LEADER COURSE (PHASE – ELS, ELC, ELD & ELP)

TARGET : JEE (MAIN) 2015

SCORE – I | **DATE : 07 - 03 - 2015**

MAJOR TEST | **Test Pattern : JEE (Main)**

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	2	1	3	3	4	3	3	2	1	1	4	3	4	2	1	2	1	2	1
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	4	1	4	2	2	3	3	2	2	2	4	1	2	4	3	1	1	2	1	4
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	2	3	2	3	4	3	4	3	4	2	2	4	4	4	3	3	2	4	4	4
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	1	3	1	1	2	1	1	2	1	2	2	4	4	1	4	4	1	4	4	2
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	2	3	2	3	2	1	3	2	1	2										

HINT – SHEET

1. $S_W > S_A$ so T_{eq} will be closer to T_W.

3. $t_1 = 2\pi\sqrt{\dfrac{M}{K}}$

$K = \dfrac{YA}{L}$

$t_2 = \dfrac{2L}{v}$

4. First we find the center of mass of each cube. It is located by symmetry: (0.5,0.5,0.5), (1.5,0.5,0.5), (0.5,1.5,0.5), (0.5,0.5,1.5)

Now we find the center of mass by treating the COM of each cube as a point particle:

$$x_{COM} = \frac{0.5+1.5+0.5+0.5}{4} = 0.75$$

$$y_{COM} = \frac{0.5+0.5+1.5+0.5}{4} = 0.75\ ;$$

$$z_{COM} = \frac{0.5+0.5+0.5+1.5}{4} = 0.75$$

5. Optical path difference between (OPD) P & Q

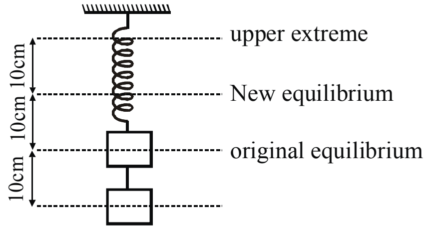
(O.P.D.) $= 2.25\,\lambda_0 \times 1 + (3.5\,\lambda_0) \times 2 + 3\lambda_0 \times 3$

$= 18.25\,\lambda_0$ and $\Delta\phi = \dfrac{2\pi}{\lambda_0} \times \Delta x = \dfrac{\pi}{2}$

6. As insect moves outwards its tangential speed (in ground frame) increases.

7. Time to move at upper extreme

$= \dfrac{T}{2} = \dfrac{\pi}{\omega} = \pi\sqrt{\dfrac{m}{k}} = \dfrac{\pi}{10}\text{sec}$



Distance moved by lower block

$= \dfrac{1}{2}gt^2 = \dfrac{1}{2} \times 10 \times \dfrac{\pi^2}{100} = 50cm$

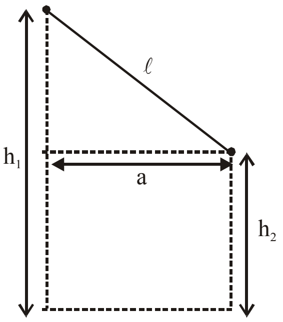
Seperation between the blocks $(20 + 10 + 50)$cm $= 80$ cm

8. $\frac{3}{2\ell}\sqrt{\frac{kx_1}{\mu}} = \frac{2}{2\ell}\sqrt{\frac{kx_2}{\mu}}$

9. Equating pressure at the surface on both vessel

$\rho_W g \times \frac{10}{100} = \rho_B g \times \frac{12}{100} \Rightarrow \rho_B = \frac{10}{12}\rho_W$

10.



relative vertical distance = $h_1 - h_2 = \sqrt{\ell^2 - a^2}$

Relative velocity = $e_1v_1 - e_2v_2 = 0.4\sqrt{2gH}$

Time = $\frac{5}{2}\sqrt{\frac{\ell^2 - a^2}{2gH}}$

11. $\Delta x = d \sin\theta = n\lambda$

$\frac{d\sin 45^\circ}{d\sin 30^\circ} = \frac{n\lambda}{n\left(\frac{\lambda}{\mu}\right)} \Rightarrow \mu = \sqrt{2}$

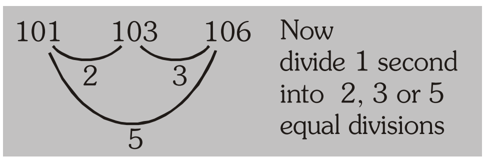
12. [ax] must be dimensionless, dim [a] = L^{-1} ; dim $[a^m e^{ax}] = L$

$\Rightarrow$ dim $(a^m) = L \Rightarrow m = -1$, dim [C] = L

13. For isothermal : $\Delta U = 0$

For adiabatic : $\Delta U = -\Delta W$

14.



