

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	1	1	3	2	1	4	2	2	1	1	2	3	2	2	4	2	1	3	4	4
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	4	1	1	4	4	2	1	1	1	1	4	3	2	1	3	1	1	3	2	2
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	2	4	4	3	1	2	1	2	4	3	3	4	2	3	4	1	3	3	3	3
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	3	3	2	1	4	1	3	1	2	1	4	2	3	3	2	1	3	1	4	1
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	4	3	4	2	1	3	3	4	3	2										

HINT - SHEET

1. Internal resistance $r = \frac{R[l-l']}{l'}$

where l = balanced length with key open.

l' = balanced length with key K closed

$$\therefore r = \frac{132.40[70-60]}{60} = \frac{132.40 \times 10}{60}$$

$$= 21.06 \Omega \approx 22.1 \Omega$$

2. Flux through

$$S_1(\phi_1) = \frac{\text{Charge enclosed within } S_1}{\epsilon_0} = \frac{q}{\epsilon_0}$$

Since the shell is conducting therefore a charge of magnitude $-q$ will be induced at the inner surface (radius R_1) of the shell.

\therefore Flux through

$$S_2(\phi_2) = \frac{\text{Charge enclosed within } S_2}{\epsilon_0} = \frac{q-q}{\epsilon_0} = 0$$

3. Since the two spheres are identical, final charge on each of them after contact

$$q = \frac{(q_1 + q_2)}{2}$$

$$F_1 = \frac{Kq_1q_2}{r^2}$$

$$F_2 = \frac{Kq^2}{r^2} = \frac{K}{r^2} \left(\frac{q_1 + q_2}{2} \right)^2$$

$$F_2 - F_1 = \frac{K}{r^2} \left\{ \left(\frac{q_1 + q_2}{2} \right)^2 - q_1q_2 \right\}$$

$$= \frac{K}{4r^2} (q_1 - q_2)^2 \Rightarrow F_2 - F_1 > 0 \text{ or } F_2 > F_1$$

4. As process is cyclic $\Rightarrow \Delta U_{ABCA} = 0$

$$\Rightarrow Q_{ABCA} = W_{ABCA}$$

$$\Rightarrow Q_{A \rightarrow B} + Q_{B \rightarrow C} + Q_{C \rightarrow A} = W_{A \rightarrow B} + W_{B \rightarrow C} + W_{C \rightarrow A} \quad (1)$$

$$\text{Given } Q_{B \rightarrow C} + Q_{C \rightarrow A} = W_{A \rightarrow B} + W_{B \rightarrow C} \quad (2)$$

Subtracting (2) from (1),

$$Q_{A \rightarrow B} = W_{C \rightarrow A} = 0$$

(as process $C \rightarrow A$ is isochoric)

\therefore process $A \rightarrow B$ is adiabatic.

5. $L_f = L_i(1 + \alpha \Delta T)$
 $L_{S_f} = L_{S_i}[1 + \alpha_S \Delta T]$
 $\Rightarrow \Delta L_{steel} = L_{S_i} \alpha_S \Delta T$
 $L_{C_f} = L_{C_i}[1 + \alpha_C \Delta T]$
 $\Rightarrow \Delta L_{copper} = L_{C_i} \alpha_C \Delta T$
 For $\Delta L_{steel} = \Delta L_{copper}$
 $\Rightarrow L_{S_i} \alpha_S \Delta T = L_{C_i} \alpha_C \Delta T$
 $\Rightarrow \frac{L_{S_i}}{L_{C_i}} = \frac{\alpha_C}{\alpha_S} = \frac{1.8 \times 10^{-5}}{1.2 \times 10^{-5}} = \frac{3}{2}$
 $\therefore \frac{L_{S_i}}{L_{C_i}} = \frac{3}{2}$ in (1) only.

