INDIAN OLYMPIAD QUALIFIER IN PHYSICS (IOQP) 2020-21

(Held On Sunday 07th FEBRUARY, 2021)

Time allowed : 60 Minutes **Max. Marks : 120**

TEST PAPER WITH ANSWER & SOLUTION (PART-1)

Attempt All The Thirty Two Questions

A-1

- **ONLY ONE OUT OF FOUR OPTIONS IS CORRECT BUBBLE THE CORRECT OPTION.**

1. If speed of light c, Planck's constant h and gravitational constant G are chosen as fundamental quantities, dimensions of time in this system of units is :-

(a) $ch^{3/2}G^{-3/2}$ (b) $c^{-2}G^{1/2}h$ (c) $c^2G^{1/2}h^{5/2}$ (d) $c^{-5/2}G^{1/2}h^{1/2}$

Ans. (d)

Sol. $t \propto c^x h^y G^z$

$M^0L^0T^1 = [LT^{-1}]^x\,[ML^2T^{-1}]^y[M^{-1}L^3T^{-2}]^z$

$M^0L^0T^1 = M^{y-z}L^{x+2y+3z}T^{-x-y-2z}$

$y - z = 0 \Rightarrow y = z$

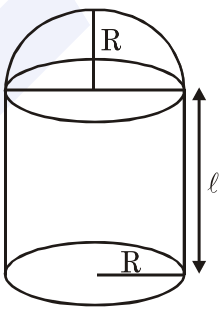
$x + 2y + 3z = 0 \Rightarrow x + 5z = 0 \Rightarrow x = -5z$

$-x - y - 2z = 1 \Rightarrow 5z - z - 2z = 1$

$\Rightarrow\ z = \frac{1}{2},\ y = \frac{1}{2},\ x = -5/2$

So $t \propto c^{-5/2}G^{1/2}h^{1/2}$

2. A solid hemisphere is cemented on the flat surface of a solid cylinder of same radius R and same material. The composite body is rotating about the axis of the cylinder of length ℓ with angular speed ω. The radius of gyration K is :-



(a) $R\sqrt{\frac{2}{5}\left(\frac{15R+8\ell}{3R+2\ell}\right)}$ (b) $R\sqrt{\frac{1}{10}\left(\frac{15\ell+8R}{3\ell+2R}\right)}$ (c) $R\sqrt{\frac{3}{10}\left(\frac{15R+8\ell}{3R+2\ell}\right)}$ (d) $R\sqrt{\frac{1}{10}\left(\frac{3\ell+2R}{15\ell+8R}\right)}$

Ans. (b)

Sol. $I = I_{cy} + I_{hemisphere}$

$$I = \frac{M_1R^2}{2} + \frac{2}{5}M_2R^2$$

$$M_1 = \rho\pi R^2\ell,\ M_2 = \rho\frac{2}{3}\pi R^3$$

$$I = \rho R^2\left[\frac{\pi R^2 \ell}{2} + \frac{2}{5}\frac{2}{3}\pi R^3\right]$$

$$(M_1 + M_2)K^2 = I = \pi\rho R^4\left[\frac{\ell}{2} + \frac{4}{15}R^3\right]$$

$$K^2\rho\pi R^2\left[\ell + \frac{2}{3}R\right] = \rho\pi R^4\left[\frac{\ell}{2} + \frac{4}{15}R\right]$$

$$K = R\sqrt{\frac{(15\ell + 8R)(1)}{(3\ell + 2R)(10)}}$$

3. The shortest period of rotation of a planet (considered to be a sphere of uniform density ρ) about its own axis, such that any mass m kept on its equator is just to fly off the surface, is :-

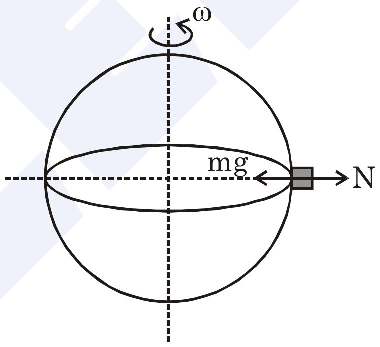
(a) $T = \sqrt{\frac{5\pi}{\rho G}}$ (b) $T = \sqrt{\frac{\pi}{3\rho G}}$ (c) $T = \sqrt{\frac{3\pi}{\rho G}}$ (d) $T = \sqrt{\frac{5\pi}{3\rho G}}$

Ans. **(c)**

Sol. $mg - N = m\omega^2 R$

$N = 0$

$$\omega = \sqrt{\frac{g}{R}}$$



$$g = \frac{GM}{R^2}$$

$$g = \frac{G}{R^2} \times \rho \times \frac{4}{3}\pi R^3 = \frac{4G\rho\pi R}{3}$$

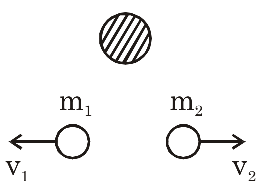
So $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3R}{4G\rho\pi R}} = \sqrt{\frac{3\pi}{G\rho}}$

4. A body of mass 10 kg at rest explodes into two fragments of masses 3kg and 7kg. If the total kinetic energy of two pieces after explosion is 1680 J, the magnitude of their relative velocity in m/s after explosion is :-

(a) 40 (b) 50 (c) 70 (d) 80

Ans. **(a)**

Sol.



