LEADER COURSE (PHASE – ELS, ELC, ELD & ELP)

TARGET : JEE (MAIN) 2015

SCORE – I | **DATE : 05 - 03 - 2015**

MAJOR TEST | **Test Pattern : JEE (Main)**

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	3	3	1	3	3	3	3	2	1	4	3	2	3	2	2	2	3	4	1	2
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	3	4	2	2	2	1	3	2	3	2	4	1	2	1	2	3	1	4	4	1
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	2	1	2	2	1	4	4	3	2	4	4	4	2	1	3	3	3	2	2	4
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	3	1	4	1	2	1	4	4	4	1	4	4	1	2	2	3	2	1	4	3
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	3	3	1	1	2	1	4	4	1	2										

HINT – SHEET

1. $= \cos^2 5° + \cos^2 10° + ... + \cos^2 85° + \cos^2 90°$

$= 8 + \frac{1}{2} = \frac{17}{2}$

2. $\tan^{-1}\left[\frac{2\sin 1\cos 1 - \cos^2 1 - \sin^2 1}{\cos^2 1 - \sin^2 1}\right]$

$= -\tan^{-1}\frac{\cos 1 - \sin 1}{\cos 1 + \sin 1}$

$= -\tan^{-1}\tan\left[\frac{\pi}{4} - 1\right]$

$= 1 - \frac{\pi}{4}$

4. Here statements

$p \to (p \leftrightarrow q)$ and $(q \to p) \to (p \to q)$ both are false only when p is true and q is false otherwise they are true.

5. Here, given observatios are a, a ...... n times, –2a, –2a ......2n times

No. of observations = 3n

mean $(\bar{X}) = \frac{n \times a + 2n \times (-2a)}{3n} = -a$

$\therefore$ Mean deviation about mean $= \frac{\Sigma|x_i - \bar{x}|}{3n}$

$\frac{n \times 2a + 2n \times a}{3n} = \frac{4a}{3}$

6. $\tan^{-1}\frac{1}{1+n(n+1)} = \tan^{-1}\frac{n+1-n}{1+n(n+1)}$

$= \tan^{-1}(n+1) - \tan^{-1}(n)$

so that L.H.S. of the given eq uation is

$\tan^{-1} 2 - \tan^{-1} 1 + \tan^{-1} 3 - \tan^{-1} 2 + \ldots + \tan^{-1}(n+1) - \tan^{-1} n.$

$= \tan^{-1}(n+1) - \tan^{-1} 1$

$= \tan^{-1}\frac{n+1-1}{1+(n+1)} = \tan^{-1}\frac{n}{n+2}$

so that $\tan^{-1}\frac{n}{n+2} = \tan^{-1}\theta \Rightarrow \theta = \frac{n}{n+2}$

7. $5\cos^2\theta - 3\sin^2\theta + 6\sin\theta\cos\theta = 7$

$$5\left(\frac{1+\cos 2\theta}{2}\right) - 3\left(\frac{1-\cos 2\theta}{2}\right) + 3\sin 2\theta = 7$$

$4\cos 2\theta + 3\sin 2\theta = 6$

but $4\cos 2\theta + 3\sin 2\theta \le \sqrt{4^2+3^2} = 5$

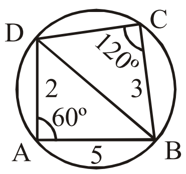
$\therefore$ Solution does not exist.

8. Weighted A.M. (भारित माध्य)

$$= \frac{2\times 60 + 1\times 70 + 1\times 70 + 2\times 80}{2+1+1+2} = 70$$

9. In ΔABD,

$$\cos 60^\circ = \frac{2^2+5^2-BD^2}{2(3)(5)}$$



$\Rightarrow BD^2 = 19$

Now, in ΔBCD

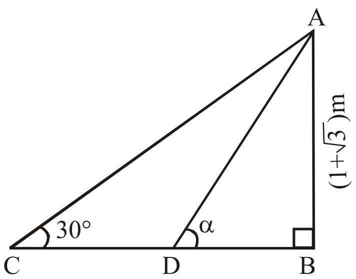
$$\cos 120^\circ = \frac{CD^2+9-19}{2(3)(CD)}$$

$\Rightarrow CD^2 + 3CD - 10 = 0$

$\Rightarrow CD = -5, 2$

$\Rightarrow CD = 2 \qquad (\because CD \ne -5)$

12. Let AB is a tower of height $(1+\sqrt{3})$ m ; BC and BD are its shadows. When the sun's elevation are 30° and α (say) respectively and CD = 2m.



