JEE (MAIN) : ENTHUSIAST COURSE

TARGET : JEE (MAIN) 2015

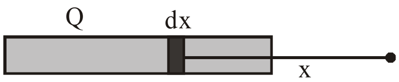
TEST # 09 (SCORE – I) DATE : 18 - 01 - 2015

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	1	1	1	3	1	2	4	2	3	1	1	1	3	2	4	2	2	2	3
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	4	3	3	3	1	2	1	3	3	1	4	4	3	3	2	3	1	2	2	2
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	3	3	2	3	1	3	1	1	3	2	2	2	1	2	2	3	4	1	2	3
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	1	2	2	3	3	1	2	3	4	1	3	2	4	1	2	2	3	2	1	4
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	2	1	4	4	2	1	2	2	3	1										

HINT – SHEET

1. $E = \int_{r-\frac{L}{2}}^{r+\frac{L}{2}} k\lambda dx / x^2$

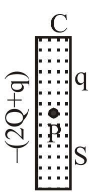


$= \frac{KQ}{L}\left(-\frac{1}{x}\right)_{r-\frac{L}{2}}^{r+\frac{L}{2}}$

$= \frac{KQ}{\left(r^2 - (L/2)^2\right)}$

F = qE

2. The net field inside plate C at say point P should be zero. Let q is charge on S. Then field at P from all the charges (plate A, B and C) is



$\frac{Q/2A}{\epsilon_0} + 0 - \frac{(2Q+q)/A}{2\epsilon_0} - \frac{q/A}{2\epsilon_0} = 0$

or Q – 2Q – q – q = 0

or q = – Q/2

3. $\because$ mg << qF

$T = \frac{2u\sin\theta}{\left(\frac{qE}{m}\right)}$

4. 6 μC, Hint : the p.d across R_2 is 4 volt. The total charge in the two parallel arms is 12 μC.
Thus in each row,
q = 6 μC.

5. $\epsilon_0 AV/d$, $-2\epsilon_0 AV/d$ (negative charge occurs on both sides of the plate 4).

6. %error $= \frac{V - V'}{V} = \frac{2 - \frac{12}{7}}{2} \times 100 \Rightarrow 14\%$.

9. $\frac{\mu_0 \omega q}{2\pi R}$ (Hint : Divide disc into rings, consider one such ring of radius x, thickness dx. Then field at its center $dB = (\mu_0 \omega q/2)\,dx$ integrate to find B).

16. Let q_m is pole strength of the magnetized steel wire. Then $M = q_m L$. When bent into a semicircular arc $\pi r = L$.
Thus new magnetic moment $M' = q_m(2r)$
(2r is the new distance between ends of semicircular arc). Thus $M' = q_m(2L/\pi) = 2M/\pi$.

17. We know that $B_H \cos\delta = B$, and $\tan\delta = B_V/B_H$
Thus $B_v = B_H \tan\delta$
$= 5 \times 10^{-5} \times \tan(30^\circ)$
$= (5/\sqrt{3}) \times 10^{-5}$
and $B = B_H/\cos\delta$
$= 5 \times 10^{-5} (\sqrt{3}/2)$
$= (10/\sqrt{3}) \times 10^{-5}$

18. The χ is positive and has a small value 0.00754. Thus it must be paramagnetic with
$\mu_r = 1 + \chi = 1 + 0.00754 = 1.00754$.

19. The work done
$W = MB_H(\cos\theta_1 - \cos\theta_2)$
where B_H is the horizontal component of the earth's magnetic field. Now $\theta_1 = 0$ and $\theta_2 = \theta$,
thus $W = MB_H(1 - \cos\theta)$.