From momentum conservation

$0 = m_1 v_1 - m_2 v_2$

$3v_1 = 7v_2$ .....(i)

$$1680 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$3360 = 3v_1^2 + 7v_2^2 \qquad ....(ii)$$

$$3360 = 3\left(\frac{7v_2}{3}\right)^2 + 7v_2^2$$

$$3360 = \frac{49v_2^2}{3} + 7v_2^2$$

$10080 = (49 + 21)v_2^2$

$10080 = 70\ v_2^2$

$v_2 = 12$ m/s

$v_1 = 7/3\ v_2 = 7/3 \times 12 = 28$ m/s

relative velocity : $v_{12} = v_1 + v_2 = 40$ m/s

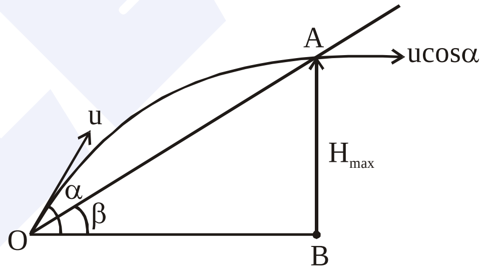
5. A shot is fired at an angle α to the horizontal up a hill (Considered to be a long straight incline plane) of inclination β to the horizontal. It will strike the hill horizontally if :-

(a) $\tan\alpha = 2\tan\beta$ (B) $\sin\alpha = \sin 2\beta$ (c) $\sin\alpha = 2\sin\beta$ (d) $\tan\alpha = 4\tan\beta$

Ans. (a)

Sol. It will hit horizontally if at A vertical velocity is zero.

$$\therefore\ OB = \frac{R}{2}$$



and $AB = H_{max}$

$$\tan\beta = \frac{AB}{OB} = \frac{H_{max}}{\frac{R}{2}}$$

$$\tan\beta = \frac{\frac{u^2\sin^2\alpha}{2g}}{\frac{1}{2}\left(\frac{u^2\sin 2\alpha}{g}\right)}$$

$$\Rightarrow \tan\beta = \frac{\sin^2\alpha}{\sin 2\alpha} = \frac{\tan\alpha}{2}$$

$\Rightarrow \tan\alpha = 2\tan\beta$

$\therefore$ correct option is (a)

6. A particle is executing Simple harmonic Motion of time period $T = 4\pi^2$ in a straight line. Starting from rest, it travels a distance 'a' in the first second and distance 'b' in the next second travelling in the same direction. The amplitude of SHM is :-

(a) $\frac{2a^2}{3a-b}$ (b) $\frac{3a^2}{3a-2b}$ (c) $\frac{2a^2}{2a-b}$ (d) none of these

Ans. (a)

Sol. $T = 4\pi^2 = \frac{2\pi}{\omega}$

$$\Rightarrow \omega = \frac{1}{2\pi}\text{rad/sec}$$

$x = A\sin(\omega t + \phi)$

$\omega = \frac{1}{2\pi}, \phi = \frac{\pi}{2}$ (as it starts from extreme position)

$\therefore x = A\sin\left(\frac{t}{2\pi} + \frac{\pi}{2}\right)$

$\Rightarrow x = A\cos(\omega t)$ $\left(\omega = \frac{1}{2\pi}\right)$

Displacement in 't' time = $A - A\cos\omega t$

For $t = 1$ $A - A\cos\omega = a$ ... (1)

For $t = 2$ $A - A\cos 2\omega = a + b$ ... (2)

$$\frac{1-\cos\omega}{1-\cos 2\omega} = \frac{a}{a+b}$$

$$\Rightarrow \frac{1-\cos\omega}{1-\left(2\cos^2\omega - 1\right)} = \frac{a}{a+b}$$

$$\frac{1-\cos\omega}{2-2\cos^2\omega} = \frac{a}{a+b}$$

$$\frac{1-\cos\omega}{1-\cos^2\omega} = \frac{2a}{a+b}$$

$$\frac{1-\cos\omega}{(1-\cos\omega)(1+\cos\omega)} = \frac{2a}{a+b}$$

$$\frac{1}{(1+\cos\omega)} = \frac{2a}{a+b}$$

$a + b = 2a + 2a\cos\omega$

$$\frac{b-a}{2a} = \cos\omega$$

as, $A - A\cos\omega = a$

$$\therefore \quad A = \frac{a}{1-\cos\omega} = \frac{a}{1-\left(\frac{b-a}{2a}\right)}$$

$$\therefore \quad A = \frac{2a^2}{2a-b+a} = \frac{2a^2}{3a-b}$$

$\therefore$ correct option is (A)

7. The kinetic energy of a particle moving along a circle of radius R depends upon the distance covered 's' as $KE = as^2$ where a is a constant. The magnitude of the force acting on the particle as a function of 's' is :-

(a) $\frac{2as^2}{R}$ (b) $\frac{2as^2}{m}$ (c) $2as$ (d) $2as\sqrt{1+\left(\frac{s}{R}\right)^2}$

Ans. (d)

Sol. Given, kinetic energy = as^2

$$\frac{1}{2}mv^2 = as^2$$

$$\Rightarrow v = s\sqrt{\frac{2a}{m}}$$

$$a_t = \frac{dv}{dt} = \frac{vdv}{ds} = s\left(\sqrt{\frac{2a}{m}}\right)\left(\sqrt{\frac{2a}{m}}\right) = \frac{2as}{m}$$

$$\Rightarrow F_t = ma_t = 2as$$

$$F_c = ma_c = \frac{mv^2}{R} = \frac{2as^2}{R}$$

$$F_{net} = \sqrt{F_c^2 + F_t^2}$$

$$= \sqrt{\left(\frac{2as^2}{R}\right)^2 + (2as)^2}$$

$$= 2as\sqrt{1 + \left(\frac{s}{R}\right)^2}$$

$\therefore$ correct answer is (d)

8. The flow of water in a horizontal pipe is stream line flow. Along the pipe, at a point, where cross sectional area is 10 cm^2, the velocity of water flow is 1.00 ms^{-1} and the pressure is 2000 Pa. The pressure of water at another point where cross-sectional area is 5 cm^2 is :-