$\therefore$ In triangle ABC,

$2 + BD = (1+\sqrt{3})\cot 30^\circ$

or $\quad 2 + BD = (1+\sqrt{3})\sqrt{3}$

or $\quad BD = (\sqrt{3}+3) - 2 = \sqrt{3}+1$

Now, in triangle ABD

$$\tan\alpha = \frac{AB}{BD} = \frac{1+\sqrt{3}}{\sqrt{3}+1} = 1 = \tan 45^\circ$$

$\therefore \alpha = 45^\circ$

13. Since, $\quad \vec{a}\cdot(\vec{b}+\vec{c}) = 0$

$\Rightarrow \quad \vec{a}\cdot\vec{b} + \vec{c}\cdot\vec{a} = 0$

Similarly, $\quad \vec{b}\cdot\vec{c} + \vec{a}\cdot\vec{b} = 0$

and $\quad \vec{c}\cdot\vec{a} + \vec{b}\cdot\vec{c} = 0$

On adding Eqs. (i), (ii) and (iii), we get

$$2\left(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}\right) = 0$$

Now, $\quad |\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$

$= 9 + 16 + 25 = 50$

$\Rightarrow \quad |\vec{a}+\vec{b}+\vec{c}| = 5\sqrt{2}$

14. Given,

$$\frac{(\vec{b}\cdot\vec{a})\vec{a}}{|\vec{a}|^2} = \frac{4}{3}(\hat{i}-\hat{j}-\hat{k})$$

$$\Rightarrow \frac{\left\{(\lambda\hat{i}-3\hat{j}+\hat{k})\cdot(\hat{i}-\hat{j}-\hat{k})\right\}(\hat{i}-\hat{j}-\hat{k})}{(1+1+1)} = \frac{4}{3}(\hat{i}-\hat{j}-\hat{k})$$

$\Rightarrow (\lambda + 3 - 1)(\hat{i}-\hat{j}-\hat{k}) = 4(\hat{i}-\hat{j}-\hat{k})$

$\Rightarrow (\lambda + 2)(\hat{i}-\hat{j}-\hat{k}) = 4(\hat{i}-\hat{j}-\hat{k})$

On equating the coefficient of $\hat{i}$, we get

$\lambda + 2 = 4 \Rightarrow \lambda = 2$

15. $$\vec{a}\cdot\vec{b}_1 = \vec{a}\cdot\left(\vec{b} - \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}\right)$$

$$= \vec{a}\cdot\vec{b} - \frac{|\vec{a}|^2(\vec{b}\cdot\vec{a})}{|\vec{a}|^2}$$

$$= \vec{a}\cdot\vec{b} - \vec{b}\cdot\vec{a} = 0$$

Similarly, $\quad \vec{a}\cdot\vec{c}_2 = \vec{b}_1\cdot\vec{c}_2 = 0$

Hence, $\{\vec{a}, \vec{b}_1, \vec{c}_2\}$ are mutually orthogonal vectors.

16. Given, $2\vec{a}+3\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow\ 2\vec{a}+3\vec{b}=-\vec{c}$

Taking cross product with $\vec{a}$ and $\vec{b}$ respectively, we get

$2(\vec{a}\times\vec{a})\ +\ 3(\vec{a}\times\vec{b})\ =-\ \vec{a}\times\vec{c}$

$\Rightarrow\quad 3(\vec{a}\times\vec{b})=\vec{c}\times\vec{a}$ ..(i)

and $\quad 2(\vec{b}\times\vec{a})+3(\vec{b}\times\vec{b})=-\vec{b}\times\vec{c}$

$\Rightarrow\quad 2(\vec{a}\times\vec{b})=\vec{b}\times\vec{c}$ ..(ii)

Now, $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$

$=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+3(\vec{a}\times\vec{b})$ [using Eq. (i)]

$=4\ (\vec{a}\times\vec{b})+\vec{b}\times\vec{c}$

$=2\ (\vec{b}\times\vec{c})+\vec{b}\times\vec{c}$ [using Eq. (ii)]

$=3(\vec{b}\times\vec{c})$

17. Let angle between $\vec{a}$ and $\vec{b}$ be θ_1, $\vec{c}$ and $\vec{d}$ be θ_2 and $\vec{a}\times\vec{b}$ and $\vec{c}\times\vec{d}$ be θ.