20. The induced emf
$E = -d\phi/dt$
$$= -\frac{d}{dt}(3t^2 + 2t + 3) \times 10^{-3}$$
(because given flux is in mWb). Thus
$E = (-6t - 2) \times 10^{-3}$
at t = 2 sec,
$E = (-6 \times 2 - 2) \times 10^{-3}$
$= -14$ mV

21. When the conductor falls, an induced emf $E = B\ell v$ is generated. The induced current $I = B\ell v/R$ flows through the conductor (clockwise). As a result a magnetic force $F = I\ell B = B^2\ell^2 v/R$ acts on the conductor in the upward direction under the influence of upward magnetic force F and downward gravitational force mg, the conductor falls. Terminal speed is attained when the two forces are equal
$$mg = \frac{B^2\ell^2 v}{R}$$
Thus $v = (mgR/B^2\ell^2)$

22. At resonance, $X_L = X_C$ $V_{OL} = V_{OC}$
but the phase difference between the voltages across L and C is π or 180°. Thus $\vec{V}_{OL} + \vec{V}_{OC} = 0$.
The resultant voltage across L – C is zero. The entire applied voltage appears across R.

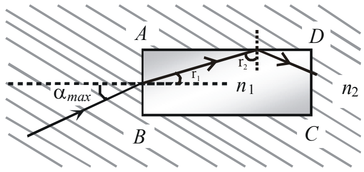
23. The current in parallel ac circuit is given by, I = EY. The admittance
$$Y = \sqrt{G^2 + (B_L - B_C)^2}$$
In fig. G = 0 (no resistance in parallel), $B_L = 1/24$. $B_C = 1/48$, Therefore
$$Y = \frac{1}{24} - \frac{1}{48} = \frac{1}{48}$$
Thus, the current in the circuit is
$I = 240 \times 1/48 = 5$

24. From the first data, resistance of the solenoid is R = 12/2 = 6 ohm. From the second data, the impedance of the solenoid is
$$Z = \frac{12V}{1A} = 12\Omega$$
Since, $Z = \sqrt{R^2 + (\omega L)^2}$
or $Z^2 = R^2 + (\omega L)^2$, we have
$12^2 = 6^2 + (2\pi \times 50\ L)^2$
or $100\pi L = \sqrt{12^2 - 6^2} = \sqrt{108}$
$= 10.4$
$$L = \frac{10.4}{314} = 0.033 \text{ henry}$$

25. $K_{max} = h\nu - h\nu_0$
$$hc = \left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$
use $hc = 1.24 \times 10^{-6}$ (eV) m
$$= 1.24 \times 10^{-6}\left(\frac{10^8}{18} - \frac{10^8}{23}\right)$$
$$= \frac{1.24 \times 100 \times (23 - 18)}{18 \times 23}$$
$= 1.49$ eV

26. The stopping potential does not depend on the intensity of light. When the source is moved away its intensity decreases, but λ and ν of the incident light remains the same. Thus, stopping potential should not change.

27. Ray comes out from CD, means rays after refraction from AB get, total internally reflected at AD



$$\frac{n_1}{n_2} = \frac{\sin\alpha_{max}}{\sin r_1} \Rightarrow \alpha_{max} = \sin^{-1}\left[\frac{n_1}{n_2}\sin r_1\right] \quad ....(i)$$

Also $r_1 + r_2 = 90° \Rightarrow r_1 = 90 - r_2 = 90 - C$

$$\Rightarrow r_1 = 90 - \sin^{-1}\left(\frac{1}{{}_2\mu_1}\right) \Rightarrow r_1 = 90 - \sin^{-1}\left(\frac{n_2}{n_1}\right) \quad ..(ii)$$

Hence from equation (i) and (ii)

$$\alpha_{max} = \sin^{-1}\left[\frac{n_1}{n_2}\sin\left(90 - \sin^{-1}\frac{n_2}{n_1}\right)\right]$$

$$= \sin^{-1}\left[\frac{n_1}{n_2}\cos\left(\sin^{-1}\frac{n_2}{n_1}\right)\right]$$