(a) 2000 Pa (b) 1500 Pa (C) 3500 Pa (d) 500 Pa

Ans. **(d)**

Sol. $A_1V_1 = A_2V_2$

$A_1 = 10cm^2$, $V_1 = 1m/s$

$A_2 = 5\ cm^2$, $V_2 = ?$

$$V_2 = \frac{A_1V_1}{A_2} = 2m/s$$

Now applying Bernoullie

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V^2$$

$$2000 + \frac{1}{2}(1000)(1)^2 = P_2 + \frac{1}{2}(1000)(2)^2$$

$\Rightarrow P_2 = 500$ Pa

$\therefore$ correct option is (d)

9. Three containers A, B and C are filled with water at different temperature. When 1 litre of water from A is mixed with 2 litre of water from B, the resulting temperature of mixture is 52°C. When 1 litre of water from B is mixed with 2 litre of water from C, the resulting temperature of mixture is 40°C. Similarly when 1 litre of water from C is mixed with 2 litre of water from A, the resulting temperature of mixture is 34°C. Temperature of mixture when one litre of water from each container is mixed (neglect the water equivalent of container) is :-

(a) 40°C (b) 42°C (c) 38°C (d) 45°C

Ans. (b)

Sol. Case-1 : $1(T_1 - 52) + 2(T_2 - 52) = 0$

Case-2 : $1(T_2 - 40) + 2(T_3 - 40) = 0$

Case-3 : $2(T_1 - 34) + 1(T_3 - 34) = 0$

Solving case-1, 2 & 3

$T_2 = 60$

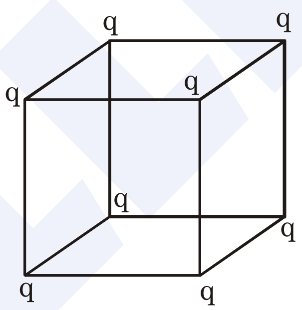
$T_3 = 30$

$T_1 = 36$

Case-4 : $(36 - T) + (30 - T) + (60 - T) = 0$

$\therefore T = 42°C$

10. Point charge q is kept at each corner of a cube of edge length ℓ. The resultant force of repulsion on any one of the charges due to all others is expressed as :-



(a) $\dfrac{\left(1+\dfrac{1}{2\sqrt{2}}+\dfrac{1}{3\sqrt{3}}\right)q^2}{\pi\varepsilon_0\ell^2}$

(b) $\dfrac{\left(\dfrac{1}{2\sqrt{2}}-1+\dfrac{1}{3\sqrt{3}}\right)q^2}{\pi\varepsilon_0\ell^2}$

(c) $\dfrac{(1-0.1775)q^2}{\pi\varepsilon_0\ell^2}$

(d) none of these

Ans. (c)

Sol. $\vec{F} = \dfrac{kq^2}{\ell^2}\left(\hat{i}+\hat{j}+\hat{k}\right)$

$\dfrac{kq^2}{\sqrt{2}\ell^2}\left(\hat{i}+\hat{j}+\hat{k}\right)$

$\dfrac{kq^2}{3\sqrt{3}\ell^2}\left(\hat{i}+\hat{j}+\hat{k}\right) = \dfrac{kq^2}{\ell^2}\left(\hat{i}+\hat{j}+\hat{k}\right)\left[1+\dfrac{1}{\sqrt{2}}+\dfrac{1}{3\sqrt{3}}\right]$

11. In an experiment with potentiometer, the balancing length is 250 cm for a cell. When the cell is shunted by a resistance of 7.5 Ω, balancing point is shifted by 25 cm. If the cell is shunted by a resistance of 20 Ω, the balancing length will be nearly :-

(a) 240 cm (b) 236 cm (c) 232 cm (d) 230 cm

Ans. (a)

Sol. $E = E_0 \frac{250}{\ell}$ ... (1)

$\frac{E(7.5)}{7.5+r} = E_0 \frac{225}{\ell}$ ... (2)

$\frac{1+r}{7.5} = \frac{7.5+r}{7.5} = \frac{250}{225} = \frac{50}{45} = \frac{10}{9}$

$\Rightarrow r = \frac{7.5}{9} = \frac{15}{18} = \frac{5}{6}\Omega$

$\frac{E(20)}{20+\frac{5}{6}} = E_0 \frac{x'}{\ell}$ ... (3)

$\frac{20+\frac{5}{6}}{20} = \frac{250}{x'}$

$x' = \frac{250\times 20}{20+\frac{5}{6}} = 250\times\frac{20}{125}\times 6$

= 240 cm

12. One mole of a gas with $\gamma = \frac{5}{3}$ is mixed with two moles of another non-interacting gas with $\gamma = \frac{7}{5}$. The ratio of specific heats $\gamma = \frac{C_P}{C_v}$ of mixture is approximately

(a) 1.50 (b) 1.46 (c) 1.49 (d) 1.53

Ans. (b)

Sol. $\gamma = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}} = \frac{1\times\frac{5}{2} + 2\times\frac{7}{2}}{1\times\frac{3}{2}+2\times\frac{5}{2}}$

For $\gamma = \frac{5}{3}$: $C_P = \frac{\gamma R}{\gamma - 1} = \frac{\frac{5}{3}}{\frac{5}{3}-1} = \frac{5}{2}$, $C_V = \frac{R}{\gamma-1} = \frac{3}{2}$

For $\gamma = \frac{7}{5}$: $C_P = \frac{\frac{7}{5}}{\frac{2}{5}} = \frac{7}{2}$, $C_V = \frac{1}{\frac{7}{5}-1} = \frac{5}{2}$

13. An ideal gas is expanding such that PT^3 = constant. The coefficient of volume expansion of the gas is:

(a) 1/T (b) 2/T (c) 3/T (d) 4/T

Ans. (d)

