LEADER COURSE (PHASE – ELS)

TARGET : JEE (MAIN) 2015

SCORE – I | DATE : 22 - 02 - 2015

MAJOR TEST | Test Pattern : JEE (Main)

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	1	3	2	1	4	1	1	3	2	4	1	4	4	2	3	2	3	4	3
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	4	3	4	3	1	3	1	4	2	2	2	1	2	1	2	1	3	3	1	4
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	4	3	2	2	4	3	1	4	4	2	1	2	4	1	4	4	1	3	2	2
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	2	1	3	4	3	2	1	4	1	1	1	4	3	3	3	3	4	1	2	2
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	3	3	1	2	4	3	2	4	2	2										

HINT – SHEET

1. positive charge flow outside
so, charge decreases

thus voltage $= \frac{Q}{C}$ also decreases and $v = \frac{-Ldi}{dt}$

so $\frac{di}{dt}$ increases

3. $\tau = RC$,

$\therefore \omega = \frac{1}{RC}$;

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \sqrt{R^2 + R^2} = \sqrt{2R}$$

4. Sensitivity $= \frac{BNA}{K}$, by increasing B (stronger magnet) and decreasing torsion constant, k (weaker springs)

6. At any instant $|v| = v_0$ & component along y-axis must remain constant.

7. According to Mosley's law, frequency of K_α X–ray $v = \eta(Z-1)^2$

Hence $\eta \rightarrow$ constant $\frac{\nu_{cd}}{\nu_{ca}} = \left(\frac{47}{19}\right)^2$

$$\Rightarrow \nu_{cd} = \left(\frac{47}{19}\right)^2 \nu_{ca} ; \nu_{cd} = 5.46 \times 10^{18} \text{ Hz}$$

8. In forward bias potential barrier decreases

9. We know that

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.96}{1-0.96} = 24$$

The collector current I_C is given by

$$I_C = \frac{V_C}{R} = \frac{0.5}{800} = 0.625 \times 10^{-3} \text{ A}$$

Further $I_B = \frac{I_C}{\beta} = \frac{0.625 \times 10^{-3}}{24}$
$= 26 \times 10^{-6}$ A $= 26$ μA

11. The wavelength order of the given types of waves are given below

Waves Wavelength Range (in metre)

Gamma rays	$10^{-14} - 10^{-10}$
IR-rays	$7 \times 10^{-7} = 10^{-3}$
UV-rays	$10^{-9} - 4 \times 10^{-7}$
Microwaves	$10^{-4} - 10^{0}$

Hence, statements (1) and (4) are correct.

12. From boolen algebra we have
$A\,.\,(A + B) = A$

13. $u = \frac{1}{2}\in_0 E^2 = \frac{1}{2}\in_0\left(\frac{q}{4\pi\in_0 r}\right)^2 = \frac{q^2}{32\pi^2\in_0 r^2}$

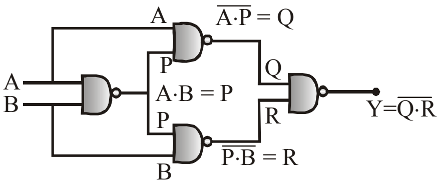
$$\Rightarrow \log u = \log\left(\frac{q^2}{32\pi^2\in_0 r^2}\right) = \log k - 2\log r$$

14. Total force necessary to hold the two parts is
= (Cross – section area) × Pressure

$$= \pi(R^2 - h^2) \times \frac{\sigma^2}{2\in_0} = \pi\left(\frac{R^2 - h^2}{2\in_0}\right)\times\left(\frac{Q}{4\pi R^2}\right)^2$$

$$= \frac{(R^2 - h^2)Q^2}{32\pi\in_0 R^4}$$

16. distance $= \sqrt{2hR}$

17. 

