(m)=b$ & $f\text{o}g(x)$ be an identity function then $f'(b)$ is -

(A) 4
(B) 4/3
(C) 1/4
(D) 3/4

29. If $f(x)=\sqrt{x^2+1}$, $g(x)=\dfrac{x+1}{x^2+1}$ and h(x) = 2x – 3, then $f'(h'(g'(x)))=$

(A) 0
(B) $\dfrac{1}{\sqrt{x^2+1}}$
(C) $\dfrac{2}{\sqrt{5}}$
(D) $\dfrac{x}{\sqrt{x^2+1}}$

30. The value of $f(0)$ so that $f(x)=\dfrac{-e^x+2^x}{x}$ may be continuous at x = 0 is -

(A) $-\ell n2$
(B) 0
(C) 4
(D) $\ell n\left(\dfrac{2}{e}\right)$

31. If $I=\int\dfrac{x^5dx}{\sqrt{1+x^3}}$, then I equals to -

(A) $\dfrac{2}{9}(1+x^3)^{5/2}+\dfrac{2}{3}(1+x^3)^{3/2}+c$
(B) $\ell n\left|\sqrt{x}+\sqrt{1+x^3}\right|+c$
(C) $\ell n\left|\sqrt{x}-\sqrt{1+x^3}\right|+c$
(D) $\dfrac{2}{9}(1+x^3)^{3/2}-\dfrac{2}{3}(1+x^3)^{1/2}+c$

32. If $f(x)=\ell n(-x+\sqrt{x^2+1})$, then $\int f''(x)dx$ is equal to -

(A) $\dfrac{1}{(x+\sqrt{x^2+1})}+c$
(B) $\dfrac{-1}{\sqrt{x^2+1}}+c$
(C) $-\sqrt{x^2+1}+c$
(D) $\ell n(x+\sqrt{x^2+1})$

33. Let $x^2\neq n\pi-1$, $n\in N$. Then the value of $\int x\sqrt{\dfrac{2\sin(x^2+1)-\sin 2(x^2+1)}{2\sin(x^2+1)+\sin 2(x^2+1)}}dx$ is -

(A) $\ell n\left|\dfrac{1}{2}\sec(x^2+1)\right|+c$
(B) $\ell n\left|\sec\left(\dfrac{x^2+1}{2}\right)\right|+c$
(C) $\dfrac{1}{2}\ell n\left|\sec(x^2+1)\right|+c$
(D) $\dfrac{1}{2}\ell n\left|\dfrac{2}{\sec(x^2+1)}\right|+c$

27.	Ⓐ Ⓑ Ⓒ Ⓓ	**28.**	Ⓐ Ⓑ Ⓒ Ⓓ	**29.**	Ⓐ Ⓑ Ⓒ Ⓓ	**30.**	Ⓐ Ⓑ Ⓒ Ⓓ
31.	Ⓐ Ⓑ Ⓒ Ⓓ	**32.**	Ⓐ Ⓑ Ⓒ Ⓓ	**33.**	Ⓐ Ⓑ Ⓒ Ⓓ		

34. $\int \frac{\sec x .\operatorname{cosec} x}{2\cot x - \sec x \operatorname{cosec} x} dx$

(A) $\frac{1}{2}\ell n|\sec 2x + \tan 2x| + c$ (B) $\ell n|\sec x + \operatorname{cosec} x| + c$

(C) $\ell n|\sec x + \tan x| + c$ (D) $\frac{1}{2}\ell n|\sec x + \operatorname{cosec} x| + c$

35. If $\ell = \lim\limits_{x\to 1^+} 2^{-2^{\frac{1}{1-x}}}$ and $m = \lim\limits_{x\to 1^+} \frac{x\sin(x-[x])}{x-1}$, where [.] denotes greatest integer function, then $\int_{\ell}^{m} \frac{\ell n(x+\sqrt{1+x^2})}{\sqrt{1+x^2}} dx$ is equal to -

(A) 1 (B) $\frac{1}{2}$ (C) $\ell n\frac{1}{2}$ (D) 0

36. If $f(x)$ is an even function which is also periodic with the period T and $\int_0^a f(x)dx = 3$ and $\int_{-T/2}^{3T/2} f(x)dx = 18$, then $\int_{-a}^{a+5T} f(x)dx$ is equal to -

(A) 96 (B) 93 (C) 51 (D) 48

37. If antiderivative of $\frac{x^3}{\sqrt{1+2x^2}}$ which passes through (1, 2) is $\frac{1}{m}(1+2x^2)^{1/2}(x^2-1)+n$. Then value of m+n is equal to -

(A) 8 (B) 5 (C) 6 (D) 7

38. If $|f(x) + 6 - x^2| = |f(x)| + |4 - x^2| + 2$, then $f(x)$ is necessarily non-negative in -

(A) $(-\sqrt{6}, \sqrt{6})$ (B) $(-\infty, -2) \cup (2, \infty)$ (C) $[-2, 2]$ (D) none of these

39. If $f(8-t) = f(t)$ and $\int_0^4 f(\alpha)d\alpha = 8$, then $\int_0^8 f(\gamma)d\gamma$ is -

(A) 4 (B) 8 (C) 16 (D) 32

40. If $f(x) \geq 0 \ \forall \ x \in R$ and area bounded by the curve $y = f(x)$, $x = 0$, $x = a$ and x-axis is $\tan^{-1}a$, then the number of solutions of the equation $f(x) - 1 = \tan^2 x$ is -

(A) 0 (B) 1 (C) 2 (D) infinitely many

41. If $f(x) = \int \left(\tan(\ell nx) + \frac{1}{2}\right)^2 dx$ & $f(1) = 0$, then $f(e^{\pi/4})$ is -

(A) $\frac{3-e^{\pi/4}}{4}$ (B) $-\frac{3-e^{\pi/4}}{4}$ (C) $-\frac{(3+e^{\pi/4})}{4}$ (D) $\frac{e^{\pi/4}+3}{4}$

34.	Ⓐ Ⓑ Ⓒ Ⓓ	35.	Ⓐ Ⓑ Ⓒ Ⓓ	36.	Ⓐ Ⓑ Ⓒ Ⓓ	37.	Ⓐ Ⓑ Ⓒ Ⓓ
38.	Ⓐ Ⓑ Ⓒ Ⓓ	39.	Ⓐ Ⓑ Ⓒ Ⓓ	40.	Ⓐ Ⓑ Ⓒ Ⓓ	41.	Ⓐ Ⓑ Ⓒ Ⓓ

42. If $x = \int_0^{t^2} e^{\sqrt{z}} \left\{ \frac{2\tan\sqrt{z} + 1 - \tan^2 z}{2\sqrt{z}\sec^2\sqrt{z}} \right\} dz$ & $y = \int_0^{t^2} e^{\sqrt{z}} \left\{ \frac{1 - \tan^2\sqrt{z} - 2\tan\sqrt{z}}{2\sqrt{z}\sec^2\sqrt{z}} \right\} dz$.

Then the inclination of the tangent to the curve at $t = \frac{\pi}{4}$ is -

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{4}$

43. Let $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $g(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$. The derivative of $f(x)$ with respect to g(x) at $x = -\frac{1}{2}$ is -

(A) $-\frac{1}{2}$ (B) –1 (C) $\frac{1}{2}$ (D) 1

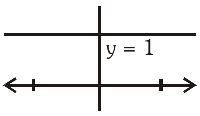
44. $\lim_{x \to 0} |x|^{\tan x}$ is equal to -

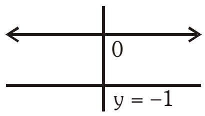
(A) 1 (B) 2 (C) 3 (D) 0

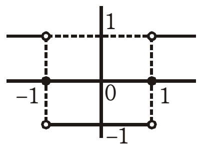
45. $I = \int \frac{2}{(2-x)^2} \sqrt[3]{\frac{2-x}{2+x}}\, dx = \frac{3}{4}\sqrt[3]{f^2(x)} + c$, then $f'(1)$ is equal to -

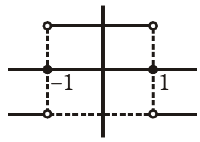
(A) 2 (B) 1 (C) 4 (D) 3

46. Which of the following represents the graph of the function $f(x) = \lim_{n \to \infty} \frac{x^{2n} - 1}{x^{2n} + 1}$?