6. Range on the inclined plane
 $= \sqrt{h_{\max}^2 + \left(\frac{R}{2}\right)^2} = \frac{u^2}{g} \frac{\sqrt{21}}{8}$

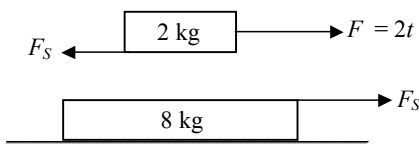
7. Current through the inductor before closing the switch = 1 A
 Current through the inductor after closing the switch (in steady state)
 $I = \frac{20}{5} = 4 \text{ A}$
 $\therefore \Delta \phi = LI = 1.5 \text{ wb}$

8. We can consider a rolling ring as a rod of length $2R$ rotating with angular velocity ω . Drawing the circuit



9. As the source is not an a.c. source, the ideal inductance will act like a short circuit, so whole of the current will pass through the inductance and no current will flow through the bulb. Therefore, the bulb will not glow.

10. Let common acceleration be $a \text{ m/s}^2$
 $2t - F_S = 2a$ (i)
 $F_S = 8a$ (ii)
 $F_S = \frac{2t}{10} \times 8 \leq 0.2 \times 2 \times 10$



$t = \frac{5}{2} \text{ s}, a = \frac{t}{5}$

$\frac{dv}{dt} = \frac{t}{5} \Rightarrow v = \frac{t^2}{10} = \frac{dx}{dt} \Rightarrow x = \frac{t^3}{30} \text{ m}$

11. $K = \frac{P^2}{2m}$

From graph, $4 = \frac{4^2}{2m}$

$\therefore m = 2 \text{ kg}$

12. $F \times \Delta t = P$

Angular impulse about center of mass

$= F \times \Delta t \times \frac{l}{2} = \frac{P \times l}{2}$

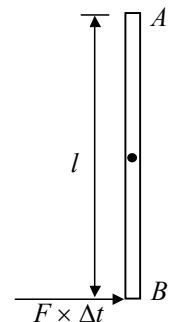
Angular impulse = Change in angular momentum

$P \times \frac{l}{2} = \frac{ml^2}{12} \omega$

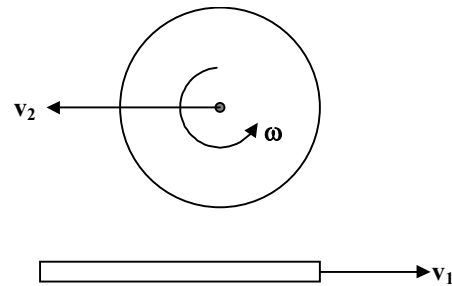
$\omega = \frac{6P}{ml}$

Time taken by rod to turn by 90° is (Time period)/4.

$= \frac{1}{4} \times \frac{2\pi}{\omega} = \frac{\pi}{2\omega} = \frac{\pi ml}{12P}$



13.



The condition for no slipping here will be

$R\omega - v_2 = v_1$

(\because point of contact remains at rest)

In terms of displacement

$R\Delta\theta - s_2 = s_1$

$\therefore \Delta\theta = \frac{s_1 + s_2}{R} = \frac{100 + 75}{150} = \frac{7}{6} \text{ rad}$

14.
$$mgl \sin \theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2} \frac{mR^2}{2} \frac{v^2}{R^2} = \frac{3}{4}mv^2$$

$$\Rightarrow v = \sqrt{\frac{4gl \sin \theta}{3}}$$

$$I = I_{cm} \omega + mvR = \frac{MR^2}{2} \frac{v}{R} + MvR = \frac{3MvR}{2}$$

$$= \frac{3}{2}mR \sqrt{\frac{4gl \sin \theta}{3}} = \sqrt{3m^2 R^2 gl \sin \theta}$$

15.
$$mg \cos \theta - qE \sin \theta = \frac{mv^2}{R} \quad (1)$$

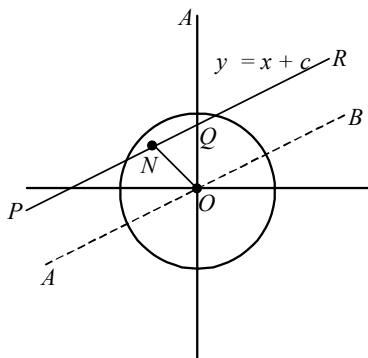
Applying work energy theorem

$$\frac{1}{2}mv^2 = mgR(1 - \cos \theta) + qER \sin \theta \quad (2)$$

Solving (1) and (2)

$$\frac{qE}{mg} = \frac{3 - 2\sqrt{2}}{3}$$

16.