$\therefore\ Y = \bar{Q} + \bar{R} = \overline{\overline{A\cdot P}} + \overline{\overline{P\cdot B}} = A\cdot P + P\cdot B$

$= P\cdot(A+B) = \overline{A\cdot B}\cdot(A+B)$

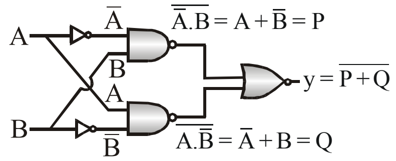
$= (\bar{A}+\bar{B})\cdot(A+B) = \bar{A}\cdot B + A\cdot\bar{B}$

18. $\Delta V_{in} = 10mV$, $\Delta I_B = 50\mu A$, $\Delta I_C = 5mA$
$R_L = 5k\Omega$, $\Delta V_0 = ?$

$\frac{\Delta V_0}{\Delta V_i} = \beta\frac{R_L}{R_{in}}$, where $\beta = \frac{\Delta I_C}{\Delta I_B}$ & $R_{in} = \frac{\Delta V_{in}}{\Delta I_B}$

$\therefore \frac{\Delta V_0}{10mV} = \left(\frac{5mA}{50\mu A}\right)\left(\frac{5k\Omega\times 50\mu A}{10mV}\right)$

$\Rightarrow \Delta V_0 = 25V$

19. 

$\therefore\ y = \bar{P}.\bar{Q} = \left(\overline{A+\bar{B}}\right).\left(\overline{\bar{A}+B}\right)$

$y = \bar{A}.B.A.\bar{B} = (\bar{A}.A).(B.\bar{B}) = 0.0$

$y = 0$

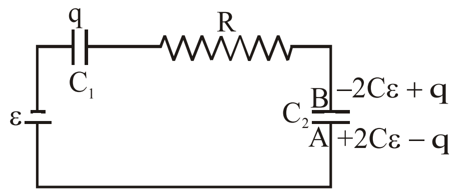
20. Force on dipole $F = p\frac{\partial E}{\partial r}$; $\left|\frac{\partial E}{\partial r}\right|$ is maximum for 3rd dipole.
So $F_3 > F_2 = F_4 > F_1 = F_5$

21. $\frac{4}{3}\pi r^3\left(\rho_{Hg}\right)g = qE$

22. $\tan 45° = \frac{qE}{mg} \Rightarrow w = \frac{V}{d}q \Rightarrow q = \frac{wd}{V}$

23. $Q_1 = EC$, $Q_n = E(nC) = nQ_1$

$\Rightarrow \frac{Q_1}{Q_n} = \frac{1}{n}$

24. 

Apply KVL

$\frac{2C\varepsilon - q}{C} - iR - \frac{q}{C} - \varepsilon = 0$

$\frac{C\varepsilon - 2q}{C} = iR \Rightarrow \frac{dq}{dt} = \frac{C\varepsilon - 2q}{RC}$

$\int_0^q \frac{dq}{C\varepsilon - 2q} = \int_0^t \frac{dt}{RC}$

$\Rightarrow -\frac{1}{2}\ell n\left(\frac{C\varepsilon - 2q}{C\varepsilon}\right) = \frac{t}{RC}$

$q = \frac{C\varepsilon}{2}\left(1 - e^{-\frac{2t}{RC}}\right)$

Charge on plate B = $-2C\varepsilon + q$

$-2C\varepsilon + \frac{C\varepsilon}{2}\left(1 - e^{-\frac{2t}{RC}}\right) = -\frac{3C\varepsilon}{2} - \frac{C\varepsilon}{2}e^{-\frac{2t}{RC}}$

26. Along y $v_y^2 = 2 \times \frac{qE}{m} \times y$

Net speed = $\sqrt{v^2 + v_y^2}$

27. Steady-state current in $L = i_0 = \frac{\varepsilon}{R_1}$.

Energy stored in $L = \frac{1}{2}Li_0^2 = \frac{1}{2}L\left(\frac{\varepsilon^2}{R_1^2}\right)$

= heat produced in R_2 during discharge.