(A) 

(B) 

(C) 

(D) 

47. If the non-negative solution set of the equation $[x]^2 = [x + 6]$ is given by [a, b), then the value of a + b is ([.] is greatest integer function)

(A) 3 (B) 5 (C) 7 (D) 4

48. If $L = \lim_{x \to 0^+} (\text{cosec}\, x)^{1/\ell nx}$, then the value of ℓnL is -

(A)1 (B) –1 (C) 0 (D) none of these

49. If $p = \lim_{n \to \infty} \frac{\left(\prod_{r=1}^{n}(r+n)\right)^{1/n}}{n}$, then $\log_2(ep)$ is equal to -

(A) 4 (B) 3 (C) 2 (D) 1

50. If $D^*f(x) = 2f(x)\, D(f(x))$, then $(D^*(x\ell nx))_{x=e}$ is -

(A) 4 (B) 4e (C) 2 (D) 2e

51. If $f(x) = x^3 + 4x^2 + 6x$, then $f^{-1}(-4)$ is -

(A) –2 (B) 1 (C) –1 (D) does not exist

42. Ⓐ Ⓑ Ⓒ Ⓓ 43. Ⓐ Ⓑ Ⓒ Ⓓ 44. Ⓐ Ⓑ Ⓒ Ⓓ 45. Ⓐ Ⓑ Ⓒ Ⓓ

46. Ⓐ Ⓑ Ⓒ Ⓓ 47. Ⓐ Ⓑ Ⓒ Ⓓ 48. Ⓐ Ⓑ Ⓒ Ⓓ 49. Ⓐ Ⓑ Ⓒ Ⓓ

50. Ⓐ Ⓑ Ⓒ Ⓓ 51. Ⓐ Ⓑ Ⓒ Ⓓ

52. The complete solution set of the inequality $[\text{cosec}^{-1}x] > [\sec^{-1}x]$, where [.] is greatest integer function, is -

(A) [1, cosec1] (B) [1, sec1] (C) [cosec1, sec1] (D) none of these

53. If $f(x) = \begin{cases} (\cot x)^{\cos x}.(\cos x)^{\cot x} & , \quad x \neq \frac{\pi}{2} \\ k & , \quad x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then k =

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2

54. If $f(x) = \sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{(x-1)}}$, then $f'(x)$ at x = 1.5 is -

(A) 0 (B) $-\sqrt{2}$ (C) $-\sqrt{3}$ (D) –4

55. $\int (\sec 2x + 1)\sqrt{\sec^2 x - 1}\,dx$ is equal to -

(A) $\frac{\ell n \sec x}{2} + c$ (B) $\frac{\ell n \sec 2x}{2} + c$ (C) $\ell n \sec 2x + c$ (D) $\ell n \sec x + c$

56. $\int x^x \left((\ell nx)^2 + \ell nx + \frac{1}{x}\right) dx$ is equal to -

(A) $x^x\left((\ell nx)^2 - \frac{1}{x}\right) + c$ (B) $x^x(\ell nx - x) + c$ (C) $-x^x \frac{(\ell nx)^2}{2} + c$ (D) $x^x \ell nx + c$

57. If $f(x) = \int \frac{\ell n^2 x + \ell n(ex)}{\ell n^2(ex)} dx$; $f(1) = 0$ and $g(x) = 1 - \frac{f(x)}{x}$. Then the domain of g(x) is -

(A) $(0,\infty)$ (B) $(0,1)\cup(1,\infty)$ (C) $\left(0,\frac{1}{e}\right)\cup\left(\frac{1}{e},\infty\right)$ (D) $(1,\infty)$

58. If $\int f(x)\cos x\,dx = \frac{1}{2}f^2(x) + C$, where $f(x)$ is not a cosntant function & $f\left(\frac{\pi}{2}\right) = 0$, then the period of $g(x) = \frac{f(x)}{f(x)}$ is -

(A) π (B) 2π (C) 4π (D) not defined

59. Let for k > 0, $f(x) = \begin{cases} \frac{k^x + k^{-x} - 2}{x^2} & \text{if} \quad x > 0 \\ 3\ell n(k-x) - 2 & \text{if} \quad x \le 0 \end{cases}$, if $f(x)$ is continuous at x = 0, then k is equal to -

(A) e or 1 (B) 1 or 2 (C) e or e^2 (D) only e^2

52.	Ⓐ Ⓑ Ⓒ Ⓓ	53.	Ⓐ Ⓑ Ⓒ Ⓓ	54.	Ⓐ Ⓑ Ⓒ Ⓓ	55.	Ⓐ Ⓑ Ⓒ Ⓓ
56.	Ⓐ Ⓑ Ⓒ Ⓓ	57.	Ⓐ Ⓑ Ⓒ Ⓓ	58.	Ⓐ Ⓑ Ⓒ Ⓓ	59.	Ⓐ Ⓑ Ⓒ Ⓓ

60. Let $A = \begin{bmatrix} x^2 & 6 & 0 \\ 1 & -5 & 1 \\ 2 & 0 & x \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{bmatrix}$. If a function is defined as $f(x) = \text{tr}(AB)$, then $\int \frac{3dx}{f(x)}$ is equal to -

(A) $\frac{1}{4}\ell n\left|\frac{2x-1}{2x+5}\right|+c$ (B) $\frac{1}{4}\ell n\left|\frac{2x+5}{2x-1}\right|+c$ (C) $\frac{1}{3}\ell n\left|\frac{1-2x}{2x+5}\right|+c$ (D) $\frac{1}{3}\ell n\left|\frac{1-2x}{2x+3}\right|+c$

61. Let $F(x) = \int e^{\sin^{-1}x}\left(1-\frac{x}{\sqrt{1-x^2}}\right)dx$ and $F(0) = 1$. If $F\left(\frac{1}{2}\right) = \frac{k\sqrt{3}e^{\frac{\pi}{6}}}{\pi}$, then k is equal to -

(A) 2π (B) π (C) $3\pi/2$ (D) $\pi/2$

62. If $\int \frac{\sqrt{9-x^2}}{x^4}dx = k.\frac{(9-x^2)^{3/2}}{9x^3}+c$, then the value of 'k' is -

(A) $-\frac{1}{3}$ (B) $\frac{1}{3}$ (C) $\frac{1}{27}$ (D) $-\frac{1}{27}$

63. The value of $\left[\lim_{n\to\infty}\frac{2}{n}\sum_{r=1}^{n}\frac{1}{1+\cos\left(\frac{2r}{n}\right)}\right]$, where [.] represents greatest integer function, is -

(A) 0 (B)1 (C) 2 (D) 3

64. If $f(x) = \frac{1}{x^2-17x+66}$, then $f\left(\frac{2}{x-2}\right)$ is discontinuous at x =

(A) $2, \frac{7}{3}, \frac{25}{11}$ (B) $2, \frac{8}{3}, \frac{24}{11}$ (C) $2, \frac{7}{3}, \frac{24}{11}$ (D) $2, \frac{3}{7}, \frac{24}{11}$

65. $\int \frac{x^{9/2}}{\sqrt{1+x^{11}}}dx$ is equal to -

(A) $\frac{2}{11}\log\left(x^{7/2}+\sqrt{x^7+1}\right)+c$ (B) $\frac{1}{2}\log\left(\frac{x^{11}+1}{x^{11}-1}\right)+c$

(C) $2\sqrt{1+x^{11}}+c$ (D) $\frac{2}{11}\log\left(x^{11/2}+\sqrt{1+x^{11}}\right)+c$

66. $\int_1^{e^{35}} \frac{\pi\sin(\pi\ell nx)}{x}dx$ is equal to -

(A) 2 (B) –2 (C) $\frac{2}{\pi}$ (D) 2π

60.	Ⓐ Ⓑ Ⓒ Ⓓ	**61.**	Ⓐ Ⓑ Ⓒ Ⓓ	**62.**	Ⓐ Ⓑ Ⓒ Ⓓ	**63.**	Ⓐ Ⓑ Ⓒ Ⓓ
64.	Ⓐ Ⓑ Ⓒ Ⓓ	**65.**	Ⓐ Ⓑ Ⓒ Ⓓ	**66.**	Ⓐ Ⓑ Ⓒ Ⓓ		

67. The value of $\int_{-1}^{1} \max(2-x,\ 2,\ 2+x)dx$ is -

(A) 4 (B) 5 (C) 2 (D) 3

68. If for a continuous function f, $f(0) = f(1) = 0$ & $f'(1) = 2$ and $g(x) = f(e^x) \cdot e^{f(x)}$, then $g'(0)$ is equal to -

(A) 0 (B) 1 (C) 2 (D) 4

69. $\int \frac{\cos\{\log(f(x)) + \log(g(x))\}}{f(x).g(x)} \{f(x) \cdot g'(x) + g(x) \cdot f'(x)\}\, dx$ is equal to -

(A) $\sin\{\log(f(x).\ g(x))\} + C$ (B) $\frac{g(x)}{f(x)} \sin\{\log(f(x).\ g(x))\} + C$