$$I_{PQR} = I_{AOB} + M(ON)^2$$

$$I_{PQR} = \frac{1}{4}MR^2 + M\left(\frac{C}{\sqrt{2}}\right)^2 \quad (1)$$

But
$$I_{PQR} = \frac{1}{2}MR^2 \quad (2)$$

From (1) and (2),

$$C = \frac{R}{\sqrt{2}}$$

17.
$$\frac{\text{Volume submerged}}{\text{Volume}} = \frac{\rho_s}{\rho_L}$$
 and is independent of g_{eff}

18. At A: Acceleration = $g \sin \theta$ (only tangential)
 at B :
 Acceleration = $\frac{v^2}{l} = 2g(1 - \cos \theta)$ (only normal)
 $\therefore g \sin \theta = 2g(1 - \cos \theta)$
 $\Rightarrow \cos \theta = 3/5$

19.
$$h = \frac{2T \cos \theta}{rpg} \Rightarrow \frac{T_w}{T_m} \times 13.6 \times (-\sqrt{2}) = \frac{10}{-3.42}$$

$$\Rightarrow \frac{T_w}{T_m} \cong \frac{1}{6.5}$$

20.
$$a_t = \frac{2v_2}{v_1 + v_2} a_i \Rightarrow \frac{a_t}{a_i} = \frac{2 \times 100}{200 + 100} = \frac{2}{3}$$

21.
$$L = \frac{nv}{4f} = 25n \text{ cm with } n = 1, 3, 5, \dots$$

 i.e.
 $L = 25 \text{ cm, } 75 \text{ cm, } 125 \text{ cm, } \dots$
 Now $L_{\min} = 120 - L_{\max} = 120 - 75 = 45 \text{ cm}$

22.
$$13.6 Z^2 \left(\frac{1}{2^2} - \frac{1}{\infty^2} \right) = 13.6 \left(\frac{1}{1^2} - \frac{1}{\infty^2} \right)$$

$$\Rightarrow Z = 2$$

23. Pitch = $\frac{1.5}{5} = 0.3 \text{ mm}$
 Least count = $\frac{0.3}{50} = 0.006 \text{ mm}$

24. Required power = $16 \times 2 = 32 \text{ MW} = 32 \times 10^6 \text{ W}$
 Hence number of fission of Uranium nuclei per second

$$= \frac{32 \times 10^6}{200 \times 1.6 \times 10^{-13}} = 1 \times 10^{18}$$

25. Increasing frequency increases the KE_{\max} and therefore stopping potential. Decreasing intensity results in less number of photons striking the surface per unit time and therefore less photocurrent.

26.
$$p^x q^y c^z = (ML^{-1}T^{-2})^x \left(\frac{ML^2T^{-2}}{L^2T} \right)^y (LT^{-1})^z$$

$$= M^{x+y} L^{-x+z} T^{-2x-3y-z}$$
 As the quantity is dimensionless, therefore
 $x + y = 0; \quad \therefore x = -y$
 $-x + z = 0; \quad \therefore x = z$
 $-2x - 3y - z = 0$, which satisfy this equation.
 Hence $x = -y = z$

27. Here $a_1 : a_2 :: 2 : 1$

$$\therefore \frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{9}{1}$$

28. $\frac{1}{v} - \frac{2}{-15} = \frac{1-2}{-10}$

$$\frac{1}{v} = \frac{1}{10} - \frac{2}{15} = \frac{3-4}{30} = -\frac{1}{30}$$

31. $\text{H}_2\text{CO}_3(\text{aq.}) \rightleftharpoons 2\text{H}^+(\text{aq.}) + \text{CO}_3^{--}(\text{aq.})$

$$K_{\text{overall}} = K_{a_1} \times K_{a_2} = 10^{-5} \times 10^{-8} = 10^{-13}$$

$$\therefore [\text{CO}_3^{--}] = 0.01 \text{ M} \ \& \ [\text{H}_2\text{CO}_3] = 0.1 \text{ M}$$

$$\therefore K_{\text{overall}} = \frac{[\text{H}^+]^2[\text{CO}_3^{--}]}{[\text{H}_2\text{CO}_3]} = 10^{-13}$$

$$[\text{H}^+] = \sqrt{\frac{10^{-13} \times 0.1}{0.01}} = 10^{-6}$$

$$\therefore \text{pH} = 6$$

32. In ideal rock salt structure anions (R) are forming FCC crystal lattice & cations (r) are present in each octahedral void.