28. Gravitional potential energy of star

$$U = -\frac{3GM^2}{5R}$$

Volume $V = \frac{4}{3}\pi R^3 \Rightarrow R = \left(\frac{3V}{4\pi}\right)^{1/3}$

$$\Rightarrow U = -\frac{3}{5}\left(\frac{4\pi}{3}\right)^{1/3} GM^2V^{-1/3}$$

As dW = PdV=–dU

$$\Rightarrow P = -\frac{dU}{dV} = \frac{1}{5}\left(\frac{4\pi}{3}\right)^{1/3} V^{-4/3}$$

29. When magnet is divided into two equal parts, the magnetic dipole moment

M' = pole strength $\times \frac{1}{2} = \frac{M}{2}$

(pole strength remains same)

Also, the mass magnet becomes half, ie,

$$m' = \frac{m}{2}$$

Moment of inertia of magnet

$$I = \frac{m\ell^2}{12}$$

New moment of inertia

$$I' = \frac{1}{12}\left(\frac{m}{2}\right)\left(\frac{\ell}{2}\right)^2 = \frac{m\ell^2}{12 \times 8}$$

$\therefore$ $I' = \frac{1}{8} \times I$

Now, $T = 2\pi\sqrt{\left(\frac{I}{MB_H}\right)}$

$$T' = 2\pi\sqrt{\left(\frac{I'}{M'B_H}\right)} = 2\pi\sqrt{\left(\frac{I/8}{MB_H/2}\right)}$$

$\therefore$ $T' = \frac{T}{2} \Rightarrow \frac{T'}{T} = \frac{1}{2}$

30. Root mean square value

$$<V> = \sqrt{\frac{\int_0^{T/4} V_0^2 dt + \int_{T/4}^{T} (0) dt}{\int_0^T dt}}$$

$$= \sqrt{\frac{V_0^2\left(\frac{T}{4}\right)}{T}} = \sqrt{\frac{V_0^2}{4}} = \frac{V_0}{2}$$

31. $n = \frac{4.48}{22.4} = 0.2$

$$C_V = \frac{\Delta U}{n.\Delta T} = \frac{12}{0.2 \times 15} = 4\,\text{cal}$$

$C_P = C_V + R = 4 + 2 = 6$ cal.

32. $[Mg^{2+}] = 10^{-2}M$

$Ksp = [Mg^{2+}]\,[OH^-]^2$

$10^{-12} = 0.01 \times [OH^{-2}]$

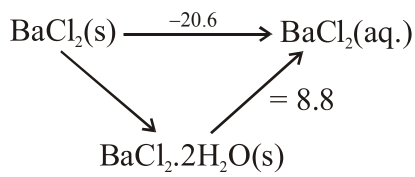
$[OH^-] = 10^{-5}$, pOH = 5, pH = 9

34. $pH = pK_a + \log\left[\frac{CH_3COO^-}{CH_3COOH}\right]$

$[CH_3COO^-] = [CH_3COOH]$

$\therefore$ $pH = pK_a = 4.74$

36.



–20.6 = X + 8.8

X = –20.6 –8.8 = –29.4 kJ

37. $n \times 6 = \left(3 - \frac{2}{0.92}\right) \times 0.92 \times 1$

$$n = \left(\frac{2.76 - 2}{6}\right)$$

$$= \frac{0.76}{6}$$

38. $U_{avg} = \sqrt{\dfrac{8RT}{\pi M}}$

$= \sqrt{\dfrac{8R}{\pi M} \times \dfrac{PV}{nR}} = \sqrt{\dfrac{8PV}{\pi Mn}}$

$= \sqrt{\dfrac{8 \times \pi \times 10 \times 10^5 \times 8 \times 10^{-3}}{\pi \times 32 \times 10^{-3} \times 2}} = 10^3 \dfrac{m}{s}$

39. 6C (graphite) + $3H_2(g) \rightarrow C_6H_6$ ΔH = ?