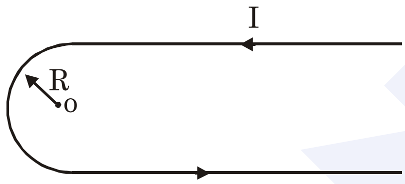
Sol. $PT^3 = K$

$$\frac{nRT}{V}T^3 = K \Rightarrow V = CT^4 \Rightarrow \frac{dV}{dT} = 4CT^3$$

vol. exp. coeff. = $\gamma = \frac{dV}{VdT}$

$$\gamma = \frac{4CT^3}{CT^4} = \frac{4}{T}$$

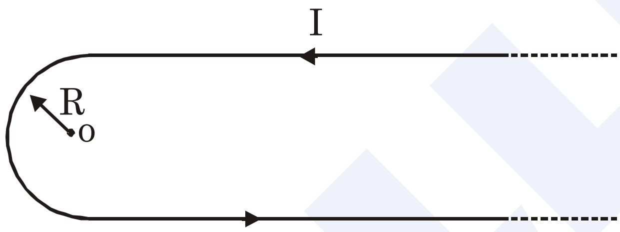
14. What is the magnetic induction B at the centre O of the semicircular arc if a current carrying wire has shape of an hair pin as shown in figure? The radius of the curved part of the wire is R, the linear parts are assumed to be very long.



(a) $B = \frac{\mu_0 I}{4\pi R}(2+\pi)$ (b) $B = \frac{\mu_0 I}{4R}(2+\pi)$ (c) $B = \frac{3\mu_0 I}{4R}(2+\pi)$ (d) $B = \frac{\mu_0}{4\pi}\frac{2I}{R}$

Ans. (a)

Sol.

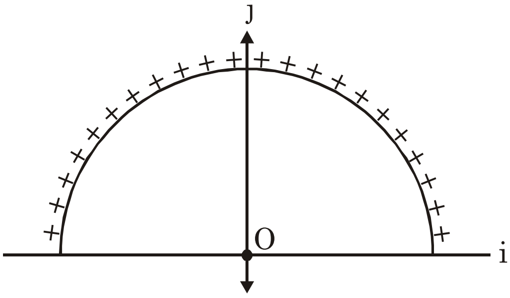


The wire has 2 long straight parts & one semicircular part

$B_{net} = 2\,B_{(straight)} + B_{(semicircle)}$

$$= 2\frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4R}$$

15. A thin semi-circular metal ring of radius R has a positive charge q distributed uniformly over its curved length. The resultant electric field $\vec{E}$ at the centre O is :



(a) $-\hat{j}\frac{q}{2\pi^2\varepsilon_0 R^2}$ (b) $+\hat{j}\frac{q}{2\pi^2\varepsilon_0 R^2}$ (c) $+\hat{j}\frac{q}{4\pi^2\varepsilon_0 R^2}$ (d) $-\hat{j}\frac{q}{4\pi^2\varepsilon_0 R^2}$

Ans. (a)

Sol. E.F. at centre due to semicircle

$$E = \frac{2k\lambda}{R}$$

$$\because \lambda = \frac{q}{\pi R}$$

$$E = \frac{2q}{4\pi\varepsilon_0 \pi R^2}$$

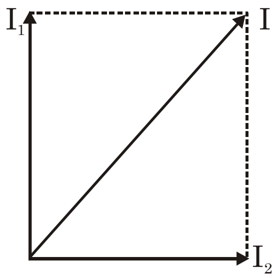
$$\vec{E} = -\hat{j}\frac{q}{2\pi^2\varepsilon_0 R^2}$$

16. An alternating current is expressed as $i = i_1 \cos\omega t + i_2 \sin\omega t$. The RMS value of current is

(a) $\sqrt{\frac{(i_1+i_2)^2}{2}}$ (b) $\sqrt{\frac{i_1 i_2}{2}}$ (c) $\sqrt{\frac{(i_1^2+i_2^2)}{2}}$ (d) $\sqrt{\frac{(i_1+i_2)^2}{2}}$

Ans. **(c)**

Sol.



$$i = i_1 \sin\left(\frac{\pi}{2}+\omega t\right) + i_2 \sin\omega t$$

adding by phasor

$$I = \sqrt{I_1^2 + I_2^2}$$

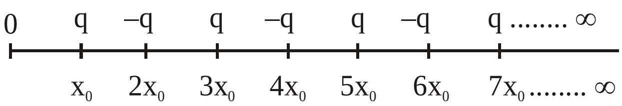
$$i_{rms} = \frac{I}{\sqrt{2}} = \sqrt{\frac{I_1^2+I_2^2}{2}}$$

17. A charge +q is placed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$, $x = 7x_0$ .... ∞ on the x-axis and a charge –q is placed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$, $x = 8x_0$ .... ∞ here x_0 is a positive constant. Take the electric potential at a point due to charge q at a distance r from it to be $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$. The electric potential at the origin due to the above system of charges is :

(a) zero (b) $\frac{1}{4\pi\varepsilon_0}\frac{q}{x_0}\ln 2$ (c) $\frac{1}{4\pi\varepsilon_0}\frac{q}{x_0 2\ln 2}$ (d) infinite

Ans. **(b)**

Sol.