$$\frac{r}{R} = 0.414 \ \& \ \sqrt{2}a = 4R \ \text{(a = edge length of cube)}$$

$$\text{Length of body diagonal} = \sqrt{3}a$$

$$\text{Length of void space} = \sqrt{3}a - (2r + 2R)$$

$$\text{Fraction of void space} = \frac{\sqrt{3}a - (2r + 2R)}{\sqrt{3}a}$$

$$= \frac{\sqrt{3} \times \frac{4}{\sqrt{2}}R - (2 \times 0.414R + 2R)}{\sqrt{3} \times \frac{4}{\sqrt{2}}R}$$

$$= 0.42$$

34. $P_1 = 750 \text{ torr} \ \& \ P_2 = 700 \text{ torr}$

$$V_1 = 127.4 \text{ ml} \quad V_2$$

$$T_1 = 273 \text{ K} \quad T_2 = 300 \text{ K}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

35. $\text{A} + \text{B} \xrightarrow{k} \text{product}$

$$\text{Rate law} = k [\text{A}]^1 [\text{B}]^0$$

$$\text{Overall order} = 1$$

So, variation of concentration of A & B will govern by 1st order.

36. Z for A atoms = $8 \times \frac{1}{8} = 1$

$$Z \text{ for O atoms} = 6 \times \frac{1}{2} + 12 \times \frac{1}{4} = 6$$

39. C = 1 (P = 1, V = 1, initially)

$$w = -\int_1^2 PdV = -\int_1^2 \frac{1}{V^n} dV \Rightarrow$$

$$-\left[\frac{V^{-n+1}}{-n+1} \right]_1^2 = \frac{1}{n-1} \left[\frac{1}{2^{n-1}} - 1 \right] = -0.375 \text{ L-atm}$$

$$V^2 T = C$$

$$T_f = 75 \text{ K}$$

$$\Delta E = \frac{1}{24.63} \times \frac{3}{2} \times 0.0821 [75 - 300]$$

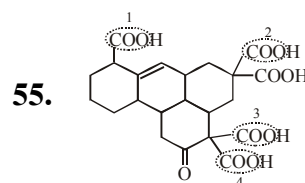
$$= -1.125 \text{ L-atm}$$

$$\Delta E = q + w$$

$$= -1.125 \text{ L-atm} = -0.375 + q$$

$$\Rightarrow q = -0.75 \text{ L-atm} = 75.9 \text{ J}$$

40. $\frac{7x \times 4 - (2x + 2x)}{x} = \frac{28 - 4}{1} = 24$



61. a, b, c are in AP.

$$\lambda a, \lambda b, \lambda c \text{ will be in AP}$$

$$\Rightarrow \sin A, \sin B, \sin C \text{ in AP}$$

$$\text{Also } h_1 = \frac{2\Delta}{a}; h_2 = \frac{2\Delta}{b}; h_3 = \frac{2\Delta}{c} \text{ will be in HP.}$$

$$a, b, c \text{ in AP} \Rightarrow s - a, s - b, s - c \text{ in AP}$$

$$\Rightarrow \frac{\Delta}{s-a}, \frac{\Delta}{s-b}, \frac{\Delta}{s-c} \text{ will be in HP.}$$

$$\Rightarrow r_1, r_2, r_3 \text{ in H.P.}$$

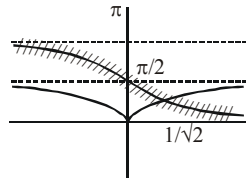
$$\text{Also } \frac{\Delta}{s(s-a)}, \frac{\Delta}{s(s-b)}, \frac{\Delta}{s(s-c)} \text{ will be in HP.}$$

