JEE (MAIN) : ENTHUSIAST COURSE

TARGET : JEE (MAIN) 2015

TEST # 07 (SCORE – I) | **DATE : 06 - 01 - 2015**

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	4	2	3	4	1	4	3	2	3	4	4	3	4	4	3	2	4	1	1	3
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	2	1	3	4	2	3	1	3	4	2	1	4	2	2	4	3	3	4	2	2
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	2	4	1	1	4	1	2	4	4	3	4	4	1	4	4	3	4	2	4	3
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	2	4	3	2	4	2	1	3	3	1	3	2	3	4	4	1	4	2	4	4
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	4	3	3	2	2	2	3	2	4	3										

HINT – SHEET

5. $\dfrac{A_1T_1^4}{A_2T_2^4} = 1$

6. $Q \propto R^2T^4$

7. $H = k4\pi r^2 \dfrac{dT}{dr}$ = constant

8. $\dfrac{s\left(A_{sun}\right)T^4}{4\pi r^2} \times \pi r_0^2$

9. $\dfrac{k_1\left(T_1 - T\right)}{\ell_1} = \dfrac{k_2\left(T - T_2\right)}{\ell_2}$

10. in previous question $k_1 = k_2 = k$ and $\ell_1 = x$ and $\ell_2 = \ell - x$

$\Rightarrow \theta = -mx + C$

11. $\sqrt{\dfrac{3RT_{H_2}}{M_{H_2}}} = \sqrt{\dfrac{3RT_{O_2}}{M_{O_2}}}$

14. $ms\Delta T$

17. For adiabatic process,

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$T_1 (L_1A)^{\frac{5}{3}-1} = T_2 (L_2A)^{\frac{5}{3}-1}$

$\dfrac{T_1}{T_2} = \left(\dfrac{L_2}{L_1}\right)^{\frac{2}{3}}$

18. dR = dU + dW

$5 = 0 + 10 (2 - 1) + 0 + W_{CA}$

$W_{CA} = 5 - 10$

$= -5$ Joule

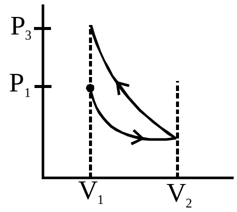
19. At constant temperature.

$\dfrac{dP}{dV} = -\dfrac{P}{V}$

$\dfrac{-dV/dP}{V} = \dfrac{1}{P}$

$\beta \propto \dfrac{1}{P}$

20.



$P_3 > P_1$

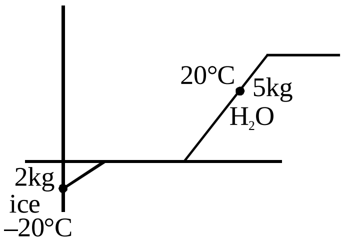
$W < 0$

21. $\lambda_{m_1} < \lambda_{m_3} < \lambda_{m_2}$

But $\lambda_m \propto \frac{1}{T}$

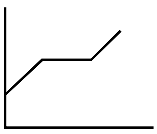
$T_1 > T_3 > T_2$

22.



$Q_1 = 2000 \times 0.5 \times 20 + 2000 \times 80$
$= 180000$
$Q_2 = 5000 \times 1 \times 20$
$= 100000$
T = 0°C
Q = mL
80000 = m × 80
m = 1000 gm
= 1 kg
Ice = 2 – 1 = 1 kg
$H_2O = 5 + 1 = 6$ kg

23. Graph correct represents is



24. $$\frac{dH}{dt_{parallel}} = (KA + KA)\frac{(T_1 - T_2)}{\ell}$$

$$= \frac{2KA(T_1 - T_2)}{\ell}$$

$$\frac{dH}{dt_{series}} = \frac{KKA(T_1 - T_2)}{K\ell + K\ell} = \frac{K^2A(T_1 - T_2)}{2K\ell}$$

$$\frac{dH/dt_{series}}{dH/dt_{parallel}} = \frac{1}{4}$$

28. Approaching source n' = n $\left[\frac{V}{V - V_S}\right]$ constant

Receading source n' = $\left[\frac{V}{V + V_S}\right]$ constant.