$C_6H_6 + \dfrac{15}{2}O_2 \rightarrow 6CO_2 + 3H_2O$ Q_1

$C + O_2 \rightarrow CO_2$ Q_2

$H_2 + \dfrac{1}{2}O_2 \rightarrow H_2O$ Q_3

$\Delta H = 6Q_2 + 3Q_3 - Q_1$

61. $\text{Arg}(-i) + \dfrac{\pi}{9} + 2\text{Arg}(z)$

$+2\left[\text{Arg}(i) - \dfrac{\pi}{18} + \text{Arg}(\overline{z})\right]$

$= -\dfrac{\pi}{2} + \dfrac{\pi}{9} + \pi - \dfrac{\pi}{9} + 2(\text{Arg}z + \text{Arg}\overline{z})$

$= \dfrac{\pi}{2} + 2(0) = \dfrac{\pi}{2}$

62. $f(x).f(f(x)) = x^2 + 1$

put x = 1, $f(1)f(2) = 2$

$\Rightarrow f(2) = 1$

Differentiating

$f'(x).f(f(x)) + f(x).f'(f(x)).f'(x) = 2x$

Put x = 1

$f'(1)f(f(1)) + f(1)f'(f(1)).f'(1) = 2$

$\Rightarrow kf(2) + 2f'(2).k = 2$

$k + 2f'(2).k = 2$

$f'(2) = \dfrac{2-k}{2k} = \dfrac{1}{k} - \dfrac{1}{2}$

63. ${}^2C_1 \times {}^6C_1 \times {}^6C_1 + {}^3C_1 \times {}^6C_1 \times {}^6C_1$

72 + 108 = 180

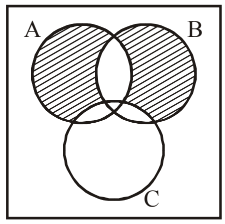
64. $D = 4b^2 - 4(a + b) > 0$

$b^2 - b > a$

$\therefore$ for $y = x^2 - x$ & (a, b)

$S_1 > 0$

65.

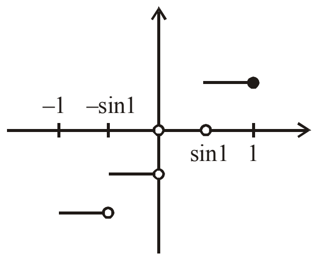


$P([(A \cap \overline{B}) \cup (\overline{A} \cap B)] \cap \overline{C})$

$= \dfrac{3}{4} - \left\{\dfrac{1}{3} + \dfrac{1}{6}\right\} = \dfrac{1}{4}$

66. $f(x)$ is discontinuous when ever $[\sin^{-1}x]$ is discontinuous

$\therefore \alpha + \beta = 0$



67. $B(AB) = (BA)B = B^2$

$= B(AB) = BA = B$

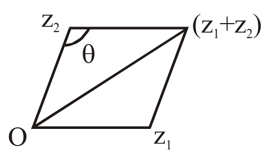
$\therefore B^2 = B$ & $A^2 = A$

$A^4 = A, B^4 = B$

$A^{10} = A, B^{10} = B$

$\therefore X - Y = O$ $\therefore$ singular

68. $\cos\theta = -\dfrac{(5 - 2\sqrt{3}) + 2 + 3}{2\sqrt{6}}$



$= \dfrac{2\sqrt{3}}{2\sqrt{2}\sqrt{3}} = \dfrac{1}{\sqrt{2}}$

$\theta = \dfrac{\pi}{4} \Rightarrow \arg\left(\dfrac{z_1}{z_2}\right) = \dfrac{3\pi}{4}$

69. $|A| = xyz$

$|ABC| = xyz \times 36 \times 4 = 1152$

$xyz = 8$

$\dfrac{x + y + z}{3} \geq \sqrt[3]{8}$

$x + y + z \geq 6$

70. $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} + \begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 1 & a-b & b-c \end{vmatrix} = 0$