28. There exists two channels for the decay. Therefore, the mean life is obtained from

$$\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2}$$

or $\tau = \frac{\tau_1\tau_2}{\tau_1 + \tau_2}$

$$\frac{1620 \times 520}{1620 + 520} = 394\,y$$

The half life $T = 0.693\,\tau$
$= 0.693 \times 394$
$= 273$

29. The energy produced per second is
$= 1000 \times 10^3$ joule

$$= \frac{10^6}{1.6 \times 10^{-19}} eV$$

$= 6.25 \times 10^{24}$ eV

The number of fissions should be, thus :

$$\text{number} = \frac{6.25 \times 10^{24}}{200 \times 10^6}$$

$= 3.125 \times 10^{16}$

30. Given $\alpha = \frac{I_c}{I_E} = 0.95$

Thus $I_c = 0.95 \times I_E$
$= 0.95 \times 2$ mA
$= 1.9$ mA

The base current is, therefore,
$I_B = I_E - I_C$
$= 2 - 1.9$ mA
$= 0.1$ mA

32. $E_{Cell} = E^{\circ}_{Cell} - \frac{0.0591}{n}\log\left[\frac{\text{Product}}{\text{Reactant}}\right]$

$$E_{Cell} = E^{\circ}_{Cell} - \frac{0.0591}{n}\log\left[\frac{1}{(10^{-2})^{14}}\right]$$

[$\therefore$ pH = 2 then $H^+ = 10^{-2}$]

$$= 1.33 - \frac{0.0591}{6}\log\frac{1}{(10^{-2})^{14}} = 1.052V$$

37.

	A	→	2B	+	C
t = 0	P		0		0
t = 10min	(P – P')		2P'		P'
t = ∞ min	(P – P)		2P		P

So, Total pressure after 10 min = (P + 2P') = 160

Total pressure at the end of reaction, 3P = 300

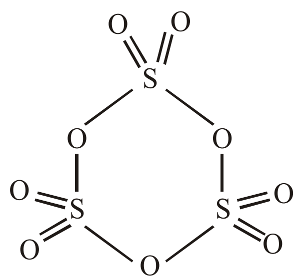
P = 100 mm

$\therefore$ Partial pressure of A after 10 min

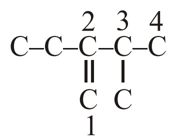
= 100 – 30 = 70mm.

38. KNO_3 dissociates completely while CH_3COOH dissociates to a small extent hence, $P_1 > P_2$

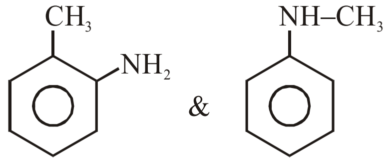
43.



51.



52. $-CH_3$ is electron releasing gps. which makes the compound more reactive.

53. 

O/m/P

54. In alkynes gem-dihalides are formed.

55. double bond is more reactive than alkyne due to formation of more stable carbocation towards any electrophile.

56. always acid products are preferred on strong oxidation.

57. $-NO_2$ by pulling electrons from o/p-positions makes C–Cl bond weak.

58. Compounds having electron releasing groups can give friends craft reactions.

59. ketones can not give silver mirror test.

61. $A-A^T$ will be skew symmetric matrix of odd order, so $|A-A^T| = 0$

62. This is the case of dearrangement of 4things. Therefore number of ways = 9.

63. All the possible number are ${}^9C_5$ (none containing the digit 0) = 126

Total numbers starting with

$1 = {}^8C_4 = 70$

(using 2, 3, 4, 5, 6, 7, 8, 9)

Total starting with

$23 = {}^6C_3 = 20$

2	3			

(using 4, 5, 6, 7, 8, 9)

Total numbers starting with

$245 = {}^4C_2 = 6$

2	4	5		

97^{th} number =

2	4	6	7	8

64. Let E_1, E_2 and W be the events defined by

$E_1 \rightarrow$ Bag from the first group

$E_2 \rightarrow$ Bag from the second group

$W \rightarrow$ Drawn ball is white

$\therefore\ P(E_1) = \frac{3}{5}$, $P(E_2) = \frac{2}{5}$

Now $P\left(\frac{W}{E_1}\right) = \frac{5}{8}$ and $P\left(\frac{W}{E_2}\right) = \frac{2}{6} = \frac{1}{3}$