$\frac{1}{2}$ $\frac{2}{2}$

$\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$

$\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{5}{5}$

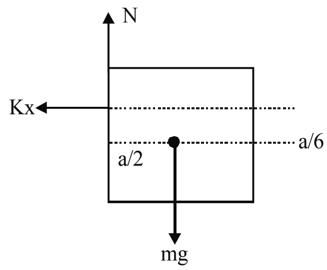
Eliminate common time instants. Total Maxima in one second 2 + 2 + 4 = 8

15. Bulk modulus $\beta = -\frac{\Delta P}{\frac{\Delta v}{v}} = \frac{100\times 10^3}{\frac{5\times 10^{-3}}{100}} = 2\times 10^9$

speed $v = \sqrt{\frac{\beta}{\rho}} = \sqrt{\frac{2\times 10^9}{10^3}} \approx 1414 \text{ m/s}$

16. As depth increases, density of water increases so rate of increase of pressure with depth will be positive.

17.



$\tau = 0,\ Kx\left(\frac{a}{6}\right) = \frac{Na}{2} \Rightarrow Kx\frac{a}{6} = mg\frac{a}{2} \Rightarrow x = \frac{3mg}{K}$

18. $\Delta\ell = \ell\alpha\Delta T = 2\text{mm}$

$\Delta\ell_{actual} = 2 - 1 = 1\text{mm}$

$\sigma = Y\left(\frac{\Delta\ell_{actual}}{\ell}\right)$

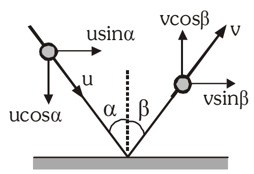
19. KE = TE $\propto pA^2\omega^2$

PE = TE $\propto qA^2$

$PA^2/\omega^2 = qA^2$

$T = 2\pi\sqrt{\frac{p}{q}}$

22. $v\sin\beta = u\sin\alpha$ and $e = \frac{v\cos\beta}{u\cos\alpha} \Rightarrow e = \frac{\tan\alpha}{\tan\beta}$



23. path difference $(\Delta x) = d\sin\theta + d\sin\phi$

so phase differnece = $k\Delta x$

for constructive interference phase difference should be integral multiple of 2π

24. $v_{average} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{ut_2 + \frac{1}{2}at_2^2 - \left(ut_1 + \frac{1}{2}at_1^2\right)}{t_2 - t_1}$

$= \frac{u(t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)(t_2 + t_1)}{t_2 - t_1}$

$= \frac{2u + a(t_2 + t_1)}{2} = \frac{(u + at_1) + (u + at_2)}{2} = \frac{v_1 + v_2}{2}$

$= \frac{\tan\theta_1 + \tan\theta_2}{2} = \frac{1+3}{2} = 2ms^{-1}$

OR

For uniformly acceleration motion

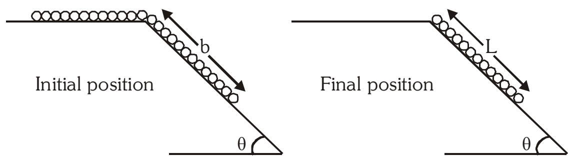
$v_{average} = \frac{v_1 + v_2}{2} = \frac{1+3}{2} = 2ms^{-1}$

25. $\frac{T}{4} = \frac{1}{f \times 4} = \frac{1}{256 \times 4} = 10^{-4} \times 9.8$ sec

$\frac{3T}{4} = 2.9 \times 10^{-3}$

$\frac{5T}{4} = 4.9 \times 10^{-3}$.