29. (a) True
(b) String become straight
(c) amplitude depends on position.
(d) Nor for every plane but for node plane only
(e) Nodes

30. $$\frac{T}{4} = \frac{1}{t \times 4} = \frac{1}{256 \times 4} = 10^{-4} \times 9.8 \text{ sec}$$

$\frac{3T}{4} = 2.9 \times 10^{-3}$

$\frac{5T}{4} = 4.9 \times 10^{-3}$.

61. $$\int \left(\frac{ae^3}{bc}\right)^x dx = \frac{\left(\frac{ae^3}{bc}\right)^x}{\log\left(\frac{ae^3}{bc}\right)} + K$$

$\therefore$ P = $\log\left(\frac{ae^3}{bc}\right)$

= log ae^3 – log bc
= log a + 3 – log bc

62. I = $\frac{1}{2}\int \sin 2x \cos 2x \cos 4x \cos 8x \, dx$

$= \frac{1}{4}\int \sin 4x \cos 4x \cos 8x \, dx$

$= \frac{1}{8}\int \sin 8x \cos 8x \, dx = \frac{1}{16}\int \sin 16x \, dx$

$= \frac{-1}{256}\cos 16x + C$

63. I = $\int e^x \left(\frac{1 - 2\sin\frac{x}{2}\cos\frac{x}{2}}{2 - \sin^2\frac{x}{2}}\right) dx$

$= \int e^x \left(\frac{1}{2}\text{cosec}^2\frac{x}{2} - \cot\frac{x}{2}\right) dx$

$= -\int e^x \left(\cot\frac{x}{2} - \frac{1}{2}\text{cosec}^2\frac{x}{2}\right) dx$

$= -e^x \cdot \cot\frac{x}{2} + C$

64. Consider $\frac{x^2}{(x^2+1)(x^2+4)}$ and put $x^2 = y$

Then $\frac{x^2}{(x^2+1)(x^2+4)} = \frac{y}{(y+1)(y+4)}$

Write $\frac{y}{(y+1)(y+4)} = \frac{A}{(y+1)} + \frac{B}{(y+4)}$

so that $y = A(y + 4) + B(y + 1)$

$\therefore \ A = -\frac{1}{3}$ & $B = \frac{4}{3}$

$\therefore \ I = \int \frac{-\frac{1}{3}}{(x^2+1)} dx + \int \frac{4/3}{(x^2+4)} dx$

$= -\frac{1}{3}\tan^{-1} x + \frac{2}{3}\tan^{-1}\left(\frac{x}{2}\right) + C$

65. $I = \int_1^2 \underset{II}{e^{2x}} . \underset{I}{\frac{1}{x}} dx - \int_1^2 e^{2x} . \frac{\ell nx}{2x^2} dx$

$= \left(\frac{1}{x}.\frac{e^{2x}}{2}\right)_1^2 - \int_1^2 -\frac{1}{x^2}.\frac{e^{2x}}{2} dx - \int_1^2 e^{2x}.\frac{1}{2x^2} dx$

$I = \left(\frac{e^4}{4} - \frac{e^2}{2}\right) = \frac{e^2(e^2-2)}{4}$

66. $\int_1^e \underset{I}{\tan^{-1}} x . \underset{II}{\frac{1}{x}} dx + \int_1^e \frac{\ell n\ x}{(1+x^2)} dx$

$= (\tan^{-1}x.\ell nx)_1^e - \int_1^e \frac{1}{(1+x^2)} \ell nx\ dx + \int_1^e \frac{\ell nx}{(1+x^2)} dx$

$= \tan^{-1}(e) . \ell ne - \tan^{-1}(1) . \ell n1$

$= \tan^{-1}(e)$

67. $I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log\left\{\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)\right\} dx$