Since, $\quad(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=1$

$\Rightarrow \sin\theta_1\cdot\sin\theta_2\cdot\cos\theta=1\ (\because\ |\vec{a}|=|\vec{b}|=|\vec{c}|=|\vec{d}|=1)$

$\Rightarrow \theta_1=90°,\ \theta_2=90°,\ \theta=0°$

$\vec{a}\perp\vec{b},\ \vec{c}\perp\vec{d},\ (\vec{a}\times\vec{b})\,||\,(\vec{c}\times\vec{d})$

So, $\vec{a}\times\vec{b}=k\ (\vec{c}\times\vec{d})$ and

$(\vec{a}\times\vec{b}).\vec{c}=k(\vec{c}\times\vec{d}).\vec{c}$

$(\vec{a}\times\vec{b}).\vec{d}=k(\vec{c}\times\vec{d}).\vec{d}$

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}]=[\vec{a}\ \vec{b}\ \vec{d}]\ =0$

$\Rightarrow \vec{a},\vec{b},\vec{c}$ and $\vec{a},\vec{b},\vec{d}$ are coplanar vectors so option (1) and (2) are incorrect.

Let $\vec{b}\,||\,\vec{d}\Rightarrow\vec{b}=\pm\vec{d}$

As $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=1\Rightarrow(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{b})=\pm1$

$\Rightarrow [\vec{a}\times\vec{b}\,\vec{c}\,\vec{b}]=\pm1$

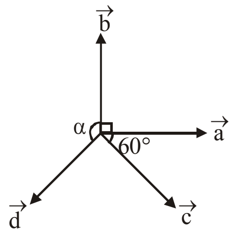
$\Rightarrow \vec{c}[\vec{b}\times(\vec{a}\times\vec{b})]=\pm1$

$\Rightarrow\ \vec{c}\cdot[\vec{a}-(\vec{b}\cdot\vec{a})\vec{b}]=\pm1$

$\Rightarrow\ \vec{c}\cdot\vec{a}=\pm1$

which is a contradiction so option (3) is correct.

Let option (4) is correct



$\Rightarrow\ \vec{d}=\pm\vec{a}$ and $\vec{c}=\pm\vec{b}$

As $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=1$

$\Rightarrow (\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{a})=\pm1$

which is a contradiction so option (4) is incorrect.

Alternate option (3) and (4) may be observed from the given in figure.

18. Given, $\quad(\vec{a}-\lambda\vec{b})\cdot(\vec{b}-2\vec{c})\times(\vec{c}+2\vec{a})=0$

$\Rightarrow (\vec{a}-\lambda\vec{b})\cdot\{\vec{b}\times\vec{c}+\vec{b}\times 2\vec{a}-4(\vec{c}\times\vec{a})\}\ =0$

$\Rightarrow \vec{a}\cdot(\vec{b}\times\vec{c})+\vec{a}\cdot(\vec{b}\times2\vec{a})-\vec{a}\cdot4(\vec{c}\times\vec{a})$

$-\lambda\ \vec{b}\cdot(\vec{b}\times\vec{c})-\lambda\vec{b}\cdot(\vec{b}\times2\vec{a})+4\lambda\vec{b}\cdot(\vec{c}\times\vec{a})=0$

$\Rightarrow\quad \vec{a}\cdot(\vec{b}\times\vec{c})+4\lambda\vec{b}\cdot(\vec{c}\times\vec{a})=0$

$\Rightarrow\quad \{\vec{a}\cdot(\vec{b}\times\vec{c})\}\,(1+4\lambda)=0$

$\Rightarrow\quad \lambda=-\dfrac{1}{4}\quad [\because\ \vec{a}\cdot(\vec{b}\times\vec{c})\neq0,\text{ given}]$

19. $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&1\\1&1&0\end{vmatrix}$

$=-\hat{i}+\hat{j}$

$\Rightarrow (\vec{a}\times\vec{b})\times\vec{c}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-1&1&0\\1&0&0\end{vmatrix}=-\hat{k}$

Now, $\quad\lambda\vec{a}+\mu\vec{b}=\lambda\ (\hat{i}+\hat{j}+\hat{k})+\mu(\hat{i}+\hat{j})$

$=(\lambda+\mu)\,\hat{i}+(\lambda+\mu)\,\hat{j}+\lambda\hat{k}$

$\because\quad \lambda\vec{a}+\mu\vec{b}=(\vec{a}\times\vec{b})\times\vec{c}$

$\Rightarrow\quad (\lambda+\mu)\,\hat{i}+(\lambda+\mu)\,\hat{j}+\lambda\,\hat{k}=-\hat{k}$

On equating the coefficient of $\hat{i}$, we get

$\lambda+\mu=0$

20. Any point on the line

$$\frac{x-1}{3}=\frac{y+2}{4}+\frac{z-3}{-2}=k$$

is (3k + 1,4k – 2, –2k + 3).