26. Loss in PE = gain in KE



$\Rightarrow mg\left(\frac{L}{2}\sin\theta\right) - \left(\frac{m}{L}b\right)g\left(\frac{b}{2}\sin\theta\right) = \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{\frac{g\sin\theta}{L}(L^2 - b^2)}$

27. $KE = \frac{1}{2}mV_C^2 + \frac{1}{2}I_C\omega^2$

$= \frac{1}{2}mV^2 + \frac{1}{2}mR^2\left(\frac{V^2}{R^2}\right) + \frac{1}{2}mV^2 +$

$\frac{1}{2}\frac{m(2R^2)}{12}\frac{V^2}{R^2}$

28. $T\sin\theta = m\omega^2\ell\sin\theta$

$T\cos\theta = mg$

$\mu(T\cos\theta + Mg) \geq T\sin\theta$

$\mu \geq \frac{m\tan\theta}{m + M}$

29. $M_1s(T_1 - T) + M_2s(T_2 - T) + M_3s(T_3 - T) = 0$

30. Suppose m kg steam is required per hour

Heat is released by steam in following three steps

(i) When 150°C steam $\xrightarrow{Q_1}$ 100°C steam

$Q_1 = mc_{steam}\Delta\theta = m \times 1\ (150–100) = 50$ m cal

(ii) When 150°C steam $\xrightarrow{Q_2}$ 100°C water

$Q_2 = mL_v = m \times 540 = 540$ m cal

(iii) When 100 °C water $\xrightarrow{Q_2}$ 90°C water

$Q_3 = mc_W\Delta\theta = m \times 1 \times (100 – 90) = 10$ m cal

Hence total heat given by the steam

$Q = Q_1 + Q_2 + Q_3 = 600$ mcal ....(i)

Heat taken by 10 kg water

$Q' = mc_W\Delta\theta = 10 \times 10^3 \times 1 \times (80–20)$

$= 600 \times 10^3$ cal

Hence $Q = Q' \Rightarrow 600\ m = 600 \times 10^3$

$\Rightarrow m = 10^3$ gm = 1 kg

31. $4NH_3(g) + 5O_2(g) \rightarrow 4NO(g) + 6H_2O(l)$

$4NO(g) + 2O_2(g) \rightarrow 4NO_2(g)$

To obtain maximum mass of NO_2 there should be no limiting reagent

	n_{NH_3} : n_{O_2}	
	4 : 7	
Mass ratio	4×17 : 7 × 32	
	17 : 56	

32. $E = \left[\frac{12400}{1240}\right]eV = 10eV$

W = 4 eV

$KE_{max} = 6\ eV = KE_{initial}$

On applying accelerating potential of 7.6V

ΔKE = charge × potential = 7.6 eV

$KE_f - KE_g = 7.6$ eV

$KE_{final} = 7.6\ eV + 6\ eV = 13.6$ eV

which is KE of electron in (n = 1) in hydrogen atom

$V = 2.188 \times 10^6$ m/s

33. $\Delta T_b = i \times K_b \times m$

$0.051 = 2 \times 0.51 \times \frac{1.685}{M} \times \frac{1000}{200}$

M = 168.5 which is CsCl

for CsCl $\frac{\sqrt{3}\,a}{2} = r^+ + r^-$

35. Each unit cell of 'A' atom contain

$8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$

Each unit cell of B atom contain $12 \times \frac{1}{4} = 3$

Each unit cell of C atom contain 1 × 1 = 1

Hence, A_4B_3C

37. $H_2(g) \rightarrow 2H^+_{anode} + 2e^-$

$2H^+_{cathode} + 2e^- \rightarrow H_2(g)$

$H_2(g) + 2H^+_{cathode} \rightarrow 2H^+_{anode} + H_2(g)$

$Q = \frac{[H^+]^2_{anode}}{[H^+]^2_{cathode}} = \frac{10^{-7}}{10^{-5}} = 10^{-2}$

n = 2

$E^o_{cell} = 0$

$E_{cell} = 0 - \frac{0.06}{2} \log 10^{-2} = 0.06$ V

38. $\lambda^\infty_{H_2O} = \lambda^\infty_{H^+} + \lambda^\infty_{OH^-} = 550$ Scm^2 eq^{-1}