Putting $\left(x+\frac{\pi}{4}\right) = \theta$; $dx = d\theta$

$= \int_0^{\frac{\pi}{2}} \log\left(\sqrt{2}\sin\theta\right)d\theta$

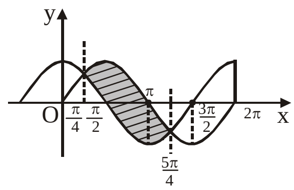
$= \int_0^{\frac{\pi}{2}} \log\sqrt{2}\ d\theta + \int_0^{\frac{\pi}{2}} \log\sin\theta\ d\theta$

$= \log\sqrt{2}\ (\theta)_0^{\pi/2} - \frac{\pi}{2}\ell n2$

$= \frac{\pi}{4}\ell n2 - \frac{\pi}{2}\ell n2 = -\frac{\pi}{4}\ell n2$

68. The point of intersection of $y = \sin x$ and $y=\cos x$ are $x=\frac{\pi}{4}, \frac{5\pi}{4}$ and $\sin x \ge \cos x$ in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

$\therefore$ Area $= \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\ dx$



$= \left(-\cos x - \sin x\right)_{\pi/4}^{5\pi/4}$

$= -\left(\cos x + \sin x\right)_{\pi/4}^{5\pi/4}$

$= -\left(\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right)$

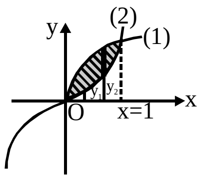
$= -(-\sqrt{2} - \sqrt{2}) = 2\sqrt{2}$

69. The curve $y = \sqrt{x}$ is $y^2 = x$; $(y \ge 0)$ ... (i)

and $y = x^3$ ... (ii)

Point of intersection are (0, 0) & (1, 1)

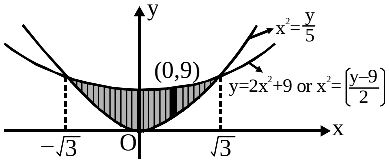
$\therefore$ Required area $= \int_0^1 (y_2 - y_1)\ dx$



$= \int_0^1 (\sqrt{x} - x^3)\ dx$

$= \frac{5}{12}$

70.



By symmetry

$$\text{Area} = 2\int_0^{\sqrt{3}} \{(2x^2+9)-5x^2\}\, dx$$

$$= 2\int_0^{\sqrt{3}} (9-3x^2)\, dx = 2\left(9x - 3\frac{x^3}{3}\right)_0^{\sqrt{3}}$$

$= 2[9\sqrt{3} - 3\sqrt{3}] = 12\sqrt{3}$ sq. units.

71. We have $x = e^{xy\, dy/dx}$

$$\Rightarrow \log x = xy\frac{dy}{dx} \Rightarrow y\, dy = \frac{\log x}{x}\, dx$$

On integration, we get

$$\frac{y^2}{2} = \frac{(\log_e x)^2}{2} + C$$

$\Rightarrow y^2 = (\log_e x)^2 + 2C$

Hence $y = \pm\sqrt{(\log_e x)^2 + 2C}$

72. Re-arranging the equation ; we have

$$dx - y\, dy + \sqrt{x^2+y^2}\,(xdx + ydy) = 0$$

$$\Rightarrow dx - ydy + \frac{1}{2}\sqrt{x^2+y^2}\, d(x^2+y^2) = 0$$

$$\Rightarrow x - \frac{y^2}{2} + \frac{1}{3}(x^2+y^2)^{3/2} = C$$

73. $(cy + d)dy = (ax + b)\, dx$

$\frac{cy^2}{2} + dy = \frac{ax^2}{2} + bx + K$ (K being the constant of integration)

The equation represents a parabola

If $c = 0,\ a \neq 0$ or $a = 0,\ c \neq 0$

74. Writing $P = \frac{dy}{dx}$ and differentiating w.r.t. x ; we have

$$P = 2P + 2x\frac{dP}{dx} + 2xP^4 + 4P^3x^2\frac{dP}{dx}$$

$$\Rightarrow 0 = P(1 + 2xP^3) + 2x\frac{dP}{dx}(1 + 2P^3x)$$

$$\Rightarrow P + 2x\frac{dP}{dx} = 0 \Rightarrow 2\frac{dP}{P} = -\frac{dx}{x}$$