(C) $\frac{f(x)}{g(x)} \sin\{\log(f(x) \cdot g(x))\} + C$ (D) none of these

70. $\int \frac{dx}{x^{1/5}(1+x^{4/5})^{1/2}}$ is equal to -

(A) $\sqrt{1+x^{\frac{4}{5}}} + K$ (B) $\frac{5}{2}\sqrt{1+x^{\frac{4}{5}}} + K$ (C) $x^{\frac{4}{5}}\left(1+x^{\frac{4}{5}}\right)^{\frac{1}{2}} + K$ (D) $\frac{2}{5}\sqrt{1+x^{\frac{4}{5}}} + K$

71. $\int_0^2 \sqrt{\frac{2+x}{2-x}}\, dx$ is equal to -

(A) $\pi + 1$ (B) $1 + \pi/2$ (C) $\pi + 3/2$ (D) $\pi + 2$

72. $\lim_{n\to\infty}\left\{\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{2n}} + \frac{1}{\sqrt{3n}} + \ldots\ldots + \frac{1}{2n}\right\}$ is equal to -

(A) 0 (B) 1 (C) 2 (D) 4

73. If $\int \frac{\operatorname{cosec}^2 x - 2009}{\cos^{2009} x} dx = -\frac{A(x)}{(B(x))^{2009}} + c$, then number of solution of the equation $\frac{A(x)}{B(x)} = \{x\}$ in $[0, 2\pi]$ is (where {.} represents fractional part function) -

(A) 0 (B) 1 (C) 2 (D) 3

74. If $g(x) = \int_0^x \ell n(\sec t \tan t - \sec^2 t + 1)dt$, then set of value of x in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which g(x) is increasing, is -

(A) $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$ (B) $\left(0, \frac{\pi}{2}\right)$ (C) $\left(-\frac{\pi}{2}, 0\right)$ (D) ϕ

67.	Ⓐ Ⓑ Ⓒ Ⓓ	68.	Ⓐ Ⓑ Ⓒ Ⓓ	69.	Ⓐ Ⓑ Ⓒ Ⓓ	70.	Ⓐ Ⓑ Ⓒ Ⓓ
71.	Ⓐ Ⓑ Ⓒ Ⓓ	72.	Ⓐ Ⓑ Ⓒ Ⓓ	73.	Ⓐ Ⓑ Ⓒ Ⓓ	74.	Ⓐ Ⓑ Ⓒ Ⓓ

75. $\int_{-\pi}^{\pi}(1-x)\sin x\cos^2 x\,dx$ is equal to -

(A) 0 (B) $\frac{2\pi^3}{3}$ (C) $\frac{2\pi}{3}$ (D) $-\frac{2\pi}{3}$

76. $\int_{6}^{8}\frac{\cos x^2}{\cos x^2+\cos(x-14)^2}dx$ is equal to -

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{1}{2}$ (D) 1

77. If $x^2 + y^2 = 5$ and $2y'' + ky^{-3} = 0$, then k is equal to -

(A) 5 (B) –5 (C) 10 (D) –10

78. The value of $\int_{-1}^{2}\left[\frac{[x]}{1+x^2}\right]dx$, where [.] denotes the greatest integer function, is -

(A) –2 (B) –1 (C) 0 (D) none of these

79. If $f(x) = e^{-x}$, $g(x) = e^x$ & $h(x) = f(g(x))$, then the value of $\ell n|h'(0)|$ is equal to -

(A) 0 (B) 1 (C) –1 (D) none of these

80. If $\int\frac{dx}{\sqrt[3]{\cos^{12}x+3\cos^{10}x+3\cos^8 x+\cos^6 x}}=f(x)-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{f(x)}{\sqrt{2}}\right)+c$, then $f(x)$ is -

(A) bounded & periodic (B) bounded & aperiodic

(C) unbounded & periodic (D) unbounded & aperiodic

81. If sinα and cosα are the roots of the equation $ax^2 - bx + c = 0$, $a \neq 0$, then $\cos^{-1}(a^2 + 2ac - b^2)$ is equal to -

(A) 0 (B) 1 (C) $\frac{\pi}{2}$ (D) π

82. If $3f\left(\frac{2x+3}{2x-3}\right)=2x-3$, then $\int f(x)dx$ is equal to -

(A) $\ell n(x-1)^2 + c$ (B) $\ell n|(x-1)^3| + c$ (C) $\ell n(x-1)^6 + c$ (D) $\ell n|(x-1)| + c$

83. The differential coefficient of $f(\ell nx)$, with respect to ℓnx, where $f(x) = \ell nx$, is -

(A) $\frac{x}{\ell nx}$ (B) $\frac{\ell nx}{x}$ (C) $\frac{1}{\ell nx}$ (D) ℓnx

84. If $f'(0) = -2$ and $\int_{0}^{\pi}(f(x)+f''(x))\cos x\,dx=3$, then $f'(\pi)$ is equal to -

(A) –1 (B) 1 (C) 0 (D) –2

75.	Ⓐ Ⓑ Ⓒ Ⓓ	76.	Ⓐ Ⓑ Ⓒ Ⓓ	77.	Ⓐ Ⓑ Ⓒ Ⓓ	78.	Ⓐ Ⓑ Ⓒ Ⓓ
79.	Ⓐ Ⓑ Ⓒ Ⓓ	80.	Ⓐ Ⓑ Ⓒ Ⓓ	81.	Ⓐ Ⓑ Ⓒ Ⓓ	82.	Ⓐ Ⓑ Ⓒ Ⓓ
83.	Ⓐ Ⓑ Ⓒ Ⓓ	84.	Ⓐ Ⓑ Ⓒ Ⓓ				

85. Let f (x) be a differentiable function satisfying the equation $\frac{f'(x)}{2} = \frac{x}{e^{f(x)}}$ $\forall$ x $\in$ R . If f '(1) = 1, then the number of solutions of the equation f (x) = f '(x) is -

(A) 1 (B) 2 (C) 3 (D) none of these

86. If $L = \lim_{n\to\infty}\left(\frac{n+2}{n^2+n+1}+\frac{n+4}{n^2+2n+4}+\frac{n+6}{n^2+3n+9}+.......+\frac{5}{7n}\right)$, then e^L is equal to -

(A) 1 (B) $\frac{1}{7}$ (C) 7 (D) e

87. Let f (x) = ax^2 + bx + c, where b, c $\in$ R, a > 0. If f (x) = 0 has two real and different positive roots α and $\beta(\alpha < \beta)$, then the value of $\int_{-\beta}^{\beta} f(|x|)+|f(|x|)|dx$ is -

(A) $4\int_0^{\alpha} f(x)dx$ (B) $4\int_{\alpha}^{\beta} f(x)dx$ (C) $4\int_0^{\beta} f(x)dx$ (D) 0

88. Let f (x) be a differentiable function such that $f^2(x) + xf(x) = 3$, then $\int \frac{3x^3+6x^2 f(x)+2f(x)}{(2f(x)+x)(x^3-2f(x))^2}dx$ equals -

(A) $\frac{1}{x^3-2f(x)}+c$ (B) $\frac{1}{2f(x)-x^3}+c$ (C) $\frac{1}{2f(x)+x}+c$ (D) $\frac{1}{x^3+2f(x)}+c$

89. If $f(x)=\frac{x-1}{x+2}$, then $\frac{df^{-1}(x)}{dx}$ is equal to -

(A) $\frac{3}{(1-x)^2}$ (B) $\frac{-3}{(1-x)^2}$ (C) $\frac{1}{(1-x)^2}$ (D) $\frac{-1}{(1-x)^2}$

90. If $\int_{-4}^{4} \cot^{-1} x\,dx = k\pi$, then k equals -

(A) 0 (B) 2 (C) 4 (D) 8

91. If $f(x)=\int x^{-2/3}(1+x^{1/3})^{1/2}dx$, then f (27) – f (0) equals -

(A) 4 (B) 12 (C) 14 (D) 16

92. Let y = f(x) be a differentiable curve satisfying $\int_2^x f(t)dt+2=\frac{x^2}{2}+\int_x^2 t^2 f(t)dt$,

then $\int_{-\pi/4}^{\pi/4}\frac{f(x)+x^9-x^3+x+1}{\cos^2 x}dx$ equals -

(A) 0 (B) 1 (C) 2 (D) 4

85. Ⓐ Ⓑ Ⓒ Ⓓ 86. Ⓐ Ⓑ Ⓒ Ⓓ 87. Ⓐ Ⓑ Ⓒ Ⓓ 88. Ⓐ Ⓑ Ⓒ Ⓓ
89. Ⓐ Ⓑ Ⓒ Ⓓ 90. Ⓐ Ⓑ Ⓒ Ⓓ 91. Ⓐ Ⓑ Ⓒ Ⓓ 92. Ⓐ Ⓑ Ⓒ Ⓓ

93. If y = f(x) is a linear function satisfying the relation $f(xy) = f(x).f(y)\ \forall\ x, y \in R$, then the curve $y^2 + \int_0^x (\sin t + a^2t^3 + bt)dt = \alpha,\ \alpha \in R^+$ cuts $y = f^{-1}(x)$ at -