Now we have to find

$$\therefore\ P(E_1/W) = \frac{P(E_1)P\left(\frac{W}{E_1}\right)}{P(E_1)P\left(\frac{W}{E_1}\right) + P(E_2)\left(\frac{W}{E_2}\right)}$$

$$P\left(\frac{E_1}{W}\right) = \frac{\frac{3}{5}\times\frac{5}{8}}{\frac{3}{5}\times\frac{5}{8}+\frac{2}{5}\times\frac{1}{3}} = \frac{45}{61}$$

65. $\log_{\frac{1}{\sqrt{3}}}\left(\frac{2|z|^2+2|z|-3}{|z|+1}\right) < -2$

$$\Rightarrow \frac{2|z|^2+2|z|-3}{|z|+1} > \frac{2|z|^2+2|z|-3}{|z|+1} > \left(\frac{1}{\sqrt{3}}\right)^{-2}$$

$\frac{2|z|^2+2|z|-3}{|z|+1} > 3 \Rightarrow 2|z|^2 - |z| - 6 > 0$

$\Rightarrow |z| > 2$

66. Let ramining root is r

then $\alpha\beta\gamma = 1$ so $\alpha\beta = \frac{1}{\gamma}$

so req. eq. is $\frac{1}{x^3} + \frac{3}{x^2} - 1 = 0$

$= 1 + 3x - x^3 = 0 \Rightarrow \boxed{x^3 - 3x - 1 = 0}$

67. Coeff of x^8

$$= \frac{(1+2+3+........10)^2 - (1^2+2^2+........10^2)}{2}$$

$= 1320$

68. $(1-x)^{30}\ (1+x+x^2)^{29}$

$(1-x)\ (1-x^3)^{29}$

$(1-x^3)^{29} - x\ (1-x^3)^{29} \longrightarrow$ Coeff of x^{37}

$O - {}^{29}C_{12}(-1)^{12} \Rightarrow -{}^{29}C_{12}$

70. **Case – 1** : $4 - x^2 \geq 0$ and $[x] + 2 > 0$

$\Rightarrow x^2 - 4 \leq 0 \Rightarrow x\ [x] > -2$

$\Rightarrow x \in [-2, 2]\ \&\ x \in [-1, \infty)$

$x \in [-1, 2]$

Case – 2 : $4 - x^2 \leq 0$ and $[x] + 2 < 0$

$\&\ [x] < -2$

$x \in (-\infty, -2] \cup [2, \infty)\ \&\ x \in (-\infty, -2)$

$\Rightarrow x \in (-\infty, -2]$

$\therefore$ answer $x \in (-\infty, -2] \cup [-1, 2]$

71. $\lim\limits_{n\to\infty} \frac{[1^2x]+[2^2x]+....+[n^2x]}{n^3}+\frac{(1^2+2^2+....+n^2)}{n^3}$

$= \frac{x}{3}+\frac{1}{3}$

72. LST $= \frac{y_1}{m}$

$x^m y^n = a^{m+n}$

$y^n = \frac{a^{m+n}}{x^m}$

$n.y^{n-1}\ y' = a^{m+n}\left(\frac{-m}{x^{m+1}}\right)$

$y' = -\frac{-m}{n}\ a^{m+n}\left[\frac{1}{x^{m+1}.y^{n-1}}\right]$

LST $= \frac{y}{-\frac{m}{n}a^{m+n}} \times \frac{1}{\left[\frac{1}{x^{m+1}.y^{n-1}}\right]}$

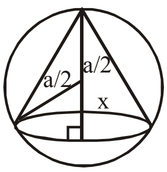
LST $= \frac{-n}{m\,a^{m+n}} \times y^n.x^m.x$

LST $= \frac{-n}{m}x$

LST $\propto$ x

73. $f(x) = \begin{cases} -\frac{(x-1)}{x^2}\ ;\ x\in(-\infty,0)\cup(0,1) \\ \frac{(x-1)}{x^2}\ ;\quad x\in(1,\infty) \end{cases}$

$f'(x) = \begin{cases} \frac{x(x-2)}{x^4}\ ;\ x\in(-\infty,0)\cup(0,1) \\ \frac{-x(x-2)}{x^4}\ ;\ x\in(1,\infty) \end{cases}$

74. 