Potential due to above system at origin will be :

$$\Rightarrow V = \frac{kq}{x_0} - \frac{kq}{2x_0} + \frac{kq}{3x_0} - \frac{kq}{4x_0} ......\infty \qquad ...(1)$$

and,

we know that

$\Rightarrow \quad \ell n2 = \left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} ......\infty\right]$ ... (2)

$\Rightarrow$ From (1) and (2)

$\Rightarrow \quad V = \frac{q}{4\pi\varepsilon_0 x_0}(\ell n2)$

Correction answer is (b)

18. The Nucleus ${}^{23}_{10}Ne$ decays by β –emission through the reaction ${}^{23}_{10}Ne \to {}^{23}_{11}Na + {}^{0}_{-1}\beta + \bar{\nu} + \text{energy}$. The atomic masses are ${}^{23}_{10}Ne$ = 22.994466u and ${}^{23}_{11}Na$ = 22.989770u, ${}^{0}_{-1}\beta$ = 0.000549u. The maximum kinetic energy that the emitted electron can ever have is :

(a) 4.374 MeV (b) 3.862 MeV (c) 2.187 MeV (d) 1.931 MeV

Ans. (a)

Sol. ${}^{23}_{10}Ne \to {}^{23}_{11}Na + {}^{0}_{-1}\beta + \bar{\nu} + \text{Energy}$

$\Rightarrow$ Maximum kinetic energy of $e^-\left({}^{0}_{-1}\beta\right)$ will be $k_{max} = \Delta mc^2$ ... (1)

$\Rightarrow$ Δm = 22.994466

– 22.989770

= 0.004696 μ ... (2)

$\Rightarrow$ We know that

(1 amu) c^2 = 931.5 MeV ... (3)

$\Rightarrow$ $k_{max} = \Delta mc^2$ = 4.374 MeV

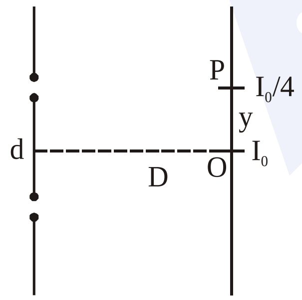
Correct answer is (a)

19. The distance between two slits in Young's double slits experiment is d = 2.5 mm and the distance of the screen from the plane of slits is D = 120 cm. The slits are illuminated with coherent beam of light of wavelength λ = 600 nm. The minimum distance (from the central maximum) of a point where the intensity reduce to 25% of maximum intensity is :

(a) 24 μm (b) 48 μm (c) 96 μm (d) 120 μm

Ans. (c)

Sol.



$\lambda = 600$ nm

$\Rightarrow$ We know that at O the intensity will be maximum.

$\Rightarrow$ Intensity at P (let's say) is $\frac{I_0}{4}$.

$\Rightarrow$ So, we solve to find "y" [The minimum value of it].

$\Rightarrow \quad \frac{I_0}{4} = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$ ... (1)

$$\Rightarrow \Delta\phi = \left(\frac{2\pi}{\lambda}\right)\cdot\left(\frac{dy}{D}\right) \Rightarrow \frac{1}{2} = \cos\left(\frac{\pi dy}{\lambda D}\right)$$

$\Rightarrow$ For minimum value,

$$\frac{\pi dy}{\lambda D} = \frac{\pi}{3}$$

$$\Rightarrow y = \frac{\lambda D}{3d} = 96\mu m$$

20. What amount of heat will be generated in a coil of resistance R (ohm) due to a total charge Q (coulomb) passing through it if the current in the coil decreases down to zero halving its value every Δt second?

(a) $\frac{1}{2}\frac{Q^2R}{\Delta t}$ (b) $\frac{Q^2R}{\Delta t}\ln 2$ (c) $\frac{1}{2}\frac{Q^2R}{\Delta t}\ln 2$ (d) $\frac{1}{4}\frac{Q^2R}{\Delta t}$

Ans. **(c)**

Sol. Obviously the current through the coil is given by

$$i = i_0\left(\frac{1}{2}\right)^{t/\Delta t}$$

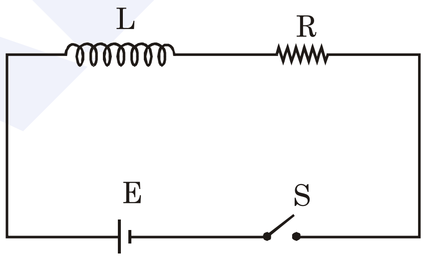
Then charge $q = \int_0^\infty idt = \int_0^\infty i_0 2^{-t/\Delta t}dt = \frac{i_0\Delta t}{\ell n2}$

So, $i_0 = \frac{q\ell n2}{\Delta t}$

And hence, heat generated in the circuit in the time interval t [0, ∞]

$$H = \int_0^\infty i^2R\,dt = \int_0^\infty\left[\frac{q\ell n2}{\Delta t}2^{-t/\Delta t}\right]^2 R\,dt = -\frac{q^2\ell n2}{2\Delta t}R$$

21. In the LR circuit shown in figure, switch S is closed at time t = 0, the charge that passes through the battery of emf E in one time constant is (e being the base of natural logarithm).



(a) $\frac{EL}{eR^2}$ (b) $\frac{EL}{eR}$ (c) $\frac{eER^2}{L}$ (d) $\frac{EL}{R}$

Ans. **(a)**

Sol. $i = \frac{E}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$

$$q = \int idt$$

$$= \int_0^\tau \frac{E}{R}\left(1 - e^{-\frac{t}{\tau}}\right)dt = \frac{E}{R}\left[t + \tau e^{-\frac{t}{\tau}}\right]_0^\tau = \frac{E}{R}\left[\left(\tau + \tau e^{-\frac{\tau}{\tau}}\right) - (0+\tau)\right] = \frac{E}{R}\left[\tau e^{-1}\right] = \frac{E}{R}\times\frac{L}{Re} = \frac{EL}{R^2e}$$

22. Natural Uranium is a mixture of $^{238}_{92}U$ and $^{235}_{92}U$ with a relative mass abundance of 140 : 1. The ratio of radioactivity contributed by the two isotopes of natural uranium, if their half-lives are 4.5×10^9 years and 7.0×10^8 years respectively is :

(a) 99.3 : 0.7 (b) 50.3 : 49.7 (c) 95.6 : 04.4 (d) cannot be estimated

Ans. (c)