$\lambda^\infty_{H_2O} = K \times \frac{1000}{C_{H^+}}$

$550 = 5.5 \times 10^{-7} \times \frac{1000}{C_{H^+}}$

$C_{H^+} = 10^{-6}$

pH = 6

39. Let mass of A_2 is X gram

B_2 is Y gram

X + Y = 10 gram

after 6 hr. $\frac{X}{8} + \frac{Y}{4} = 2$gram

X = 4 gram

Y = 6 gram

46. $_{58}$Ce : $4f^1$, $6d^1$, $5s^2$

$_{92}$U : $5f^6$, $6d^1$, $7s^2$

$_{90}$Th : $5f^0$, $6d^2$, $7s^2$

51. Order of basic strength is

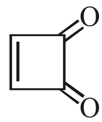
$\underbrace{CH_3–CH_2}_{+I}–NH_2 > \underbrace{Ph}_{-I}–CH_2–NH_2 > Ph–\ddot{N}H_2$ (delocalise l.p.)

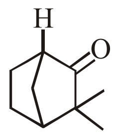
52. Nucleophilicity $\alpha \frac{1}{\text{Stability of anion}}$

order of stability of anion ⇒

$CH_3–S(=O)_2–O^\ominus > CH_3–S(=O)–O^\ominus > CH_3–O^\ominus > CH_3–CH_2^\ominus$

(3 equivalent R.S.) 2 equivalent R.S. E.N. (0 > C)

53. (i)  ⇒ Absence of α-hydrogen

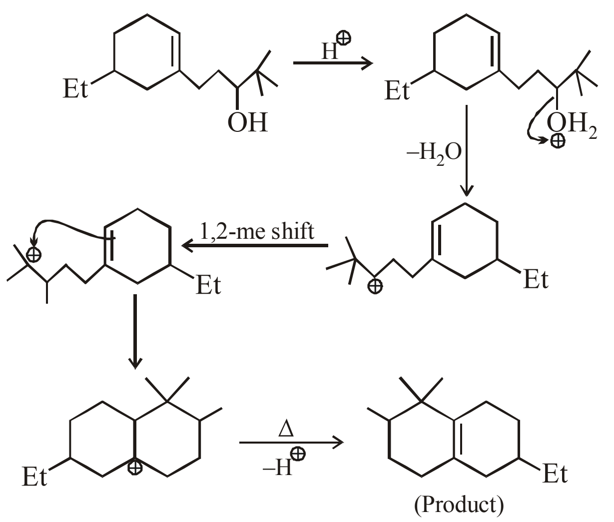
(ii)  ⇒ Due to Bredt's rule does not show Tautomerism

(iii) Ph–C(=O)–Ph ⇒ Absence of α-hydrogen

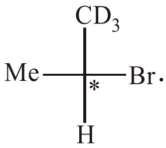
(iv) Ph–C(=O)–CH_3 ⇒ Show Tautomerism due to presence of α-hydrogen.

54. Compound I is 2-chlorobutan-1-ol and compound II is 3-chlorobutan-1-ol. Hence they are positional (Structural) isomers of each-other

55.