$v(x) = \frac{1}{3}\ \pi\left(\frac{a^2}{x^2}-x^2\right)\left(\frac{a}{2}+x\right)$

v(x) is max at $x = \frac{a}{6}$

$\therefore$ height $= \frac{a}{2}+\frac{a}{6}=\frac{2a}{3}$

75. $y = \tan^{-1}\left[\frac{(x+1)-x}{1+x(x+1)}\right]+\tan^{-1}\left[\frac{(x+2)-(x+1)}{1+(x+1)(x+2)}\right]$

$+........+\tan^{-1}\left[\frac{(x+x)-(x+n-1)}{1+(x+n)(x+n-1)}\right]$

$y = \tan^{-1}(x+n) - \tan^{-1}x$

$\frac{dy}{dx}=\frac{1}{1+(x+x)^2}-\frac{1}{1+x^2}$

76. $\frac{H}{3}\cot\alpha = d$ and $H\cot\beta = d$

or $\frac{H}{3d}=\tan\alpha$ and $\frac{H}{d}=\tan\beta$

$\tan(\beta-\alpha)=\frac{1}{2}=\frac{\frac{H}{d}-\frac{H}{3d}}{1+\frac{H^2}{3d^2}}$

$\Rightarrow 1+\frac{H^2}{3d^2}=\frac{4H}{3d}$

$\Rightarrow H^2-4dH+3d^2=0$

$\Rightarrow H^2-80H+3\ (400)=0$

$\Rightarrow$ H = 20 or 60 m

77. $\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{7\pi}{8}$

$= \frac{1}{4}\left[\left(2\sin^2\frac{\pi}{8}\right)^2+\left(2\sin^2\frac{3\pi}{8}\right)^2\right]$

$+\frac{1}{4}\left[\left(2\sin^2\frac{\pi}{8}\right)^2+\left(2\sin^2\frac{3\pi}{8}\right)^2\right]$

$= \frac{1}{4}\left[\left(1-\cos\frac{\pi}{4}\right)^2+\left(1-\cos\frac{3\pi}{4}\right)^2\right]$

$+\frac{1}{4}\left[\left(1-\cos\frac{\pi}{4}\right)^2+\left(1-\cos\frac{3\pi}{4}\right)^2\right]$

$= \frac{1}{4}\left[\left(1-\frac{1}{\sqrt{2}}\right)^2+\left(1+\frac{1}{\sqrt{2}}\right)^2\right]$

$+\frac{1}{4}\left[\left(1-\frac{1}{\sqrt{2}}\right)^2+\left(1+\frac{1}{\sqrt{2}}\right)^2\right]$

$= \frac{1}{4}(3)\ +\frac{1}{4}(3)=\frac{3}{2}$

78. Arranging the data in ascending order of magnitude, we obtain

Height (in cm)	150	152	154	155	156	160	161
Number of students	8	4	3	7	3	12	4
Cumulative frequency	8	12	12	22	25	37	41

Here, total number of items is 41 i.e., an odd number.

Hence, the median is $\frac{41+1}{2}$ th i.e., 21st item.

From cumulative frequency table, we find that median
i.e., 21st item is 155,

(All items from 16 to 22nd are equal, each 155).

79.

p	q	~p	~q	$p \wedge \sim q$	$\sim p \vee q$	$(p \wedge \sim q) \wedge (\sim p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

Clearly - $(p \wedge \sim q) \wedge (p \vee \sim q)$ is a contradiction.