Sol. Let's mass of $U^{238} = m_1$

& mass of $U^{235} = m_2$

& total mass m

$$\frac{m_1}{m_2} = \frac{140}{1}$$

$$m_1 = \frac{140}{141}m \ \ \& \ m_2 = \frac{1}{141}m$$

$$N_1 = \frac{m_1}{238}N_A \ \ \& \ N_2 = \frac{m_2}{235}N_A$$

$$\frac{A_1}{A_2} = \frac{\lambda_1}{\lambda_2}\frac{N_1}{N_2} = \frac{T_2}{T_1}\frac{N_1}{N_2} \quad \left[\lambda = \frac{\ell n2}{T}\right]$$

$$= \frac{7\times10^8}{4.5\times10^9}\times\frac{m_1}{m_2}\times\frac{235}{238} = \frac{7}{4.5\times10}\times\frac{235}{238}\times140 = 21.50 \approx \frac{95.6}{4.4}$$

23. A cylinder of length ℓ >1m filled with water $\left(\mu = \frac{4}{3}\right)$ up to the brim, kept on a horizontal table is covered at its top by an equiconvex glass (μ = 1.5) lens of focal length 25 cm when in air. At mid day, 12.00 noon, Sun is just overhead and light rays comes parallel to the principal axis of the lens. Thus sun rays will be focused

(a) 25 cm behind the lens in the water (b) 37.5 cm behind the lens in the water

(c) 50 cm behind the lens in the water (d) 100 cm behind the lens in the water

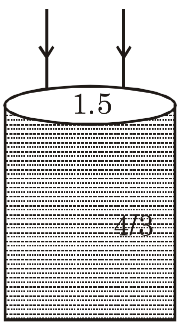
Ans. (c)

Sol. For lens

$$\frac{1}{f} = (\mu-1)\left(\frac{2}{R}\right)$$

$$\frac{1}{25} = \left(\frac{1}{2}\right)\left(\frac{2}{R}\right)$$

$\Rightarrow$ R = 25 cm



At 1st surface

$$\frac{1.5}{v_1} - \frac{1}{\infty} = \frac{1.5-1}{25}$$

$\Rightarrow v_1$ = 75 cm

At 2nd surface

$$\frac{4}{3v_2} - \frac{1.5}{75} = \frac{\frac{4}{3}-\frac{3}{2}}{-25}$$

$\Rightarrow v_2$ = 50 cm

24. Even the radiation of highest wave length in the ultraviolet region of hydrogen spectrum is just able to eject photoelectrons from a metal. The value of threshold frequency for the given metal is :

(a) 3.83×10^{15} Hz (b) 4.33×10^{14} Hz (c) 2.46×10^{15} Hz (d) 7.83×10^{14} Hz

Ans. **(c)**

Sol. Lyman series $\rightarrow$ UV region

highest $\lambda \Rightarrow n = 2 \rightarrow n = 1$

$E_2 - E_1 = 10.2$ eV

$k_{max} = 10.2 \text{ eV} - \phi$

$0 = 10.2 \text{ eV} - \phi$ (just able to eject)

$$\nu_{th} = \frac{\phi}{h} = \frac{10.2\text{eV}}{h} = 2.46 \times 10^{15} \text{ Hz}$$

ANY NUMBER OF OPTIONS d,c,b or a MAY BE CORRECT.
MARKS WILL BE AWARDED ONLY IF ALL CORRECT OPTIONS ARE BUBBLED AND NO WRONG OPTION.

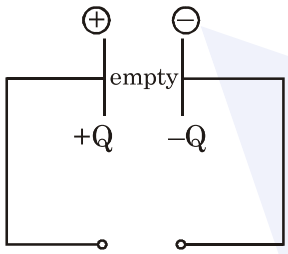
25. A parallel plate capacitor of plate area A and plate separation d is charged to potential V. Then the battery is disconnected. A slab of dielectric constant k is then inserted between the plates of the capacitor so as to fill the space between the plates completely. If Q, E and W denote respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted) and work done on the system, in question, in the process of inserting the slab, then

(a) $Q = k\varepsilon_0 AE$ (b) $Q = \frac{\varepsilon_0 kAV}{d}$ (c) $E = \frac{V}{kd}$ (d) $W = \frac{\varepsilon_0 AV^2}{2d}\left(1 - \frac{1}{k}\right)$

ALLEN Ans.(a,c)

NSEP Ans. is (a, c, d)

Sol.

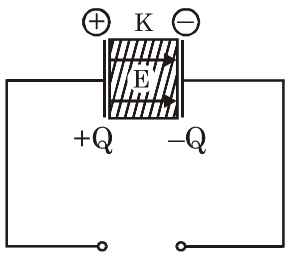


$$Q = CV = \left(\frac{\varepsilon_0 A}{d}\right)V$$

Battery is diconnected

$\Rightarrow$ Q charge = constant

Now dielectric is inserted



$$E = \frac{Q}{kA\varepsilon_0} = \frac{V}{kd}$$

Work done $= \Delta U = U_f - U_i$

$$= \frac{Q^2}{2C_f} - \frac{Q^2}{2C_i} = \frac{Q^2}{2kC} - \frac{Q^2}{2C} = \frac{Q^2}{2C}\left(\frac{1}{k}-1\right) = \frac{\left(\frac{\varepsilon_0 A}{d}V\right)^2}{2\frac{\varepsilon_0 A}{d}}\left(\frac{1}{k}-1\right)$$

$$w = \frac{\varepsilon_0 A}{2d}V^2\left(\frac{1}{k}-1\right)$$

26. The magnitudes of the gravitational field at distance r_1 and r_2 from the centre of a uniform solid sphere of radius R and mass M are F (r_1) and F (r_2) respectively. Such that :