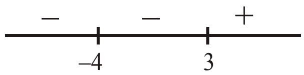
$\Rightarrow\ 0 + (a-b)(b-c) = (c-a)^2$

$\Rightarrow\ ab - ac - b^2 + bc = c^2 + a^2 - 2ac$

$\Rightarrow\ a^2 + b^2 + c^2 = ab + bc + ca$

$\Rightarrow\ a = b = c.$

71. $f'(x) = e^{x-3}(x^2+2)(x-3)(x+4)^2$



72. $(1 - x + 2x^2)^n = a_0 + a_1x + a_2x^2 + \ldots$ ......(1)

$x = 0 \Rightarrow a_0 = 1$

Diff. (1)

$n(1 - x + 2x^2)^{n-1}(-1 + 4x)$

$= a_1 + 2a_2x + \ldots\ldots\ldots\ldots$ ....(2)

put $x = 0$

$-n = a_1$

Diff. (2)

$n(n-1)(1 - x + 2x^2)^{n-2}(-1 + 4x)^2$

$+ n(1 - x + 2x^2)^{n-1}(4)$

$= 2a_2 + \ldots\ldots\ldots\ldots$ ....(3)

Put $x = 0$

$n^2 - n + 4n = 2a_2$

$2a_2 = n^2 + 3n$

$a_1 + a_0 = 2a_2$

$\Rightarrow\ -n + 1 = n^2 + 3n$

$n^2 + 4n - 1 = 0$

$\Rightarrow\ n \notin N$

73. Domain of $f(x)$ is $[-1, 1]$

$$f'(x) = \frac{-1}{\sqrt{1-x^2}} - \frac{2}{1+x^2} - 6x^2 - 4 < 0$$

$f(x)$ is decreasing function

$\therefore$ Min. value $= 0 + \frac{\pi}{2} - 2 - 4$

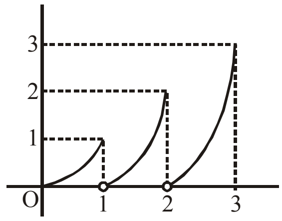
$$m = \frac{\pi}{2} - 6$$

Max value $= \pi + \frac{3\pi}{2} + 2 + 4$

$$M = 6 + \frac{5\pi}{2}$$

$m + M = 3\pi$

74.
$$f(x) = \begin{cases} 0, & x = 0 \\ x^3 & 0 < x < 1 \\ 1 & x = 1 \\ x(x-1)^2 & 1 < x < 2 \\ 2 & x = 2 \\ x(x-2)^2 & 2 < x < 3 \\ 3 & x = 3 \end{cases}$$



75. $f(g(x)) = (x+1)^3 + 3$

$f(x) = (\log x)^3 + 3$

$\therefore\ g(x) = e^{x+1}$

$\therefore\ g'(x) = e^{x+1}$

$g'(-1) = 1$

76. $b = a + 2c;\ a^2 = bc$

$\Rightarrow\ a^2 = (a + 2c)c$

$\Rightarrow\ a^2 = ac + 2c^2$

$a^2 - ac = 2c^2$ .....(1)

$\therefore$ (1) false

Also $a^2 = c(a + 2c)$

$\Rightarrow$ (2) False & (3) True.

Also, $\frac{a}{2}(a-c) = c^2$

$\therefore$ (4) false

77. $P(X + Y = 2) = P(X = 1, Y = 1)$

$+ P(X = 0, Y = 2) + P(X = 2, Y = 0)$

$$= {}^{10}C_1 \frac{1}{2^{10}} \times {}^{8}C_1 \times \frac{1}{2^8} + \frac{1}{2^{10}} \times {}^{8}C_2 . \frac{1}{2^8} + {}^{10}C_2 . \frac{1}{2^{10}} . \frac{1}{2^8}$$