(A) no point (B) exactly one point (C) atleast two points (D) infinite points

94. If $I_n = \int_1^{e^2} (\log_e x)^n d(x^2)$, then $I_n + \frac{n}{2} I_{n-1}$ is equal to -

(A) 0 (B) 1 (C) e (D) e^2

MULTIPLE OBJECTIVE TYPE

95. The value of a for which equation $\int_0^x (t^2 - 8t + 13)dt = x \sin\frac{a}{x}$ has a solution, is (are) -

(A) 3π (B) $\frac{5\pi}{2}$ (C) -9π (D) $\frac{7\pi}{6}$

96. If $f(g(x)) = x$ and $g(f(x)) = x$ then which of the following may be the functions f(x) & g(x) -

(A) $f(x) = g(x) = (7 - x^{1/3})^3$

(B) $f(x) = \frac{8x-7}{5x+8}\ x \neq -\frac{8}{5}$; $g(x) = \frac{8x+7}{8-5x}\ x \neq \frac{8}{5}$

(C) $g(x) = f(x) = \begin{cases} x, & x \in Q \\ -x, & x \notin Q \end{cases}$

(D) $f(x) = \log(x-2),\ x > 2$; $g(x) = e^x + 2,\ x \in R$

97. Let $f(x) = \ell n \cos^{-1}\sin\left(x + \frac{\pi}{3}\right)$ then -

(A) $f\left(\frac{8\pi}{9}\right) = \ell n\left(\frac{5\pi}{18}\right)$ (B) $f\left(\frac{8\pi}{9}\right) = \ell n\left(\frac{13\pi}{18}\right)$ (C) $f\left(\frac{-7\pi}{4}\right) = \ell n\left(\frac{\pi}{12}\right)$ (D) $f(0) = \frac{\pi}{6}$

98. Identify the incorrect statement(s) -

(A) $\lim_{x\to 0} \frac{\tan x - x}{x^2} = \frac{1}{3}$

(B) $\lim_{x\to 0} \left(\frac{\ell n(x+1)}{x}\right)^{\frac{1}{x}} = e^{-1/2}$

(C) $\lim_{x\to 2} \frac{x^2 + x^4 - 20}{x-2} = 32$

(D) $\lim_{x\to 0} \frac{\sqrt[3]{1+3x^2} - 1}{x} = 0$

99. The value(s) of x for which $f(x) = \frac{e^{\sin x}}{4 - \sqrt{x^2 - 9}}$ is continuous, is (are) -

(A) 3 (B) –3

(C) 5 (D) all $x \in (-\infty, -3] \cup [3, \infty)$

93.	Ⓐ Ⓑ Ⓒ Ⓓ	**94.**	Ⓐ Ⓑ Ⓒ Ⓓ	**95.**	Ⓐ Ⓑ Ⓒ Ⓓ	**96.**	Ⓐ Ⓑ Ⓒ Ⓓ
97.	Ⓐ Ⓑ Ⓒ Ⓓ	**98.**	Ⓐ Ⓑ Ⓒ Ⓓ	**99.**	Ⓐ Ⓑ Ⓒ Ⓓ		

100. One of the values of x satisfying $\tan(\sec^{-1}x) = \sin\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ is -

(A) $\frac{\sqrt{5}}{3}$ (B) $\frac{3}{\sqrt{5}}$ (C) $-\frac{\sqrt{5}}{3}$ (D) $-\frac{3}{\sqrt{5}}$

101. If $0 < x < 1$ then $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$ is equal to -

(A) $\frac{1}{2}\cos^{-1}x$ (B) $\cos^{-1}\sqrt{\frac{1+x}{2}}$ (C) $\sin^{-1}\sqrt{\frac{1-x}{2}}$ (D) $\frac{1}{2}\tan^{-1}\sqrt{\frac{1+x}{1-x}}$

102. Which of the following limits vanishes ?

(A) $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\tan x}\right)$ (B) $\lim_{x\to\infty}\left(\frac{3x^2+1}{2x^2-1}\right)^{\frac{x^3}{1-x}}$

(C) $\lim_{x\to\frac{\pi}{4}^+}\left(\tan\left(x+\frac{\pi}{8}\right)\right)^{\tan 2x}$ (D) $\lim_{x\to 1}\frac{x^3-2x^2+1}{x^3-1}$

103. If $L = \lim_{n\to\infty} n^{-n^2}\left((n+1)\left(n+\frac{1}{2}\right)\left(n+\frac{1}{2^2}\right)..........\left(n+\frac{1}{2^{n-1}}\right)\right)^n$ & $|k| = \ell nL$, then the possible value(s) of k is (are) -

(A) 2 (B) 1 (C) –2 (D) –1

104. Which of the following statement(s) are correct -

(A) If $f(x) \equiv 6x^7 + 5x^6 + 4x^5 + 3x^4 + 2x^3 + x^2 + x + 1$, then the equation $f'(x) = 0$ must have a real root.

(B) If $(x – 2)$ is a factor of the polynomial P(x) (degrees 5) repeated 3 times, then 2 is the root of the equation $P'(x) = 0$ repeated 2 times

(C) If $f(x)$ is a differentiable function, then if its graph is symmetric about origin, then the graph of $f'(x)$ will be symmetric about y-axis.

(D) If $y = \sin^{-1}(\cos(\sin^{-1}x)) + \cos^{-1}(\sin(\cos^{-1}x))$ then $\frac{dy}{dx}$ is independent of x.

105. If $I_n = \int_0^1 (1-x^2)^n dx$, then -

(A) $I_n = \frac{2n}{2n+1}I_{n-1}$ (B) $I_n = \frac{2.4.6.......2n}{3.5.7......(2n+1)}$

(C) $I_n = \frac{2^n n!}{3.5.7......(2n+1)}$ (D) $I_n = \frac{2n+1}{2n}I_{n-1}$

100. Ⓐ Ⓑ Ⓒ Ⓓ	**101.** Ⓐ Ⓑ Ⓒ Ⓓ	**102.** Ⓐ Ⓑ Ⓒ Ⓓ	**103.** Ⓐ Ⓑ Ⓒ Ⓓ
104. Ⓐ Ⓑ Ⓒ Ⓓ	**105.** Ⓐ Ⓑ Ⓒ Ⓓ		

106. Identify the correct statement(s) -

(A) $\tan^{-1}\tan\left(\frac{47}{7}\pi\right)$ is negative

(B) $\cos^{-1}(\cos(1 + \sin x)) = 1 + \sin x$ for all $x \in R$

(C) $\sin^{-1}\left(\sin\frac{23\pi}{5}\right)+\cos^{-1}\left(\cos\frac{23\pi}{5}\right)=\pi$

(D) $\tan^{-1}2 > \cot^{-1}2$

107. $\tan^{-1}\left(\frac{2x-1}{10}\right)+\tan^{-1}\frac{1}{2x}=\frac{\pi}{4}$, then x is equal to -

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 4 (D) $\frac{9}{2}$

108. Which of the following is true -

(A) $\cos^{-1}\sqrt{\frac{1+x}{2}}=\sin^{-1}\sqrt{\frac{1-x}{2}}\ \ \forall x\in[-1, 1]$

(B) $\cos^{-1}\sqrt{\frac{1+x}{2}}=2\cos^{-1}x\ \ \forall x\in[-1, 1]$

(C) $\sin^{-1}\sqrt{\frac{1-x}{2}}=\frac{1}{2}\cos^{-1}x\ \ \forall x\in[-1, 1]$

(D) $\sin^{-1}\sqrt{\frac{1-x}{2}}=\frac{1}{2}\sin^{-1}x\ \ \forall x\in[-1, 1]$

109. $\frac{d}{d(\ell nx)}(x^{\ell nx})$ is equal to -

(A) $2(\ell nx)\,.\,e^{(\ell n^2 x)}$ (B) $2(\ell nx)(x^{\ell nx})$ (C) $x\,.\,2(\ell n^2 x)(x^{\ell nx})$ (D) $2x^2(\ell n^2 x)$

110. $\int_0^{2\pi}\cos^{-1}(\cos x)dx$ is equal to -

(A) $\int_0^{2\pi}\cos^{-1}\left(\cos\left(x+\frac{\pi}{3}\right)\right)dx$

(B) $8\int_0^{\pi/2}\sin^{-1}(\sin x)dx$

(C) π^2

(D) $\int_0^{2\pi}\sin^{-1}(\sin x)dx$

111. If $f(x)=\int_1^x\frac{\ell nt}{1+t}dt$, then -

(A) $f\left(\frac{1}{x}\right)=-\int_1^x\frac{\ell nt}{t(1+t)}dt$

(B) $f\left(\frac{1}{x}\right)=\int_1^x\frac{\ell nt}{t(1+t)}dt$

(C) $f(x)+f\left(\frac{1}{x}\right)=\frac{1}{2}\ell n^2(x)$

(D) $f(x)=-f\left(\frac{1}{x}\right)$

112. Let $f(x)=\begin{cases}\int_0^x(5+|1-t|)dt, & \text{if } x>2\\ 5x+1, & \text{if } x\le 2\end{cases}$, then f (x) is -

(A) discontinuous at x = 2

(B) not differentiable at x = 2

(C) continuous at x = 2

(D) differentiable at x = 2

106.	Ⓐ Ⓑ Ⓒ Ⓓ	**107.**	Ⓐ Ⓑ Ⓒ Ⓓ	**108.**	Ⓐ Ⓑ Ⓒ Ⓓ	**109.**	Ⓐ Ⓑ Ⓒ Ⓓ
110.	Ⓐ Ⓑ Ⓒ Ⓓ	**111.**	Ⓐ Ⓑ Ⓒ Ⓓ	**112.**	Ⓐ Ⓑ Ⓒ Ⓓ		

113. Which of the followings has the value equal to the integral $\int_{1/e}^{\tan x}\frac{t}{1+t^2}dt+\int_{1/e}^{\cot x}\frac{dt}{t(1+t^2)}$?