$\Rightarrow 2\log P + \log x = \text{constant.}$

$\Rightarrow P^2x = C \Rightarrow P = \sqrt{C/x}$

Substituting this value in the given equation, we get $y = 2\sqrt{Cx} + C^2$

75. Put $\sin\phi = t$; then $\cos\phi\, d\phi = dt$

$$I = \int \frac{(3t-2)dt}{5-(1-t^2)-4t}$$

$$I = \int \frac{3t-2}{t^2-4t+4}dt = \int \frac{3t-2}{(t^2-2)^2}dt$$

$$\frac{3t-2}{(t-2)^2} = \frac{A}{(t-2)} + \frac{B}{(t-2)^2}$$

$(3t - 2) = A(t - 2) + B$

$\therefore\ A = 3,\ B = 4$

$$\therefore\ I = \int \frac{3}{t-2}dt + \int \frac{4}{(t-2)^2}dt$$

$$= 3\log|(\sin\phi - 2)| + \frac{-4}{(t-2)} + C$$

$$= 3\log(2 - \sin\phi) + \frac{4}{(2-\sin\phi)} + C$$

76. $$\int_0^9 [\sqrt{x}]\, dx + \int_0^9 2\, dx$$

$$= \int_0^1 [x]\, dx + \int_1^4 [\sqrt{x}]\, dx + \int_4^9 [\sqrt{x}]\, dx + 2(x)_0^9$$

$$= 0 + \int_0^1 1\, dx + \int_4^9 2\, dx + 18$$

$= 3 + 2.5 + 18 = 31$

77. $(2^x + 2^{-x})(3^x - 3^{-x})$ is an add function so ;

$$\int_{-4}^{4}(2^x + 2^{-x})(3^x - 3^{-x})\,dx = 0$$

80. $\int_0^{\pi/2} f'(x)\,dx = [f(x)]_0^{\frac{\pi}{2}} = f\left(\frac{\pi}{2}\right) - f(0)$

$$f(\pi/2) - f(0) = \begin{vmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{vmatrix} - \begin{vmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix} = 0$$

81. $I_2 = \int_1^{\operatorname{cosec}\theta} \frac{1}{x(x^2+1)}\,dx$

Put $x = \frac{1}{t}$; $dx = -\frac{1}{t^2}\,dt$

$$I_2 = \int_1^{t=\sin\theta} \frac{\frac{-1}{t^2}dt}{\frac{1}{t}\left(\frac{1}{t^2}+1\right)}$$

$$= \int_1^{\sin\theta} \frac{-t}{t(1+t^2)}\,dt = -\int_1^{\sin\theta} \frac{t}{(1+t^2)}\,dt$$

$I_2 = -I_1$

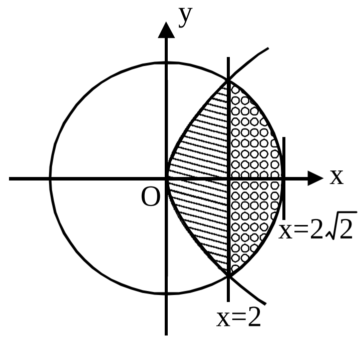
$\therefore\ I_1 + I_2 = 0$

$$\Delta = \begin{vmatrix} I_1 & I_1^2 & I_2 \\ e^0 & I_2^2 & -1 \\ 1 & I_1^2 + I_2^2 & -1 \end{vmatrix}$$

Apply $C_1 \to C_1 + C_3$; we get

$$\Delta = \begin{vmatrix} 0 & I_1^2 & I_2 \\ 0 & I_2^2 & -1 \\ 0 & I_1^2 + I_2^2 & -1 \end{vmatrix} = 0$$

87.



required area $= 2\int_0^2 \sqrt{2x}\,dx + 2\int_2^{2\sqrt{2}} \sqrt{8-x^2}\,dx$

$$= \left(2\pi + \frac{4}{3}\right).$$

88 $\int e^x \left\{\frac{(x-1)-2}{(x-1)^3}\right\}dx$

$$= \int e^x \left\{\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right\}dx$$

$$= e^x \frac{1}{(x-1)^2} + c$$