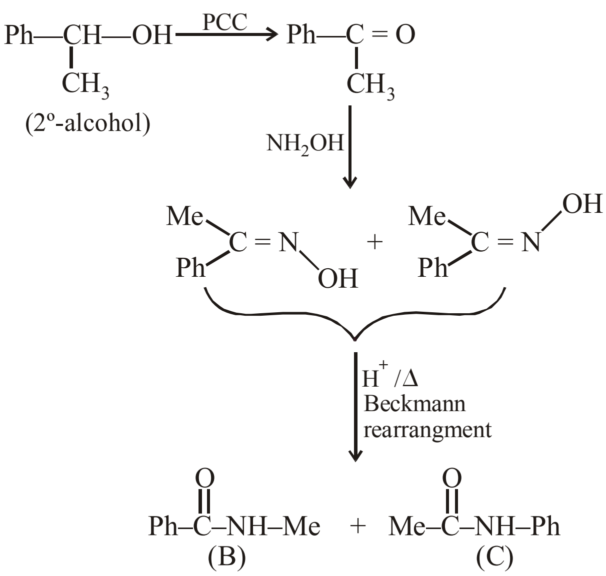


56. Chiral centre is absent in option (1), (2) and (4),

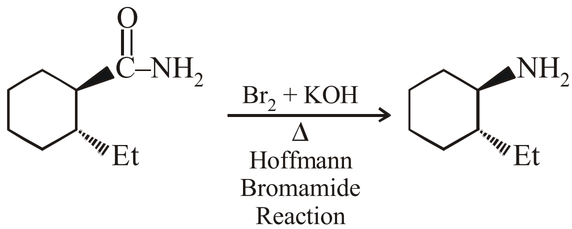
while in (3) chiral centre is present 

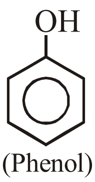
In S_N1 carbocation intermediate is formed hence both side attack of $Nu^{\ominus}$ take place. So R.M. is formed as product.

57.



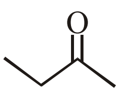
58.



59. 

(Phenol)

gives both bromine water and neutral $FeCl_3$ test

60. 

give both 2,4-DNP and I_2/NaOH test

61. Use length of LR $= \dfrac{2b^2}{a}$

$$= \left|2\left(\frac{4\sin^2\alpha}{2\cos\alpha}\right)\right| = \left|\frac{2(1-\cos 2\alpha)}{\cos\alpha}\right|$$

62. Only one arbitrary constant

$\Rightarrow$ order 1

$$2y\frac{dy}{dx} = \sqrt{c}$$

$$\therefore\ y^2 = 2y\frac{dy}{dx}\left(x + 8y^2\left(\frac{dy}{dx}\right)^2\right)$$

$\therefore$ degree = 3

63. Let $L_1 \equiv x + 4y - c = 0$

$\therefore$ Points $(c, 0)$, $\left(0, \dfrac{c}{4}\right)$, $(0, 0)$

area = 8 $\Rightarrow c^2 = 64 \Rightarrow c = 8$ or -8

$\therefore$ perimeter $= 8 + 2 + \sqrt{68} = 10 + \sqrt{68}$

64. $(2\hat{i} - \hat{j} + \hat{k}).((1-\lambda)\hat{i} + (3-\lambda)\hat{j} + (2\lambda - \lambda^2 - 3\lambda)\hat{k}) = 0$

$\Rightarrow 2 - 2\lambda - 3 + \lambda + 2\lambda - \lambda^2 - 3\lambda = 0$

$\Rightarrow -\lambda^2 - 2\lambda - 1 = 0 \quad \Rightarrow (\lambda + 1)^2 = 0$

$\Rightarrow \lambda = -1$

65. $\dfrac{\sec 8\theta - 1}{\sec 4\theta - 1}$

$$= \left(\frac{1-\cos 8\theta}{1-\cos 4\theta}\right)\frac{\cos 4\theta}{\cos 8\theta} = \left(\frac{2\sin^2 4\theta}{2\sin^2 2\theta}\right)\frac{\cos 4\theta}{\cos 8\theta}$$

$$= \frac{\tan 8\theta}{\tan 2\theta} = \frac{2\tan 4\theta}{(1-\tan^2 4\theta)\tan 2\theta}$$

Use $\tan 4\theta = \dfrac{2\tan 2\theta}{1-\tan^2 2\theta}$

66. $\dfrac{\sin(\pi\sin\theta)}{\cos(\pi\sin\theta)} = \dfrac{\cos(\pi\cos\theta)}{\sin(\pi\cos\theta)}$