(A) 1 (B) 2

(C) $\lim_{x\to 0}\frac{\int_0^{x^2}\cos t^2 dt}{x\sin x}$ (D) $\lim_{n\to\infty}\left\{\tan\frac{\pi}{2n}\tan\frac{2\pi}{2n}\tan\frac{3\pi}{2n}........\tan\frac{n\pi}{2n}\right\}^{1/n}$

REASONING TYPE

114. Let $f : R \to [0, \pi/2)$ defined by $f(x) = \tan^{-1}(x^2 + x + a)$ then -

Statement-1 : The set of values of a for which $f(x)$ is onto is $\left[\frac{1}{4}, \infty\right)$.

and

Statement-2 : Minimum value of $x^2 + x + a$ is $a-\frac{1}{4}$.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

115. **Statement-1 :** If $\lim_{x\to a}(f(x).g(x))$ exists then $\lim_{x\to a}f(x)$ and $\lim_{x\to a}g(x)$ exists finitely

and

Statement-2 : If $\lim_{x\to a}f(x)$ and $\lim_{x\to a}g(x)$ exists finitely then $\lim_{x\to a}f(x).g(x)=\lim_{x\to a}f(x).\lim_{x\to a}g(x)$

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

116. **Statement-1 :** Let α_1, α_2 and α_3 be the three real roots of the equation $ax^3 + bx^2 + cx + d = 0$ such that $|\alpha_1| \le 1, |\alpha_2| \le 1, |\alpha_3| \le 1$, $ad > 0$ and $\cos^{-1}\alpha_1 + \cos^{-1}\alpha_2 + \cos^{-1}\alpha_3 = \pi$ then the given cubic equation has exactly three negative real roots.

and

Statement-2 : If $0 \le x \le 1 \Rightarrow 0 \le \cos^{-1}x \le \frac{\pi}{2}$

If $-1 \le x < 0 \Rightarrow \frac{\pi}{2} < \cos^{-1}x \le \pi$

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

113. Ⓐ Ⓑ Ⓒ Ⓓ	**114.** Ⓐ Ⓑ Ⓒ Ⓓ	**115.** Ⓐ Ⓑ Ⓒ Ⓓ	**116.** Ⓐ Ⓑ Ⓒ Ⓓ

117. **Statement-1 :** The curves $y = 2^{\sin^2 x}$ & $y = 2^{\ell n(\cos x)}$ intersects exactly at one point in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

and

Statement-2 : $f(x) \geq g(x) \Rightarrow a^{f(x)} \geq a^{g(x)}, a \in R^+$.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

118. Let $f(x)$ is a bijective function. Then

Statement-1 : $f(x) = f^{-1}(x) \Rightarrow f^{-1}(x) = x$.

and

Statement-2 : $f^{-1}(x) = x \Rightarrow f(x) = f^{-1}(x)$.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

119. Let $f(x) = \begin{cases} Ax - B & x \leq -1 \\ 2x^2 + 3Ax + B & x \in (-1, 1] \\ 4 & x > 1 \end{cases}$

Statement-1 : $f(x)$ is continuous at all x if $A = \frac{3}{4}, B = -\frac{1}{4}$.

and

Statement-2 : Polynomial function is always continuous.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

120. Let $f : R \rightarrow R$

$f(x) = 10 \ \forall \ x \in Q$ and $f(x)$ be continuous function.

Statement-1 : $f(\pi) = 10$

and

Statement-2 : $f(x)$ is many-one into function.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

117. (A) (B) (C) (D) **118.** (A) (B) (C) (D) **119.** (A) (B) (C) (D) **120.** (A) (B) (C) (D)

121. **Statement-1 :** Number of points in (0, 5), where $f(x) = (x - 1)|x^2 - 4x + 3| + |(x - 2)^3| + \tan x$ is non-differentiable is 3.

and

Statement-2 : A function is non-differentiable at any point if it is discontinuous or its graph possesses a sharp corner at that point.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

122. Consider $f : R \to R$

Statement-1 : If $f(a) = 0$ and $\lim\limits_{x\to\infty} f(x) \to \infty \;\Rightarrow\; f(x) = 0$ has finite number of solutions.

and

Statement-2 : If $\lim\limits_{x\to\infty} f(x) \to \infty \;\Rightarrow\; f$ is aperiodic function.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

123. **Statement-1 :** $\int_{1/3}^{3} \frac{1}{x}\operatorname{cosec}^{99}\left(x - \frac{1}{x}\right)dx = 0$.

and

Statement-2 : $\int_{-a}^{a} f(x)dx = 0$ if $f(-x) = -f(x)$.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

124. Consider $f : R \to R$ be a function satisfying $f(2 - x) = f(2 + x)$ and $f(20 - x) = f(x)\ \forall\ x \in R$.

Statement-1 : If $\int_{4}^{20} f(x)dx = 10$, then $\int_{-9}^{151} f(x)dx = 90$.

and

Statement-2 : If $f(x)$ is periodic with period T, then $\int_{a}^{a+nT} f(x)dx = n\int_{0}^{T} f(x)dx$, $a \in R$ & $n \in I$.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

121. Ⓐ Ⓑ Ⓒ Ⓓ **122.** Ⓐ Ⓑ Ⓒ Ⓓ **123.** Ⓐ Ⓑ Ⓒ Ⓓ **124.** Ⓐ Ⓑ Ⓒ Ⓓ

125. Statement-1 : $\lim_{n\to\infty}\int_0^n \frac{dx}{e^{x-\{x\}}} = \frac{e}{e-1}$, $n \in N$, where {.} denotes fractional part function.

and

Statement-2 : $\int_0^{nT} f(x)dx = n\int_0^T f(x)dx$, $n \in N$, where $f(x + T) = f(x)$, $T > 0$ $\forall$ $x \in R$.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

COMPREHENSION

Paragraph for Question 126 to 128

The function whose values at any number x is the smallest integer greater than or equal to x is called the least integer function or the integer ceiling function. It is denoted by $\lceil x \rceil$.

for example $\lceil 1\cdot 1 \rceil = 2, \lceil \cdot 2 \rceil = 1, \lceil -1\cdot 2 \rceil = -1, \lceil 2 \rceil = 2$

Answer the following questions.

126. $\lceil x \rceil^2 - 3\lceil x \rceil + 2 = 0$ then x belongs to -
(A) [1, 3) (B) $(0, 1) \cup (1, 2)$ (C) {1, 2} (D) (0, 2]

127. $\lceil 1 \rceil + \lceil \sqrt{2} \rceil + \lceil \sqrt{3} \rceil + \ldots\ldots\ldots\ldots + \lceil \sqrt{10} \rceil =$
(A) 26 (B) 19 (C) 10 (D) none of these

128. The possible value(s) of $\lceil x \rceil - [x]$ where [x] is greatest integer function is (are) -
(A) {0} (B) {0, 1} (C) {0, 1, 2} (D) {1, 2}

Paragraph for Question 129 to 131

If $f(x) = \begin{cases} x+1, & \text{if } x \le 1 \\ 5-x^2, & \text{if } x > 1 \end{cases}$

$g(x) = \begin{cases} x, & \text{if } x \le 1 \\ 2-x, & \text{if } x > 1 \end{cases}$

then answer the following questions :

129. The range of $f(x)$ is -
(A) $(-\infty, 4)$ (B) $(-\infty, 5)$ (C) R (D) $(-\infty, 4]$

130. If $x \in (1, 2)$ then $g(f(x))$ is equal to -
(A) $x^2 + 3$ (B) $x^2 - 3$ (C) $5 - x^2$ (D) $1 - x$

131. Number of negative integral solutions of $g(f(x)) + 2 = 0$ are -
(A) 0 (B) 3 (C) 1 (D) 2

125. Ⓐ Ⓑ Ⓒ Ⓓ 126. Ⓐ Ⓑ Ⓒ Ⓓ 127. Ⓐ Ⓑ Ⓒ Ⓓ 128. Ⓐ Ⓑ Ⓒ Ⓓ
129. Ⓐ Ⓑ Ⓒ Ⓓ 130. Ⓐ Ⓑ Ⓒ Ⓓ 131. Ⓐ Ⓑ Ⓒ Ⓓ

Paragraph for Question 132 to 134

Consider the functions,

$$f(x) = \cos^{-1}x - \sin^{-1}x$$

$$g(x) = \sec^{-1}x - \operatorname{cosec}^{-1}x - \frac{\pi}{2}$$

$$\& \quad h(x) = mx$$

On the basis of above informations, answer the following questions.