$$= \frac{1}{2^{18}}[80 + 28 + 45] = \frac{153}{4^9}$$

78. $x_1 + x_2 + x_3 = 2a$

$x_1x_2 + x_2x_3 + x_3x_1 = 3b$

$x_1x_2x_3 = 8$

$$\frac{\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}}{3} \geq \left(\frac{1}{x_1}.\frac{1}{x_2}.\frac{1}{x_3}\right)^{\frac{1}{3}}$$

$$\frac{\frac{3b}{8}}{3} \geq \left(\frac{1}{8}\right)^{1/3}$$

$b \geq 4 \quad \Rightarrow \quad b_{min} = 4$

79. $x = \dfrac{1}{h}$

$$\lim_{h\to 0}\frac{h+1}{h^2}.\log\frac{\tan^{-1}h}{h} = \log\left(\frac{\tan^{-1}h}{h}\right)^{\frac{h+1}{h^2}}$$

$$\Rightarrow \quad \log\left(1+\left(\frac{\tan^{-1}h}{h}-1\right)\right)^{\frac{h+1}{h^2}}$$

$$\Rightarrow \quad \left(\frac{\tan^{-1}h}{h}-1\right)\frac{h+1}{h^2} = \frac{(\tan^{-1}h-h)}{h}\times\frac{h+1}{h^2}$$

Applying series of $\tan^{-1}x$

$$\lim_{h\to 0}\left[-\frac{1}{3}+\frac{h^2}{5}.....\right]\left[h+1\right] = -\frac{1}{3}$$

80. $f'(x) = 6x^2 - 6(a + 2)x + 12a$

$= 6(x^2 - ax - 2x + 2a) = 6(x - a)(x - 2)$

$\therefore$ only possible value is $a = 2$

81. $z = x + iy$

$Imz = 0 \Rightarrow y = 0$

$\therefore \quad |z| = |x|$

$$\frac{1}{\log_2|x|} - \frac{1}{\log_2|x|-1} < 1$$

$$\Rightarrow \quad \frac{-1}{(\log_2|x|)(\log_2|x|-1)} < 1$$

$$\Rightarrow \quad \frac{(\log_2|x|)(\log_2|x|-1)+1}{(\log_2|x|)(\log_2|x|-1)} > 0$$

Numerator $= t^2 - t + 1$ is +ve (where $\log_2|x| = t$)

$\therefore \quad (\log_2|x|)(\log_2|x| - 1) > 0$

$\log_2|x| < 0$ or $\log_2|x| > 1$

$0 < |x| < 1$ or $|x| > 2$

$-1 < x < 1,\ x \neq 0$ or $x < -2$ or $x > 2$

$x \in (-\infty, -2) \cup (-1, 1) \cup (2, \infty),\ x \neq 0$

82. trace $(adj(|A||B|\ AB))$

$= trace(|A|^{n-1}|B|^{n-1}(AdjB)(AdjA))$

$trace(Adj(|AB|BA))$

$= trace(|A|^{n-1}|B|^{n-1}(AdjA).(AdjB))$

83. Total words not starting and ending in vowels, but having vowels in alphabetical order

$= ({}^6C_2 \times 2!) \times {}^8C_4 \times 4! = {}^8C_4.6!$

84. $\left(x^5 + 4.3^{-\log_{\sqrt{3}}\sqrt{x^3}}\right)^{10} = (x^5 + 4.x^{-3})^{10}$