If the given line intersect the plane 2x–y+3z–1= 0, then any point on the line lies in the plane.

$\therefore$ 2(3k + 1) – (4k – 2) + 3(–2k + 3) – 1 = 0

$\Rightarrow$ k = 3

$\therefore$ Point is (9 + 1, 12 – 2, – 6 + 3) ie, (10, 10, – 3).

21. Equation of any plane passing through given line is

a (x – 1) + b (y + 1) + c (z – 3) = 0 ..(i)

Above plane is perpendicular to the plane

x + 2y + z = 12

$\therefore$ a + 2b + c = 0

Also, normal to the plane is perpendicular to the line

$\therefore$ 2a – b + 4c = 0

$$\Rightarrow \frac{a}{8+1}=\frac{b}{2-4}=\frac{c}{-1-4}$$

$$\Rightarrow \frac{a}{9}=\frac{b}{-2}=\frac{c}{-5}$$

$\therefore$ 9 (x – 1) – 2 (y + 1) – 5 (z – 3) = 0

$\Rightarrow$ 9x – 2y – 5z + 4 = 0

$\therefore$ a = 9, b = –2, c = – 5

22. Equation of plane passing through P (3, 8, 2) and parallel to 3x + 2y – 2z + 15 = 0 is

3(x – 3) + 2(y – 8) – 2 (z – 2) = 0

$\Rightarrow$ 3x + 2y – 2z – 21 = 0 ..(i)

Given line is

$$\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-2}{3}=r$$

Any point on the line is

Q(2r + 1, 4r + 3, 3r + 2)

This point is lies on the above plane.

$\therefore$ 3(2r+1) + 2 (4r+3) – 2 (3r+2) – 21 = 0

$\Rightarrow$ 8r – 16 = 0 $\Rightarrow$ r = 2

$\therefore$ Coordinate of Q (5, 11, 8)

$\therefore$ Distance between P and Q

$$= \sqrt{(5-3)^2+(11-8)^2+(8-2)^2}$$

$$= \sqrt{4+9+36}=7$$

23. Given, $\frac{x^2}{2}-\frac{y^2}{1}=1$

Here, $a^2 = 2,\ b^2 = 1$

Equation of asymptotes to the given hyperbola is

$$\frac{x}{\sqrt{2}}-\frac{y}{1}=0 \text{ and } \frac{x}{\sqrt{2}}+\frac{y}{1}=0$$

Let P $(\sqrt{2}\sec\theta, \tan\theta)$ be any point, then product of length of perpendicular

$$=\frac{\left[\frac{\sqrt{2}\sec\theta}{\sqrt{2}}-\frac{\tan\theta}{1}\right]\left[\frac{\sqrt{2}\sec\theta}{\sqrt{2}}+\frac{\tan\theta}{1}\right]}{\sqrt{\frac{1}{2}+\frac{1}{1}}\sqrt{\frac{1}{2}+\frac{1}{1}}}$$

$$=\frac{\sec^2\theta-\tan^2\theta}{\frac{3}{2}} = \frac{2}{3}$$

24. Given ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose area is π ab.

The auxiliary circle to the given ellipse is $x^2 + y^2 = a^2$ whose area is πa^2.

Given that, $\pi a^2=2\pi ab \Rightarrow a=2b$

Now , eccentricity of ellipse

$$=\sqrt{1-\frac{b^2}{a^2}}$$

$$=\sqrt{1-\frac{b^2}{4b^2}}=\frac{\sqrt{3}}{2}$$

25. Let mid point be (h, k).

$\therefore$ Equation of chord is

$$T = S_1$$

$$y\,y_1 - 2a(x + x_1) = y_1^2-4ax_1$$

Since, it passes through origin

$\therefore$ $-2ax_1 = y_1^2 - 4ax_1$

$\Rightarrow$ $y_1^2 = 2ax_1$

$\therefore$ Locus is $y^2 = 2ax$

26. Let $S \equiv 5x^2 + 9y^2 - 32$

Now, S (2, 3) ≡ 20 + 81 – 32 > 0

$\therefore$ Point (2,3) lies outside ellipse.

Thus, two tangents can be drawn.