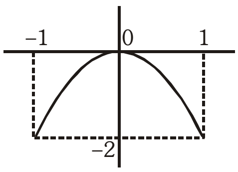
132. Number of solution(s) of the equation $f(x) = \tan^{-1}x$

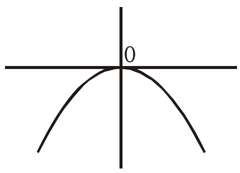
(A) 3 (B) 2 (C) 1 (D) 0

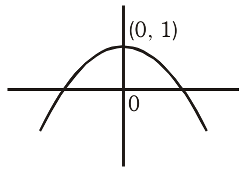
133. If the equation g(x) = h(x) has exactly 2 solutions then the range of m -

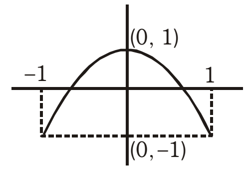
(A) $[-\pi, 0)$ (B) $(0, \pi]$ (C) $(-\infty, -\pi]$ (D) $[\pi, \infty)$

134. Which of the following best represents the graph of $y = \sin(f(x))$ -

(A) 

(B) 

(C) 

(D) 

Paragraph for Question 135 to 137

K(x) is a function such that $K(f(x)) = a + b + c + d$, where

$$a = \begin{cases} 0 & \text{if } f(x) \text{ is even} \\ -1 & \text{if } f(x) \text{ is odd} \\ 2 & \text{if } f(x) \text{ is niether even nor odd} \end{cases}$$

$$b = \begin{cases} 3 & \text{if } f(x) \text{ is periodic} \\ 4 & \text{if } f(x) \text{ is aperiodic} \end{cases}$$

$$c = \begin{cases} 5 & \text{if } f(x) \text{ is one one} \\ 6 & \text{if } f(x) \text{ is many one} \end{cases}$$

$$d = \begin{cases} 7 & \text{if } f(x) \text{ is onto} \\ 8 & \text{if } f(x) \text{ is into} \end{cases}$$

$A = \{x^2, e^x, \sin x, |x|\}$ all the functions in set A are defined from R to R

$B = \{18, 19, 16, 17\}$

$$h : R \to R, \quad h(x) = \left(\frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1}\right)$$

$$\phi : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R, \ \phi(x) = \tan x$$

On the basis of above informations, answer the following questions.

135. $K(\phi(x)) =$

(A) 15 (B) 16 (C) 17 (D) 18

132. Ⓐ Ⓑ Ⓒ Ⓓ 133. Ⓐ Ⓑ Ⓒ Ⓓ 134. Ⓐ Ⓑ Ⓒ Ⓓ 135. Ⓐ Ⓑ Ⓒ Ⓓ

136. K(h(x)) =

(A) 15 (B) 16 (C) 17 (D) 18

137. If K(x) is a function such that $K : A \to B$, y = K(x) where $x \in A$, $y \in B$ then K(x) is -

(A) one one onto (B) one one into (C) many one into (D) many one onto

Paragraph for Question 138 to 140

Consider the function y = f(x)

$$f : R - \{0\} \to R$$

The functional rule for the function y = f(x) is same as that of the functional rule for hypotenuse 'h' of the right triangle with area 25 $(\text{units})^2$ expressed as a function of its perimeter.

On the basis of above information, answer the following :

138. The function y = f(x) is -

(A) one-one onto (B) one-one into (C) many-one onto (D) many-one into

139. The value of $[\cos^{-1} \cos f\,((2\log_2 3.\ \log_3 4\ \log_4 5 ..... \log_{31} 32) + 10)]$, where $[\cdot]$ denotes the greatest integer function -

(A) 0 (B) 1 (C) 2 (D) 3

140. The sum of all the values of x at which fof(x) is discontinuous -

(A) 0 (B) 10 (C) 20 (D) 30

Paragraph for Question 141 to 143

Consider the following functions

$$f(x) = \begin{cases} 1 - x, & 1 \le x \le 2 \\ 3 - x, & 2 < x \le 3 \end{cases}$$

$g(x) = x^4 + x^2 + 1$

$h(x) = x^3$

On the basis of above informations, answer the following questions :

141. The function $gof(x)$ is -

(A) discontinuous at x = 2 (B) continuous but not derivable at x = 2

(C) continuous and derivable at x = 2 (D) non derivable at more that one point in [1, 3]

142. The function $hof(x)$ is -

(A) discontinuous at x = 2 (B) continuous but not derivable at x = 2

(C) continuous and derivable at x = 2 (D) increasing in (1, 2)

143. The domain of the function $fofogoh(x)$ is -

(A) [1, 2] (B) [2, 3] (C) [1, 3] (D) ϕ

136. Ⓐ Ⓑ Ⓒ Ⓓ	137. Ⓐ Ⓑ Ⓒ Ⓓ	138. Ⓐ Ⓑ Ⓒ Ⓓ	139. Ⓐ Ⓑ Ⓒ Ⓓ
140. Ⓐ Ⓑ Ⓒ Ⓓ	141. Ⓐ Ⓑ Ⓒ Ⓓ	142. Ⓐ Ⓑ Ⓒ Ⓓ	143. Ⓐ Ⓑ Ⓒ Ⓓ

Paragraph for Question 144 to 146

Let $g(x) = 6\sin x - 8\sin^3 x$ and $f(x) = \begin{cases} \max\left\{g(t),\ 0 \le t \le x,\ 0 \le x \le \dfrac{\pi}{3}\right\} \\ \left|x - 2 - \dfrac{\pi}{3}\right| & \dfrac{\pi}{3} < x < 3 \\ \sin^{-1}(\sin(x-1)) & 3 \le x \le 16 \end{cases}$

On the basis of above informations, answer the following questions :

144. $\lim\limits_{x \to \frac{\pi}{3}^-} f(x)$ is equal to -

(A) 2 (B) 0 (C) –2 (D) 1

145. $\int\limits_0^{\pi/3} f(x)dx$ is equal to -

(A) $\dfrac{4}{3}$ (B) $\dfrac{\pi+2}{3}$ (C) $\dfrac{\pi+1}{3}$ (D) $\dfrac{2}{3}$

146. $f(2) + f(6) + \dfrac{1}{3} f(16)$ is equal to -

(A) $\dfrac{2\pi+4}{3}$ (B) $\dfrac{30-10\pi}{3}$ (C) 1 (D) 0

Paragraph for Question 147 to 149

If $f(x) = \dfrac{x}{x^2+x+1}$, then

On the basis of above information, answer the following :

147. $\int f\left(e^x\right)dx$ is -

(A) $\tan^{-1}\left(\dfrac{2e^x+1}{\sqrt{3}}\right)+c$ (B) $\dfrac{1}{\sqrt{3}}\tan^{-1}\left(\dfrac{2e^x+1}{\sqrt{3}}\right)+c$

(C) $\dfrac{1}{2\sqrt{3}}\tan^{-1}\left(\dfrac{2e^x+1}{\sqrt{3}}\right)+c$ (D) $\dfrac{2}{\sqrt{3}}\tan^{-1}\left(\dfrac{2e^x+1}{\sqrt{3}}\right)+c$

148. If $\int f(\tan x)dx = x - \dfrac{2}{\sqrt{3}}\tan^{-1}(g(x)) + c$, then for $x \in \left(-\dfrac{\pi}{3}, \dfrac{\pi}{3}\right)$ range of g(x) is -

(A) $\left(\dfrac{1}{\sqrt{3}}-2, \dfrac{1}{\sqrt{3}}+2\right)$ (B) $\left(-\sqrt{3}, \sqrt{3}\right)$

(C) $\left(-\dfrac{\sqrt{3}}{4-\sqrt{3}}, \dfrac{\sqrt{3}}{4+\sqrt{3}}\right)$ (D) none of these