62. $\cot\left(\sin^{-1}\left(\frac{\sqrt{2}}{\sqrt{3+\cos 2x}}\right)\right) = \cot \cot^{-1}\left(\sqrt{\frac{1+\cos 2x}{2}}\right)$

$f(x) = |\cos x| \Rightarrow f'\left(\frac{2\pi}{3}\right) = \frac{\sin 2\pi}{3} = \frac{\sqrt{3}}{2}$

63. Range : $\left[\frac{\pi}{4}, \pi\right)$

Non derivable at $x = \frac{1}{\sqrt{2}}$

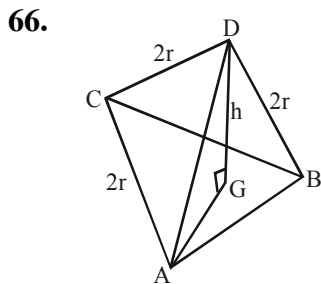


I and II are wrong

64. $\ln((1 + \sin^2 x)(5 + x^2)) = 1$
least value of LHS = $\ln 5 (> 1)$
 \therefore No solution.

65. Homogeneous equation $\Rightarrow \frac{dy}{dx} = \frac{y}{x}$

\therefore Slope = 4



BC = 2r = AD
G is centroid of ΔABC
 $AG = \frac{2}{3}(2r \sin 60)$
 $AG = \frac{2r}{\sqrt{3}}$

From ΔAGD

$AG^2 + h^2 = (2r)^2$

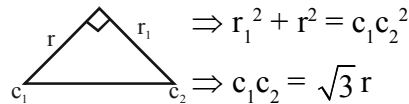
$h = \sqrt{4r^2 - \frac{4r^2}{3}} = 2r\sqrt{\frac{2}{3}}$

\therefore ABCD are centres of Balls so required height = $h + 2r$

$= 2r\left(\sqrt{\frac{2}{3}} + 1\right) = 4\left(\sqrt{\frac{2}{3}} + 1\right)$

67. (1,1), (1,2).....(1,99) $\Rightarrow 98 \times 2 + 1$ pairs
(2,2),(2,49) $\Rightarrow 47 \times 2 + 1$
(3,3),(3,33) $\Rightarrow 30 \times 2 + 1$
(4,4),(4,24) $\Rightarrow 20 \times 2 + 1$
(5,5),(5,19) $\Rightarrow 14 \times 2 + 1$
(6,6),(6,16) $\Rightarrow 10 \times 2 + 1$
(7,7),(7,14) $\Rightarrow 7 \times 2 + 1$
(8,8),(8,12) $\Rightarrow 4 \times 2 + 1$
(9,9),(9,11) $\Rightarrow 2 \times 2 + 1$ pairs
total = 946

68. $\pi r_1^2 = 2\pi r^2 \Rightarrow r_1 = \sqrt{2}r$



69. $b^2 - 4a \geq 0$

$b = 1 \Rightarrow a \in \phi$

$b = 2 \Rightarrow a \in \{1\}$ total = 7

$b = 3 \Rightarrow a \in \{1, 2\}$

$b = 4 \Rightarrow a \in \{1, 2, 3, 4\}$

70. $s = \frac{1^2}{7^0} + \frac{2^2}{7^1} + \frac{3^2}{7^2} + \frac{4^2}{7^3} + \dots \infty$

$\frac{s}{7} = \frac{1^2}{7} + \frac{2^2}{7^2} + \frac{3^2}{7^3} + \dots \infty$

$\frac{6s}{7} = 1 + \frac{3}{7} + \frac{5}{7^2} + \frac{9}{7^3} + \dots \infty$

$\frac{6s}{7^2} = \frac{1}{7} + \frac{3}{7^2} + \frac{5}{7^3} + \frac{9}{7^4} + \dots \infty$

$\therefore \frac{36s}{7^2} = 1 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \dots \infty$