(a) $\frac{F(r_1)}{F(r_2)} = \frac{r_1}{r_2}$ if $r_1 \le R$ and $r_2 \le R$

(b) $\frac{F(r_1)}{F(r_2)} = \frac{r_2^2}{r_1^2}$ if $r_1 \ge R$ and $r_2 \ge R$

(c) $\frac{F(r_1)}{F(r_2)} = \frac{r_1}{r_2}$ if $r_1 \ge R$ and $r_2 \ge R$

(d) $\frac{F(r_1)}{F(r_2)} = \frac{r_1^2}{r_2^2}$ if $r_1 \le R$ and $r_2 \le R$

Ans. **(a, b)**

Sol. For $r \le R$

$$F = \left(\frac{GM}{R^3}r\right) \propto r$$

For $r \ge R$

$$F = \frac{GM}{R^3} \propto \frac{1}{r^2}$$

27. The intensity of sound at a point P is I_0, when the sounds reach this point directly and in same phase from two identical sources S_1 and S_2. The power of S_1 is now reduced by 64% and the phase difference (ϕ) between S_1 and S_2 is varied continuously. The maximum and minimum intensities recorded at P are now I_{max} and I_{min} such that

(a) $I_{max} = 0.64\ I_0$ (b) $I_{min} = 0.36\ I_0$ (c) $\frac{I_{max}}{I_{min}} = 16$ (d) $\frac{I_{max}}{I_{min}} = \frac{16}{9}$

Ans. **(a, c)**

Sol. Let the initial intensities for both sources be I each.

So, maximum intensity $I_0 = 4I$

Now, $I_1 = (0.36)I$ & $I_2 = I$

(reduced by 64%)

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

$= (0.6 + 1)^2 I$

$= (2.56)I = (0.64)I_0$

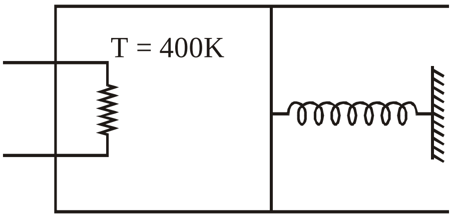
$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

$= (0.6 - 1)^2 I$

$= (0.16)I = (0.04)I_0$

$$\frac{I_{max}}{I_{min}} = \frac{16}{1}$$

28. An ideal monatomic gas is confined within a cylinder by a spring loaded piston of cross-sectional area 4×10^{-3} m^2. Initially the gas is at 400 K and occupies a volume 2×10^{-3} m^3 and the spring is in its relaxed position. The gas is heated by an electric heater for some time. During this time the gas expands and the piston moves out by a distance 0.1 m. The spring connected to the rigid wall is massless and frictionless. The force constant of the spring is 2000 Nm^{-1} and atmospheric pressure is 10^5 Nm^{-2} then



(a) The final temperature of the gas is 720 K

(b) The work done by gas in expanding is 50 J

(c) The heat supplied by heater is 190 J

(d) The heat supplied by heater is 290 J

Ans. (a,b,d)

Sol. Initially spring is relaxed hence, pressure of gas inside is equal to atmospheric pressure outside the piston.

$P_i = P_{atm} = 10^5$ N/m^2

finally spring is compressed by

$x = 0.1$m,

So for equilibrium of piston (again).

Pressure of gas inside

$$P_f = P_{atm} + \frac{kx}{A}$$

$$P_f = 10^5 + \frac{(2000)(0.1)}{4 \times 10^{-3}}$$

$P_f = 1.50 \times 10^5$ N/m^2

initial volume $V_i = 2 \times 10^{-3}$ m^3

& initial length of gas = V_i/A

$$= \frac{2 \times 10^{-3}}{4 \times 10^{-3}} = 0.5 \text{ m}$$

final length of gas = 0.5 + 0.1

(when pistion shifts by 0.1m) = 0.6 m

final volume of gas

$= (0.6)(4 \times 10^{-3})$

$= 2.4 \times 10^{-3}$m^3

$$\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f}$$ Conservation of moles.

$$\frac{(10^5)(2 \times 10^{-3})}{400} = \frac{(1.5 \times 10^5)(2.4 \times 10^{-3})}{T_f}$$

$T_f = 720$ K

Work done by gas $= \int PdV$

$$= \int \left(P_{atm} + \frac{kx}{A}\right) dV$$

$$= P_{atm} \int dV + k \int x\,dx$$

$$= P_{atm}(\Delta V) + \frac{kx^2}{2}$$

$$= (10^5)(0.4 \times 10^{-3}) + \frac{2000}{2}(0.1)^2 = 50J$$

Change in internal energy

$$\Delta U = \frac{f}{2} nR\Delta T = \frac{3}{2}(P_f V_f - P_i V_i)$$

$$= \frac{3}{2}[(1.50 \times 10^5)(2.4 \times 10^{-3}) - (10^5)(2 \times 10^{-3})] = 240 \text{ J}$$

So, heat supplied by heater

$\Delta Q = \Delta U + W$

$= 240 + 50$

$= 290$ J

29. A particle of mass m is located in a one dimensional potential field $U(x) = U_0(1 - \cos ax)$; U_0 and a are constants. Which of the following statement/s is/are correct ?

(a) The particle will execute Simple Harmonic Motion for small displacements.

(b) The stable equilibrium condition is $x = 0$

(c) The time period of small oscillations is $\frac{2\pi}{a}\sqrt{\frac{m}{U_0}}$

(d) The angular frequency for small oscillations is $\omega = a\sqrt{\frac{U_0}{m}}$

Ans. (a,b,c,d)

Sol. Given, $U = U_0(1 - \cos ax)$

$$F = -\frac{\partial U}{\partial x} = -U_0 a \sin ax$$

For equilibrium, $F = 0$

$\therefore$ equilibrium position is at $x = 0$

For small displacement,

$F = -U_0 a^2 x$ [$\sin ax \approx ax$]

$$\therefore \omega = \sqrt{\frac{U_0 a^2}{m}} = a\sqrt{\frac{U_0}{m}}$$

$$\therefore T = \frac{2\pi}{a}\sqrt{\frac{m}{U_0}}$$

30. A ray of light is incident on an equilateral prism made of flint glass (refractive index 1.6) placed in air.

(a) The ray suffers a minimum deviation if it is incident at angle 53°

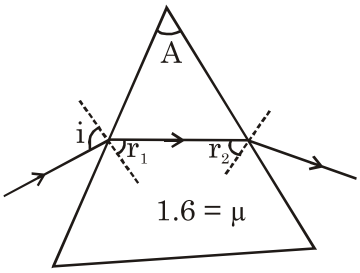
(b) The minimum angle of deviation suffered by the ray is 46°.