144. Ⓐ Ⓑ Ⓒ Ⓓ **145.** Ⓐ Ⓑ Ⓒ Ⓓ **146.** Ⓐ Ⓑ Ⓒ Ⓓ **147.** Ⓐ Ⓑ Ⓒ Ⓓ

148. Ⓐ Ⓑ Ⓒ Ⓓ

149. If $\int f(x^2)dx + \int f\left(\frac{1}{x^2}\right)d\left(\frac{1}{x}\right) = \lambda \ell n\left|\frac{x^2-x+1}{x^2+x+1}\right| + c$, then λ is -

(A) 1 (B) –1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

MATCH THE COLUMN

150. Column-I **Column-II**

(A) Solution set of the inequality

$\left(\text{cosec}^{-1}x\right)^2 - 2\text{cosec}^{-1}x \geq \frac{\pi}{6}\left(\text{cosec}^{-1}x - 2\right)$

(B) Solution set of the inequality $|\sin^{-1}x| \geq \cos^{-1}|x|$

(C) Domain of the function

$f(x) = \frac{1}{\sqrt{\ell n(\sin^{-1}x) + 2\ell n2 - \ell n\pi}}$

(D) Domain of the function

$f(x) = \sqrt{\sin^{-1}(\sin(\sin^{-1}(\sin(\sin^{-1}(\sin(\sin(\sin^{-1}x)))))))}$

(P) $\left[-1, \frac{-1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, 1\right]$

(Q) $\left(\frac{1}{\sqrt{2}}, 1\right]$

(R) $(-\infty, -1] \cup [2, \infty)$

(S) $[0, 1]$

151. Column-I **Column-II**

(A) If $f(x) = \max\left(\frac{1}{2} - \frac{3x^2}{4}, \frac{5x^2}{4}\right)$ then minimum value of $f(x)$ is

(B) Let $f(x)$ be a function such that $f(x + y) = f(x)f(y)\ \forall\ x, y \in R$. If $f(x)$ is not identically zero then $f(x)f(-x) =$

(C) Sum of the squares of all the solution(s) of $2\sin^{-1}(x + 2) = \cos^{-1}(x + 3)$

(D) Let P and Q be polynomials such that P(x) and Q(P(Q(x))) have the same roots. If the degree of P is 8 then degree of Q is

(P) 1

(Q) 5/16

(R) 8

(S) $\frac{41}{4}$

149. (A) (B) (C) (D)

150. (A) (P) (Q) (R) (S) (T)
(B) (P) (Q) (R) (S) (T)
(C) (P) (Q) (R) (S) (T)
(D) (P) (Q) (R) (S) (T)

151. (A) (P) (Q) (R) (S) (T)
(B) (P) (Q) (R) (S) (T)
(C) (P) (Q) (R) (S) (T)
(D) (P) (Q) (R) (S) (T)

152. Column-I **Column-II**

(A) If $f(x) = \dfrac{e^{x\cos x} - 1 - x}{\tan x^2}$ for $x \neq 0$ is continuous at $x = 0$ then $f(0)$ must be — (P) 4

(Q) 1/2

(B) $e^{\lim\limits_{x\to\infty}\left(\ell nx - \ell n(\sqrt{x^2-1}+x)\right)} =$ — (R) 2

(C) $\lim\limits_{x\to 0}\dfrac{\sin^{-1}2x - 2\tan^{-1}x}{x^3}$ — (S) 1

153. Column-I **Column-II**

(A) Number of integers in the range of the function $f(x) = \cos 2x - 10\cos x - 11$ is — (P) 10

(B) The period of the function — (Q) 21

$$f(x) = \left(\sec^2\left(\frac{\pi x}{10}\right) - \tan^2\left(\frac{\pi x}{10}\right)\right)^{\cos^4 4\pi x + 100x - [100x]}$$ is

where [·] denotes greatest integer function — (R) not defined

(C) If the number of solution of the equation $\sin^{-1}(\sin x) - mx = 0$ is 5, then 70m equals to (where 'm' is positive number)

(D) The period of the function $f : (0, \infty) \to R$ — (S) 14

$f(x) = \sum\limits_{r=1}^{n}\left[r^2 + e^{-x} + r - 1\right]$ is

where [·] represents greatest integer function

154. Column-I **Column-II**

(A) The number of the values of x in $(0, 2\pi)$, where the function $f(x) = \dfrac{\tan x + \cot x}{2} - \left|\dfrac{\tan x - \cot x}{2}\right|$ is continuous but not differentiable is — (P) 2

(Q) 0

(B) The number of points where the function $f(x) = \min\{1, 1 + x^3, x^2 - 3x + 3\}$ is non-derivable

(C) The number of points where $f(x) = (x + 4)^{1/3}$ is non-differentiable is — (R) 4

(D) Consider $f(x) = \begin{cases} -\dfrac{\pi}{2}\ell n\left(\dfrac{x.2}{\pi}\right) + \dfrac{\pi}{2}, & 0 < x \le \dfrac{\pi}{2} \\ \sin^{-1}\sin x, & \dfrac{\pi}{2} < x < \dfrac{3\pi}{2} \end{cases}$ — (S) 1

Number of points in $\left(0, \dfrac{3\pi}{2}\right)$, where $f(x)$ is non-differentiable is

152.		153.		154.	
(A)	P Q R S T	(A)	P Q R S T	(A)	P Q R S T
(B)	P Q R S T	(B)	P Q R S T	(B)	P Q R S T
(C)	P Q R S T	(C)	P Q R S T	(C)	P Q R S T
		(D)	P Q R S T	(D)	P Q R S T

155. **Column-I** | **Column-II**

(A) If range of $f(x) = \cos^{-1}\frac{x}{2} + x^2 - 4x + 1$ is $[a, \pi + b]$, then a + b is equal to — (P) 1

(B) Length of the tangent drawn from a point on the circle $x^2 + y^2 = 25$ to the circle $x^2 + y^2 = 21$ is — (Q) 2

(C) If $\int_0^6 e^{3x-[3x]}.dx = p(e-q)$, where [.] denotes greatest integer function, then p + q is — (R) 7

(D) Minimum possible number of positive roots of the quadratic equation $x^2 - (1 + b)x + b - 2 = 0$, is — (S) 10

156. **Column-I** | **Column-II**

(A) Period of the function $f(x) = 6x - [6x + 7] + \cos\pi x - \frac{e^{2x}}{e^{[2x]}}$, where [.] denotes greatest integer function, is — (P) 5

(B) If $f(x) = \int \frac{(1-\cos 2x)\sin 5x}{x^2 \tan 3x}dx$, then $\lim_{x\to 0}\left(\frac{3}{2}f'(x)\right)$ is equal to — (Q) –3

(C) If $f(x) = \ell n(8\cos^3 x - 6\cos x)$, then $f'\left(\frac{\pi}{4}\right)$ is equal to — (R) non-existent

(S) 2

157. **Column-I** | **Column-II**

(A) $\int_{-1}^{1} \frac{dx}{(1+x^2)(3^x+1)}$ is equal to — (P) $-\frac{\pi}{4}$

(B) $\lim_{n\to\infty}\left(\frac{(2.1+n)}{1^2+n.1+n^2}+\frac{(2.2+n)}{2^2+n.2+n^2}+\frac{(2.3+n)}{3^2+n.3+n^2}+.....+\frac{(2.n+n)}{3n^2}\right)$ is equal to — (Q) does not exist

(C) $\lim_{x\to 0}\frac{\pi\int_0^x \sin^{-1}\sqrt{t^2}dt}{2x^2}$ is equal to — (R) $\ell n3$

(D) The set of values of x satisfying the inequality $\sqrt{\sec^{-1}\left(x+\frac{1}{x}\right)}(8x^2+2\pi x-\pi^2) \le 0$ is [a, b] – {c} then a + b + c is equal to — (S) $\frac{\pi}{4}$

155.						156.						157.					
(A)	P	Q	R	S	T	(A)	P	Q	R	S	T	(A)	P	Q	R	S	T
(B)	P	Q	R	S	T	(B)	P	Q	R	S	T	(B)	P	Q	R	S	T
(C)	P	Q	R	S	T	(C)	P	Q	R	S	T	(C)	P	Q	R	S	T
(D)	P	Q	R	S	T							(D)	P	Q	R	S	T

158. Column-I | **Column-II**

(A) Range of the function $f(x) = \log_3\left(\frac{\sin^2 x - \sin x + 1}{\sin^2 x + \sin x + 1}\right)$ is [m, M] then m + M is equal to — (P) 2

(B) If the tangent at every point to the curve $f(x) = x^3 + 3ax^2 + 3x + 5$ is inclined with positive direction of x-axis at non-zero acute angle then number of integral values of 'a' is — (Q) 5

(C) If $\cot^{-1}\left(\frac{n}{\pi}\right) > \frac{\pi}{6},\ n \in N,$ then maximum value of n is — (R) 1