$\Rightarrow \frac{36s}{49} = \frac{4}{3} \Rightarrow s = \frac{49}{27}$

71. $f(x)$ is continuous for all non integers
for integers

$f(I^+) = I - 0 = I$

$f(I^-) = I - 1 + \sqrt{1} = I$

$\therefore f(x)$ is continuous for integers

$\therefore f(x)$ is continuous $\forall x \in \mathbb{R}$

72. $\int_{-3\pi}^{3\pi} \sin^2 \theta \sin^2 2\theta d\theta = 2 \int_0^{3\pi} \sin^2 \theta \sin^2 2\theta d\theta$

$= 8 \int_0^{\pi} \sin^4 \theta \cos^2 \theta d\theta = 24 \int_0^{\pi} \sin^4 \theta \cos^2 \theta d\theta$

$= 48 \int_0^{\pi/2} \sin^4 \theta \cos^2 \theta d\theta$

$= \frac{48 \cdot (3 \cdot 1) \cdot (1)}{6 \cdot 4 \cdot 2} \cdot \frac{\pi}{2} = \frac{3\pi}{2}$

$$73. \quad \vec{a} = 2\hat{i} + 4\hat{j} + 5\hat{k} \quad ; \quad \vec{p} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k} \quad ; \quad \vec{q} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$SD = \frac{|(\vec{b} - \vec{a}) \cdot (\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|}$$

$$= \frac{|-(\hat{i} + 2\hat{j} + 2\hat{k}) \times (\hat{i} - 2\hat{j} + \hat{k})|}{|\hat{i} - 2\hat{j} + \hat{k}|} = \frac{1}{\sqrt{6}}$$

$$74. \quad \sec x = \frac{1}{1 - \cos x} \Rightarrow \cos x = 1 - \cos x$$

$$\Rightarrow \cos x = \frac{1}{2}$$

2 solutions in $[0, 2\pi]$

\therefore 100 solutions.

$$75. \quad r = \frac{2}{2} \cot \frac{\pi}{10} \quad \therefore \pi R^2 - \pi r^2 = \pi \text{ sq. units}$$

$$R = \frac{2}{2} \operatorname{cosec} \frac{\pi}{10}$$

Note that it is independent of 'n'.

$$76. \quad (\vec{a} - \vec{d}) \times \vec{b} = 0 \Rightarrow \vec{a} = \vec{d} + \lambda \vec{b}$$

Dot with \vec{c}

$$2 + 3 + 1 = 8 + \lambda(1 - 1 + 1)$$

$$\Rightarrow \lambda = -2$$

$$\therefore \vec{d} = \vec{a} + 2\vec{b}$$

$$= 4\hat{i} + \hat{j} + 3\hat{k}$$

$$\therefore \vec{a} \cdot \vec{d} = 4 - 1 + 3 = 6$$

$$77. \quad f'(x) = f(x); \quad f(1) = 0$$

$$\therefore \frac{f'(x)}{f(x)} = 1 \Rightarrow \ln(f(x)) = x + c$$

$$\Rightarrow f(x) = k.e^x \Rightarrow k = 0$$

$$\therefore f(x) = 0.$$

$$78. \quad [\vec{a} \vec{b} \vec{c}] = 12$$

$$\frac{1}{6}([\vec{a} - \vec{b} \quad \vec{b} - \vec{c} \quad \vec{a} - \vec{c} + \vec{b}]) = \frac{1}{6}[\vec{a} \vec{b} \vec{c}] = 2$$

79. Let mid point be (h, k)
 chord of hyperbola : $hx - ky = h^2 - k^2$
 this is tangent to $x^2 = 4by$
 form of tangent for parabola : $y = mx - bm^2$
 comparing we get

$$m = \frac{h}{k}; \quad -bm^2 = -\left(\frac{h^2 - k^2}{k}\right)$$

$$\therefore b\left(\frac{h^2}{k^2}\right) = -\left(\frac{h^2 - k^2}{k}\right)$$

clearly, locus is dependent on b , but not a

$$80. \quad 2xydx + x^2dy = \frac{ydx - xdy}{y^2}$$

$$\Rightarrow d(x^2y) = d\left(\frac{x}{y}\right)$$

$$\Rightarrow x^2y = \frac{x}{y} + c$$

$$\text{at } x = 2; y = 1 \Rightarrow 4 = 2 + c \Rightarrow c = 2$$

$$\text{at } x = -1; y = -\frac{1}{y} + 2 \Rightarrow y = 1$$

$$81. \quad y = mx + \frac{2}{m}; y = mx \pm \sqrt{32m^2 + 8}$$

$$\therefore \frac{2}{m} = \pm \sqrt{32m^2 + 8}$$

$$\Rightarrow \frac{4}{m^2} = 32m^2 + 8 \Rightarrow 1 = 8m^4 + 2m^2$$

$$\Rightarrow m^2 = -\frac{1}{2}; m^2 = \frac{1}{4} \Rightarrow m = \frac{1}{2} \text{ or } -\frac{1}{2}$$