27. Here a = 2, m = – 1

$\therefore$ Required point is $(am^2, -2am) = (2,4)$

28. Let the point is

$(3t^2, 6t)$

$\therefore$ Focal distance $= 3t^2 + 3$

$\Rightarrow \quad 3t^2 + 3 = 12$

$\Rightarrow \quad 3t^2 = 9$

$\Rightarrow \quad t^2 = 3$

$\Rightarrow \quad t = \sqrt{3}$

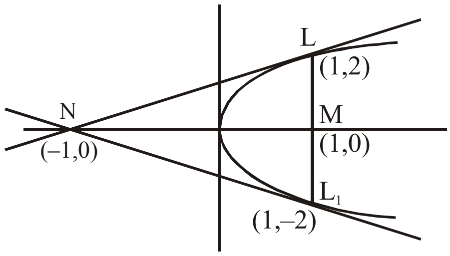
Hence, the required point is

$$\left(9, 6\sqrt{3}\right)$$

29. Since, the line $y = -\frac{a}{b}x - \frac{c}{b}$ is tangent to the parabola $y^2 = 4ax$, then

$$-\frac{c}{b} = \frac{a}{-\frac{a}{b}} \Rightarrow c = b^2$$

30. The coordinates at the ends of the latursectum of the parabola $y^2 = 4x$ are L (1,2) and L_1 (1,–2). Equation of tangent at L and L_1 are $2y = 2(x + 1)$ and $-2y = 2(x+1)$, which givens $x = -1$, $y = 0$



73. For fcc,

$a\sqrt{2} = 4r$

$\Rightarrow r = \frac{a\sqrt{2}}{4} = 127.6$ pm

74. If Number of O^{-2} ions = n

then number of O.H.V. = n

$\therefore$ Number of M^{+n} ions = 2/3 n

$\therefore$ M : O = 2/3 n : n = 2 : 3

$\therefore$ Simplest formula of solid = M_2O_3

75. $\rho = \frac{z \times \text{Atomic mass}}{a^3 \times N_A}$

$= \frac{4 \times 58.5}{(5.64 \times 10^{-8})^3 \times 6.02 \times 10^{23}}$

$= 2.16$ g/cm^3

80. $\because N_t = N_0\left(\frac{1}{2}\right)^n$

$\therefore 25 = 200 \times \left(\frac{1}{2}\right)^n$

$\Rightarrow \frac{1}{8} = \left(\frac{1}{2}\right)^n \Rightarrow \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^n$

$\Rightarrow n = 3$

$\therefore$ Total time = n × half life

= 3 × 4650

= 13950 years

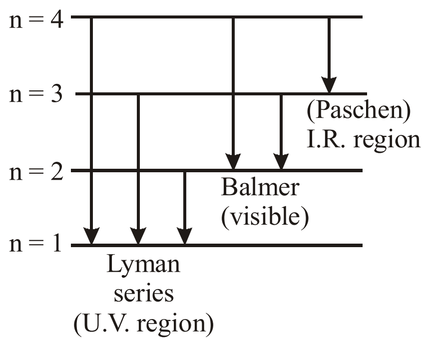
81. $\lambda = h/p$

$\Rightarrow p = \frac{h}{\lambda}$

$\Rightarrow \frac{6.6 \times 10^{-34}}{6630 \times 10^{-10}} = 9.95 \times 10^{-28}$ kg m/sec

$= 1 \times 10^{-27}$ kg m/sec

82.



83. Number of radial Nodes = n – ℓ – 1

84. E_n for H atom $= \frac{E_1}{n^2}$

$\therefore E_n = \frac{-13.6}{n^2} \Rightarrow n^2 = \frac{-13.6}{E_n}$

$\Rightarrow n^2 = \frac{-13.6}{-3.4} = 4$

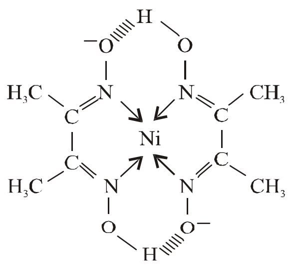
$\Rightarrow n = 2$

$\because$ angular momentum $= \frac{nh}{2\pi} = \frac{2 \times h}{2\pi} = \frac{h}{\pi}$

$= \frac{6.6 \times 10^{-34}}{3.14}$

$= 2.1 \times 10^{-34}$ Jsec

90. $Ni^{2+} + 2DMG \rightarrow$



Red precipitate

A = DMG

x = 2

C = Red