(D) If $f(x) = x^2 \ell nx$, where $f : [1, e] \to R$ and the maximum value of $f(x)$ is M, then ℓnM is — (S) 0

159. Column I | **Column II**

(A) If $\int \frac{2x^2 - 3x + 3}{x^3 - 2x^2 + x}dx = a\ell n|x| - \frac{b}{x-1} - \ell n|x-1| + C$ then a + b is equal to — (P) 3

(B) If $\tan\left(\sin^{-1}\sqrt{\frac{1-x}{2}} + \cos^{-1}\sqrt{\frac{1+x}{2}}\right) = \sin\left(\tan^{-1} 2\right)$ where x is a positive real number then $\frac{27x^2}{5}$ is equal to — (Q) 5

(C) If $f(x) = x\ \ell n2x - x$, where $x \in \left[\frac{1}{2e}, \frac{e}{2}\right]$, then range of $f(x)$ is $\left[-\frac{1}{a}, b\right]$. The value of a+b is — (R) 6

(D) If $\int \tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right)dx = \frac{x}{a}\tan^{-1}x - \frac{1}{b}\ell n(1+x^2) + C$ then a + b is equal to — (S) 2

158.					
(A)	P	Q	R	S	T
(B)	P	Q	R	S	T
(C)	P	Q	R	S	T
(D)	P	Q	R	S	T

159.					
(A)	P	Q	R	S	T
(B)	P	Q	R	S	T
(C)	P	Q	R	S	T
(D)	P	Q	R	S	T

160. **Column–I** **Column–II**

(A) Number of solutions of the equation (P) 1

$$\sum_{n=0}^{\infty}(\sin^2 x)^n + \sum_{n=0}^{\infty}(\cos^2 x)^n = 4 \text{ in } (0, 2\pi)$$

(B) If the range of m for which the equation $\operatorname{cosec}^{-1}x = mx$ has exactly two solutions is $\left(0, \frac{\lambda\pi}{10}\right]$, then λ is equal to (Q) 2

(C) Sum of integral solutions of the equality $[x]^2 = [x+6]$ is, where [.] represent greatest integer function (R) 4

(D) The number of solutions to the equation (S) 5

$$\left|\operatorname{cosec}\left(\sin\left(\sin^{-1}x\right)\right)\right| = \left(\frac{d}{dx}\left(\frac{x^{10}}{10} + \frac{x^8}{8} + x^2 - x^9 - \frac{9x^7}{7}\right)\right)_{x=9}$$

161. **Column-I** **Column-II**

(A) If $\ell = \lim_{x\to 0}\left(\int_0^x \sin t^2 dt\right)^{\tan x}$, then $\frac{\pi\ell}{2}$ is equal to (P) $-\pi$

(B) If $f(x) = \lim_{n\to\infty}\left(\frac{x^n + \left(\frac{\pi}{2}\right)^n}{x^{n-1} + \left(\frac{\pi}{2}\right)^{n-1}}\right)$, then $\pi\int_0^{\pi} f(x)\cos x dx$ is equal to (Q) $-\frac{\pi}{4}$

(C) $\left(\int_0^{\pi/2} \frac{d\theta}{\tan\theta + \cot\theta + \operatorname{cosec}\theta + \sec\theta}\right) - 1$ is equal to (R) π

(D) $\frac{32\int_0^{\pi} \cos^4 2x \sin^2 4x dx}{5}$ is equal to (S) $\frac{\pi}{2}$

160. (A) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ
(B) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ
(C) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ
(D) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ

161. (A) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ
(B) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ
(C) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ
(D) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ

INTEGER TYPE / SUBJECTIVE TYPE

162. If f be a function such that f : R→ R, $f(x) = \dfrac{3x^2+mx+n}{x^2+1}$ and the range of f (x) is [–4, 3) then find $m^4 + n^4$.

163. If $\tan\left(\cos^{-1}\dfrac{4}{5}+\sin^{-1}\dfrac{2}{\sqrt{13}}\right)+8\tan^{-1}\dfrac{1}{3}+4\tan^{-1}\dfrac{1}{7}=\dfrac{a}{b}+c.\pi$

where a, b & c are co-prime numbers. Then find the value of a + b + c.

164. If y = $\sin^2x$ + cosx & $\left.\dfrac{d^2x}{dy^2}\right|_{x=\frac{\pi}{4}} = a+b\sqrt{c},\ a,b,c\in I,\ \forall\ a+b+c=$ ....

165. If $\begin{bmatrix}1&0&0\\6&2&0\\5&4&3\end{bmatrix}\begin{bmatrix}x\\x^2\\1\end{bmatrix}=\begin{bmatrix}x\\2ax+bx^2\\5x+cx^2+3\end{bmatrix}$ $\forall\ x\in R$ and f (x) is a differentiable function satisfying $f(x) + f(y) = f\left(\dfrac{x+y}{1-xy}\right)$ for all x, y ∈ R, (xy ≠ 1) and $\lim\limits_{x\to 0}\dfrac{f(x)}{x}=2$, then find the value of $\left[\displaystyle\int_0^1 \frac{ax^2+bx+c}{f(1)}dx\right]$, where [.] denotes greatest integer function.

166. Let $x.g(f(x)).f'(g(x)).g'(x)=f(g(x)).g'(f(x)).f'(x)\ \forall\ x\in R$. f is nonnegative & g is positive. Also $\displaystyle\int_0^a f(g(x))dx = 1-\frac{e^{-2a}}{2}\ \ \forall\ a\in R$. Given that g($f$ (0)) = 1, then the value of $|\ell n(g(f(4)))|$ is equal to........

162.		**163.**		**164.**		**165.**	
166.							

ANSWER KEY

1. D	**2.** C	**3.** A	**4.** A	**5.** B	**6.** D	**7.** B	**8.** C	**9.** B	**10.** C
11. D	**12.** B	**13.** C	**14.** A	**15.** C	**16.** B	**17.** B	**18.** B	**19.** A	**20.** A
21. D	**22.** A	**23.** C	**24.** A	**25.** C	**26.** B	**27.** D	**28.** C	**29.** C	**30.** D
31. D	**32.** B	**33.** B	**34.** A	**35.** D	**36.** C	**37.** A	**38.** C	**39.** C	**40.** B
41. D	**42.** D	**43.** B	**44.** A	**45.** C	**46.** C	**47.** C	**48.** B	**49.** C	**50.** B
51. A	**52.** A	**53.** C	**54.** B	**55.** B	**56.** D	**57.** C	**58.** B	**59.** C	**60.** A
61. D	**62.** A	**63.** B	**64.** C	**65.** D	**66.** A	**67.** B	**68.** C	**69.** A	**70.** B
71. D	**72.** D	**73.** A	**74.** D	**75.** D	**76.** D	**77.** C	**78.** B	**79.** C	**80.** C
81. C	**82.** A	**83.** C	**84.** A	**85.** B	**86.** C	**87.** A	**88.** B	**89.** A	**90.** C
91. C	**92.** C	**93.** C	**94.** D	**95.** A,C		**96.** A,B,C,D		**97.** B,C	

98. A,C **99.** A,B **100.** B **101.** A,B,C **102.** A,B,C

103. A,C **104.** B,C,D **105.** A,B,C **106.** A,B,C,D **107.** A,D

108. A,C **109.** A,B **110.** A,B,C **111.** B,C **112.** B,C

113. A,C,D **114.** D **115.** D **116.** D **117.** C **118.** D **119.** B **120.** B **121.** A

122. D **123.** B **124.** D **125.** B **126.** D **127.** A **128.** B **129.** A **130.** B **131.** C

132. C **133.** A **134.** D **135.** A **136.** D **137.** C **138.** C **139.** C **140.** A **141.** B

142. A **143.** D **144.** A **145.** B **146.** D **147.** D **148.** A **149.** C

150. (A)→(R); (B)→(P); (C)→(Q); (D)→(S)

151. (A)→(Q); (B)→(P); (C)→(S); (D)→(P)

152. (A)→(Q); (B)→(Q); (C)→(R)

153. (A)→(Q), (B)→(P), (C)→(S), (D)→(R)

154. (A)→(R), (B)→(P), (C)→(S), (D)→(Q)

155. (A)→(S), (B)→(Q), (C)→(R), (D)→(P)

156. (A)→(S), (B)→(P), (C)→(R)

157. (A)→(S), (B)→(R), (C)→(Q), (D)→(P)

158. (A)→(S), (B)→(R), (C)→(Q), (D)→(P)

159. (A)→(Q); (B)→(P); (C)→(S); (D)→(R)

160. (A)→(R); (B)→(S); (C)→(P); (D)→(Q)

161. (A)→(S); (B)→(P); (C)→(Q); (D)→(R)

162. 256 **163.** 24 **164.** 26 **165.** 3 **166.** 16