80. Since a, b, c are the roots of the equation
$x^3 - 3x^2 + x + \lambda = 0$
$\Rightarrow\ a + b + c = 3 \qquad \Rightarrow\ a + b = 3 - c$
Now area of the triangle will be

$$A = \frac{1}{2} \times \frac{1}{a+b} \times 1 = \frac{1}{2(a+b)} = \frac{1}{2(3-c)}$$

$$\Rightarrow \frac{dA}{dc} = \frac{1}{2(3-c)^2} > 0$$

As A is an increasing function & $c \in [1,2]$

$\therefore\ A_{max} = \frac{1}{2}$ sq. units.

81. Let P(h, k) be the point from which two tangents are drawn to $y^2 = 4x$. Any tangent to the parabola $y^2 = 4x$ is

$$y = mx + \frac{1}{m}$$

If it passes through P(h, k), then

$$k = mh + \frac{1}{m} \Rightarrow m^2 h - mk + 1 = 0$$

Let m_1, m_2 be the roots of this equation. Then,

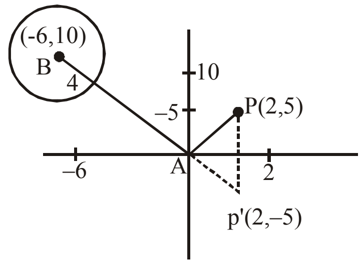
$$m_1 + m_2 = \frac{k}{h} \text{ and } m_1 m_2 = \frac{1}{h}$$

$$\Rightarrow 3m_2 = \frac{k}{h} \text{ and } 2m_2^2 = \frac{1}{h}\ [\because m_1 = 2m_2(\text{given})]$$

$$\Rightarrow 2\left(\frac{k}{3h}\right)^2 = \frac{1}{h} \Rightarrow 2k^2 = 9h$$

Hence, P(h, k) lies on $2y^2 = 9x$

82.



BP'=17

83. $$AB = \sqrt{(PQ)^2 - \left(r + \frac{r}{2}\right)^2}$$

$$= \sqrt{9r^2 - \frac{9r^2}{4}}$$

$$= \frac{3\sqrt{3}}{2} r$$

84. $$\left.\frac{dy}{dx}\right|_{(x_1 y_1)} = \frac{4x_1}{y_1} = 4 \Rightarrow y_1 = x_1$$

(x_1, y_1) lies on $4x^2 - y^2 = 12$
$\Rightarrow x_1 = y_1 = \pm 2$
$\therefore$ Equation of tangent lines with slope 4 are
$y = 4x + 6$ & $y = 4x - 6$
$\Rightarrow |c_1 - c_2| = 12$

85. Eq of plane O Q R is

$$\begin{vmatrix} x & y & z \\ 1 & 3 & 4 \\ 2 & 1 & -2 \end{vmatrix} = 0$$

$\Rightarrow 2x - 2y + z = 0$

distance of P from plane = $\left|\dfrac{6+4-1}{\sqrt{4+4+1}}\right| = 3$

86. Required plane is

$[r-(2\hat{i}+\hat{j}-3\hat{k})].(2\hat{i}+\hat{j}-3\hat{k})=0$

87. $\displaystyle\int \frac{x^{5m-1}+2x^{4m-1}}{(x^{2m}+x^m+1)^3}dx = \int \frac{x^{-m-1}+2x^{-2m-1}}{\left(1+\frac{1}{x^m}+\frac{1}{x^{2m}}\right)^3}dx$

Put $1+\dfrac{1}{x^m}+\dfrac{1}{x^{2m}} = t$

$$-\frac{1}{m}\int\frac{dt}{t^3} = -\frac{1}{m}.\left(\frac{t^{-2}}{-2}\right)+c$$

$$= \frac{x^{4m}}{2m(x^{2m}+x^m+1)^2}+c$$

88. $\displaystyle\int_{-1}^{0} -1dx + \int_{0}^{1} odx + \int_{1}^{2} odx = -1$

90. $\dfrac{dy}{dx} = ke^{\sin^{-1}x} \cdot \dfrac{1}{\sqrt{1-x^2}}$

$$\frac{dy}{dx} = (y-3)\cdot\frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\,\frac{dy}{dx} = y-3$$