(c) If prism is immersed in water ($\mu = \frac{4}{3}$) the minimum deviation produced by the prism is 14°.

(d) The minimum deviation produced by the prism is 23.6° if it is immersed in a liquid of refractive index $\mu = 1.2$

Ans. (a, b, c, d)

Sol. For minimum deviation, $r_1 = r_2 = \frac{A}{2} = 30°$



$\therefore$ Applying snell's law,

$$1 \sin i = 1.6 \sin r \Rightarrow \sin i = 1.6 \times \frac{1}{2} = 0.8 = \frac{4}{5}$$

$\therefore i = 53°$

$\delta_{min} = 2i - A = (2 \times 53 - 60) = 46°$

when prism is immersed in water,

$$\frac{4}{3}\sin i' = 1.6 \times \frac{1}{2} \Rightarrow \sin i' = \frac{3}{5} \Rightarrow i' = 37°$$

$\delta_{min} = 2i - A = (2 \times 37° - 60°) = 14°$

If $\delta_{min} = 23.6° \Rightarrow i = 41.8°$

$\therefore \mu \sin i = 1.6 \sin r$

$$\Rightarrow \mu = \frac{1.6 \times 0.5}{0.67} \approx 1.2$$

31. In a p-n junction diode, the current (i) varies with applied biasing voltage (V) and can be expressed as $i = i_0\left(e^{qV/kT} - 1\right)$ where $i_0 = 5 \times 10^{-12}$ A is reverse saturation current, k is Boltzmann constant and q is the charge on the electron.

At Absolute Temperature T = 300 K

(a) The forward current is approximately 59.5 mA for a forward bias of 0.6 volt

(b) The current increases approximately by 2.75 A if the biasing voltage changes from 0.6 V to 0.7V

(c) The dynamic resistance of p-n junction is approximately 435 mΩ at the biasing voltage of 0.6V

(d) The change in reverse bias current when biasing voltage change from –1 volt to –2 volt happens to be practically zero.

ALLEN Ans. (b, c, d)

NSEP Ans. is (a, b, c, d)

Sol. Given,

$$i = i_0\left(e^{\frac{qV}{kT}} - 1\right)$$

$\therefore$ when V = 0.6 volt,

$$i = 5 \times 10^{-12}\left(e^{\frac{1.6\times10^{-19}\times0.6}{1.38\times10^{-23}\times300}} - 1\right) \approx 58.8 \text{ mA}$$

when V = 0.7 volt,

i = 2.8 mA

$$R = \frac{dV}{di}$$

$$\therefore \quad \frac{di}{dV} = \frac{i_0 q}{kT} e^{\frac{qV}{RT}} = \frac{5\times10^{-12}\times1.6\times10^{-19}}{1.38\times10^{-23}\times300} e^{\frac{1.6\times10^{-19}\times0.6}{1.38\times10^{-23}\times300}} \approx 2.27$$

$\therefore$ R $\approx$ 435 mΩ

Current at V = –1 $\approx$ Current at V = –2 $\approx$ 0

Note :- Option A its coming out to be 58.8 mA

32. A charged oil (density 880 kg m^{-3}) drop is held stationary between two parallel horizontal metal plates 6.0 mm apart when a potential difference of V = 103 volt is applied between the two plates. When the electric field is switched off, the drop falls. At a certain time the drop is seen to fall a distance of 0.2 mm in 35.7 s and next 1.2 mm in 21.4 s. (The upper plate in the experiment is at higher potential).

Given that the viscosity of air = 1.80 $\times$ 10^{-5} Nsm^{-2} and density of air = 1.29 kg m^{-3}

(a) The radius of the drop is a = 7.25 $\times$ 10^{-7} m

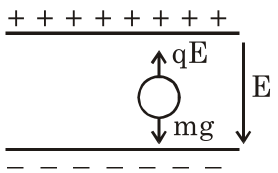
(b) The charge on the drop is q = 8.0 $\times$ 10^{-19} C

(c) The terminal velocity of the oil drop, under its free fall, is 5.6 $\times$ 10^{-5} ms^{-1}

(d) The oil drop carries 5 excess electrons

Ans. **(a, c, d)**

Sol.



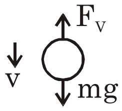
$$\Delta V = Ed$$

$$E = \frac{\Delta V}{d} = \frac{103}{6 \times 10^{-3}}$$

As qE is upwards so charge on drop should be negative.

As ρ_{air} is very less so it can be neglected while writing Buoyant force

When electric field is switched off



for terminal velocity

$F_V = mg$

$$6\pi\eta rv = \rho\frac{4}{3}\pi r^3 g$$

$$v = \frac{2}{9}\frac{\rho g r^2}{\eta} \quad ...(1)$$

Now, we have speed, $v = \dfrac{\text{distance}}{\text{time}}$

(Note : since speed calculated from both interval is same hence terminal speed)

$$v = \frac{2\times10^{-3}}{35.7} = 5.60\times10^{-5}\,m/s$$

Put v in equation (1) to get r

on solving we get $r = 7.254 \times 10^{-7}$ m

we have

mg = qE, in equilibrium

$$\rho.\frac{4}{3}\pi r^3 g = qE$$

on solving, $q = 8 \times 10^{-19}$C

so charge $= -8 \times 10^{-19}$ C, so option (b) is wrong

as charge is negative

$q = ne \Rightarrow n = 5$

$\therefore$ correct option is a,c,d