$$\therefore \text{Product} = -\frac{1}{4}$$

$$82. \quad f'(x) = 0 \text{ at } x = 0, \pm\sqrt{2}$$

$$f(x)_{\max} = \frac{4}{e^2}; f(x)_{\min} = 0$$

$$83. \quad \left|z - \frac{6}{z}\right| \geq \left|z - \frac{6}{|z|}\right| \Rightarrow 5 \geq \left|z - \frac{6}{|z|}\right|$$

$$\Rightarrow |z|^2 - 5|z| - 6 \leq 0$$

$$\Rightarrow |z| \in [-1, 6]$$

$$\therefore \text{maximum value} = 6$$

84. Area will be equal to area bounded by $f(x)$ between $y = 1$ and $y = 1 + e$ at $y = 1$; $x = 0$; at $y = 1 + e$; $x = 1$.

$$\therefore \text{Area} = \int_0^1 x + e^x dx = \left. \frac{x^2}{2} + e^x \right|_0^1$$

$$= \frac{1}{2} + e - (1) = e - \frac{1}{2}$$

$$\text{required area} = (1 + e) - \left(e - \frac{1}{2} \right) = \frac{3}{2}$$

85. Number of ways to choose four non consecutive numbers

$$= {}^{20-4+1}C_4$$

$$\text{Total ways} = {}^{20}C_4$$

$$\text{Probability} = \frac{{}^{17}C_4}{{}^{20}C_4} = \frac{17 \cdot 16 \cdot 15 \cdot 14}{20 \cdot 19 \cdot 18 \cdot 17} = \frac{28}{57}$$

86. $|A| = 2, n = 4$

$$|\text{Adj}(\text{Adj } 2A)| = |2A|^{(4-1)^2} = |2A|^9$$

$$= (2^4 \cdot |A|)^9 = 2^{36} \cdot 2^9 = 2^{45}$$

87. $\int \frac{\log x}{x} \left(\frac{1 - \log x}{x^2} \right) dx + \int \frac{dx}{x}$

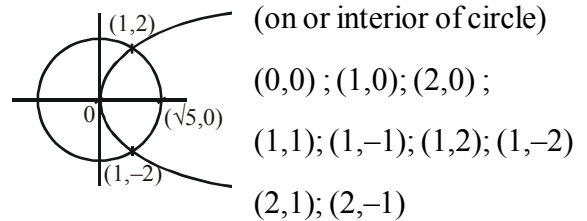
$$\text{Put } \frac{\log x}{x} = t \Rightarrow \int t dt + \ln x + c$$

$$\Rightarrow \frac{1}{2} \left(\frac{\log x}{x} \right)^2 + \log x + c$$

88. $\left(\frac{z - \bar{z}}{zi} \right)^2 \leq 2(z + \bar{z}) \quad z = x + iy$

$$\Rightarrow y^2 \leq 4x \quad (\text{on or interior of parabola})$$

$$\text{Also } |z| \leq \sqrt{5} \Rightarrow x^2 + y^2 \leq 5$$



Total 9 points.

89. $\text{Var}(ax_i + b) = a^2 \text{var}(x_i)$

Variance on doubling each observation

$$= 2^2 \times 16 = 64$$

$$\text{Std. deviation} = \sqrt{\text{var}} = 8$$

90. $\sim (p \wedge (\sim q \vee \sim r)) \Rightarrow \sim p \vee \sim (\sim q \vee \sim r)$

$$\Rightarrow \sim p \vee (q \wedge r) \Rightarrow \sim p \vee q \wedge (\sim p \vee r)$$