$T_{r+1} = {}^{10}C_r(x^5)^{10-r}(4x^{-3})^r$

$= {}^{10}C_r 4^r x^{50-5r-3r}$

$= {}^{10}C_r 4^r x^{50-8r}$

$50 - 8r = 2 \Rightarrow r = 6$

& $\ 50 - 8r = 10 \Rightarrow r = 5$

Required ratio

$$= \frac{{}^{10}C_6 4^6}{{}^{10}C_5 4^5} = \frac{10}{3}$$

85. $f(1 - x) + f(x)$

$$= 2\left(\frac{\sin\pi x}{1-x}+\frac{\sin\pi x}{x}\right)$$

$$= 2\frac{\sin\pi x}{x(1-x)} = 4.\frac{\sin\frac{\pi x}{2}.\cos\frac{\pi x}{2}}{x(1-x)}$$

$$= \frac{2\sin\frac{\pi x}{2}}{4.\frac{x}{2}}.\frac{\sin\frac{\pi(1-x)}{2}}{\frac{(1-x)}{2}} = \frac{1}{2}.\frac{1}{2}f\left(\frac{x}{2}\right).\frac{1}{2}f\left(\frac{1-x}{2}\right)$$

$$= \frac{1}{8}f\left(\frac{x}{2}\right).f\left(\frac{1-x}{2}\right)$$

86. $\cot^{-1}\left(\frac{2x^2-1}{2x\sqrt{1-x^2}}\right)$

put $x=\cos\theta$ $\because$ $\theta\in\left(0,\frac{\pi}{2}\right)$ $\therefore$ $0<x<1$

$\Rightarrow$ $\cot^{-1}\left(\frac{\cos 2\theta}{2\cos\theta|\sin\theta|}\right)$

$=\cot^{-1}(\cot 2\theta)=2\theta=2\cos^{-1}x.$

$\because$ $2\theta\in(0,\pi)$

87. Consider a cubic equation whose roots are x_1, x_2 and x_3

$f(x)=x^3+\alpha x^2+\beta x+\gamma$

$=(x-x_1)(x-x_2)(x-x_3)=0$

Given $(x_1x_2+x_2x_3+x_3x_1)(x_1+x_2+x_3)=x_1x_2x_3$

$\beta(-\alpha)=-\gamma$

$\gamma=\alpha\beta$

$\therefore$ $f(x)=0=x^3+\alpha x^2+\beta x+\alpha\beta$

$=x^2(x+\alpha)+\beta(x+\alpha)=(x+\alpha)(x^2+\beta)$

$\therefore$ $x=-\alpha$

Let $x_1=-\alpha$; $x_1+x_2+x_3=-\alpha$ $\Rightarrow$ $x_2+x_3=0$

$\Rightarrow$ $x_2=-x_3$

$\therefore$ $\frac{1}{x_1^n+x_2^n+x_3^n}=\frac{1}{x_1^n+(x_2)^n+(-x_2)^n}$

$=\frac{1}{x_1{}^n}+\frac{1}{x_2{}^n}+\frac{1}{(-x_2)^n}$

Which is true for all odd integer n.

88. If $x\in(0,1)$ $\{x\}=x$

$\{\sin x\}=\sin x$ & $\{x\sin x\}=x\sin x$

$\therefore$ $f(x)=2\sin x$

$\therefore$ differentiable in $x\in(0,1)$

89. $\frac{1}{2}\sum_{r=1}^{19}{}^{20}C_{r+1}\left(\frac{-1}{4}\right)^r=\frac{1}{2}\sum_{r=2}^{20}{}^{20}C_r\left(-\frac{1}{4}\right)^{r-1}$

$=-2\sum_{r=2}^{20}{}^{20}C_r\left(-\frac{1}{4}\right)^r$

$=-2\left\{\left(1-\frac{1}{4}\right)^{20}-\left({}^{20}C_0\left(-\frac{1}{4}\right)^0+{}^{20}C_1\left(-\frac{1}{4}\right)^1\right)\right\}$

$=-2\left\{\left(\frac{3}{4}\right)^{20}+4\right\}$

90. $f(x)=x\left\{\sin^2 x.\frac{1}{x}+\sin 2x.\log x\right\}$

$=x(\sin^2x.\log x)'$

$g(x)=\sin^2x.\log x$

$g(1)=0$, $g(\pi)=0$, $g(2\pi)=0$

$\therefore$ by Rolle's theorem

$g'(x)=0$ has at least two root in $(1,2\pi)$