$\Rightarrow \cos(\pi\cos\theta + \pi\sin\theta) = 0$

$\Rightarrow (\cos\theta + \sin\theta)\pi = \pm\dfrac{\pi}{2}$

$\Rightarrow \cos\theta + \sin\theta = \pm\dfrac{1}{2}$

$\Rightarrow \cos\left(\theta - \dfrac{\pi}{4}\right) = \pm\dfrac{1}{2\sqrt{2}}$

$\Rightarrow \cot\left(\theta - \dfrac{\pi}{4}\right) = \pm\dfrac{1}{\sqrt{7}}$

67. $(2+\cos x)\dfrac{dz}{dx} = -\sin x(-1+z)$

$$\frac{dz}{z-1} = \frac{-\sin x}{2+\cos x}dx$$

$\ell n|z-1| = \ell n|2+\cos x| + c$

$\because \quad z\left(\dfrac{\pi}{2}\right) = 3 \quad \Rightarrow \quad c = 0$

$\therefore \quad z\left(\dfrac{\pi}{3}\right) = \dfrac{5}{2}$

68. $\dfrac{\tan(\theta+\alpha)}{\tan(\theta-\alpha)} = \dfrac{3}{1}$

by using componendo and dividendo we get

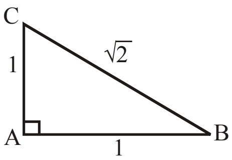
$\sin 2\theta = 2\sin 2\alpha$

this equation has no solution if $|\sin 2\alpha| > \dfrac{1}{2}$

69. $8.\dfrac{1}{2}bc\sin A = (b+c)(bc+1)$

$$4\sin A = (b+c)\left(1+\frac{1}{bc}\right) = b+\frac{1}{b}+c+\frac{1}{c}$$

it is possible if b = c = 1 = sinA



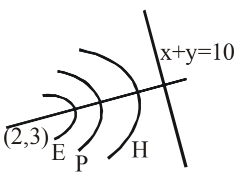
$R = \dfrac{1}{\sqrt{2}} = \sqrt{\Delta}$

70. $x = 2\displaystyle\int_0^y \frac{dt}{\sqrt{1+9t^2}} \Rightarrow \frac{dx}{dy} = \frac{2}{\sqrt{1+9y^2}}$

$$\frac{dy}{dx} = \frac{\sqrt{1+9y^2}}{2}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2}.\frac{1}{2}\frac{18y}{\sqrt{1+9y^2}}.\frac{\sqrt{1+9y^2}}{2} = \frac{9}{4}y$$

71.



72. $\displaystyle\sum_{n=1}^{\infty}\sin^n x + \sum_{n=1}^{\infty}\cos^n x = \frac{\sin x}{1-\sin x} + \frac{\cos x}{1-\cos x}$

$$= \frac{[(\sin x+\cos x) - \sin 2x]}{1-(\sin x+\cos x)+\sin x\cos x}$$

Put $\sin 2x = a^2 - 1$ & $\sin x + \cos x = a$

73. $g(x) = \dfrac{x}{(1+7x^7)^{1/7}}$

$$\therefore \quad I = \int\frac{x^6}{(1+7x^7)^{1/7}}dx$$

$$= \frac{1}{49}\int\frac{49x^6}{(1+7x^7)^{1/7}}dx$$

$$= \frac{1}{49}\int\frac{d(1+7x^7)}{(1+7x^7)^{1/7}}dx = \frac{1}{42}(1+7x^7)^{6/7} + C$$

74. Total elements (with repeation) in

$$\bigcup_{r=1}^{24} X_r = 5\times 24 = 120$$

but each element lie in $10x_r$'s

$\Rightarrow$ Total different elements = $\dfrac{120}{10} = 12$ ......(i)

Similarly from $\displaystyle\bigcup_{r=1}^{n} Y_r$

we get total different elements = $\dfrac{4n}{6}$ ......(ii)

from (i) and (ii), $12 = \dfrac{4n}{6} \quad \Rightarrow \quad n = 18$

75. $= \displaystyle\int_{-1}^{1}\frac{x^3}{(|x|+1)(|x|+3)} + \int_{-1}^{1}\frac{|x|+3}{(|x|+1)(|x|+3)}dx$

$$= 0 + 2\int_0^1 \frac{1}{(x)+1}dx = 2\ell n2$$

Use $\displaystyle\int_0^{\pi/2}\log(\sin\alpha)d\alpha = \int_0^{\pi/2}\log(\cos\alpha)d\alpha = -\frac{\pi}{2}\ell n2$

77. It is always true for $n \geq 7$.

78. For linearly dependent vectors

$[\vec{P}\ \vec{Q}\ \vec{R}] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0$$

$\Rightarrow \beta = 1$, α can be any real number.

79. Converse of $p \to q$, is $q \to p$

80. Point $(4, \sin\alpha, 1)$ lie in plane

$\Rightarrow\ 8 - \sin\alpha\sin\beta + \cos\beta = k$ ....(1)

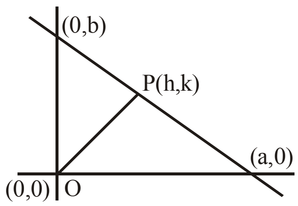
Line lie in plane

$\Rightarrow\ \sin\beta - \sin\beta + \cos\alpha\cos\beta = 0$ .....(2)

(1) & (2) are satisfied by all real values of α

$\therefore\ \cos\beta = 0 \quad \Rightarrow\ k = 8 + \sin\alpha$

81. $OP = \left|\dfrac{1}{\sqrt{\dfrac{1}{a^2}+\dfrac{1}{b^2}}}\right|$



$\Rightarrow\ h^2 + k^2 = 2c^2$

$\Rightarrow\ x^2 + y^2 = 2c^2$

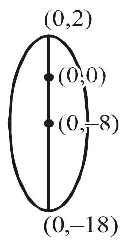
82. $x(y + z) = 0$

$x = 0$ or $y + z = 0$

$\therefore$ there are two perpendicular planes.

83. $b = 10$

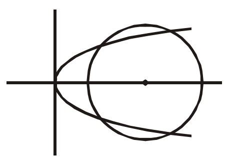
& $be = 8 \Rightarrow e = \dfrac{4}{5}$ and $a = 6$



let ellipse $\dfrac{x^2}{a^2} + \dfrac{(y+8)^2}{b^2} = 100$

$b = 10$ & $a = 6$

84. A circle can intersect a parabola at 4 points



85. Cicumcentre $(-1, 2)$

radius $= \dfrac{2}{3}$(length of median)$= \dfrac{4b}{3}$

$\therefore$ Circle $(x + 1)^2 + (y - 2)^2 = \dfrac{16b^2}{9}$

86. If $y = mx + c$ is normal to parabola $y^2 = 4ax$

the $c = -2am - am^3$

$$\Rightarrow\ -\frac{c}{b} = 2(3)\left(-\frac{1}{b}\right) - (3)\left(-\frac{1}{b}\right)^3$$

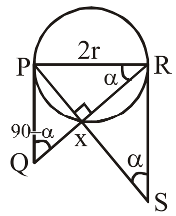
$$\Rightarrow\ -\frac{c}{b} = \frac{6}{b} + \frac{3}{b^3} \quad \Rightarrow\ (6 + c)b^2 + 3 = 0$$

$$\Rightarrow\ b^2 = \frac{-3}{6+c}$$

it is possible $\Rightarrow\ 6 + c < 0$

$c < -6$

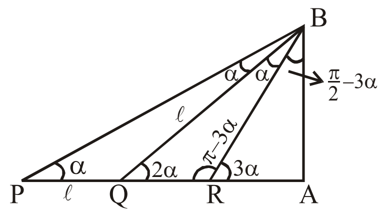
87. $\dfrac{PQ}{RP} = \dfrac{PR}{RS}$



$PQ.RS = (PR)^2$

$= (PX)^2 + (RX)^2$

88.



$AB = RB \sin 3\alpha$

$= \dfrac{1}{\sin 3\alpha}.\sin 2\alpha.\sin 3\alpha$ (use sine rule in ΔBQR)

$= \ell \sin 2\alpha = \ell \sin\beta$

89. $A = \displaystyle\int_{-1}^{1/\sqrt{2}} \cos^{-1} x dx + \int_{1/\sqrt{2}}^{1} \sin^{-1} x dx$

$$= \int_{-1}^{1/\sqrt{2}} \frac{\pi}{2} dx - \int_{-1}^{1/\sqrt{2}} \sin^{-1} x dx + \int_{1/\sqrt{2}}^{1} \sin^{-1} x dx$$

$$= \frac{3\pi}{2} - \sqrt{2}$$

90. Statement 3 and 2 are true.

$$h(-x) = \int_0^{-x} g(t) dt$$

put $-t = u$

we get $h(-x) = \displaystyle\int_0^{x} g(u) du = h(x)$